ALMUSTAQBAL UNIVERSITY

DEPARTMENT OF BUILDING & CONSTRUCTION ENGINEERING TECHNOLOGY

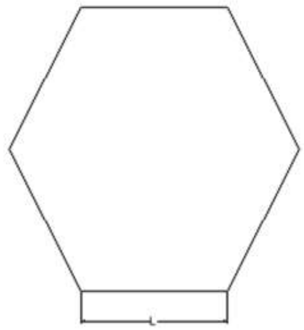
ANALYSIS AND DESIGN OF REINFORCED CONCRETE STRUCTURES II

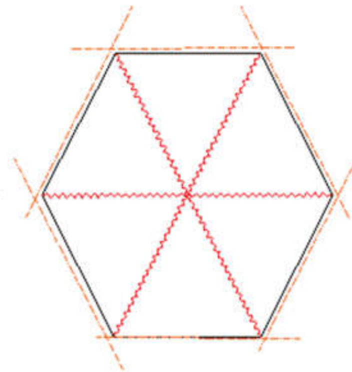
YIELD LINE THEORY SOLVED EXAMPLES

III

EXAMPLE 18: For the simply supported hexagon shaped two-way slab shown in the figure below, determine the ultimate resisting moment per linear meter (m) required to resist a uniformly distributed load of w.

SOLUTION:





$$W_E = W_I$$

$$W_E = w \times \left(\frac{1}{2} \times l \times \frac{3}{2\sqrt{3}} \times l \times \frac{1}{3} \times 6\right) = \frac{3wl^2}{2\sqrt{3}}$$

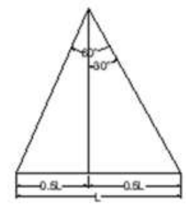
$$W_I = m \times l \times \theta$$

$$= m \times l \times \frac{1}{\frac{3}{2\sqrt{3}}l} \times 6 = 4\sqrt{3}m$$

$$\therefore m = \frac{wl^2}{8}$$

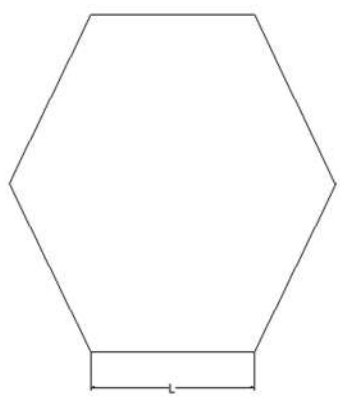
$$\tan 30 = \frac{0.5l}{x}$$

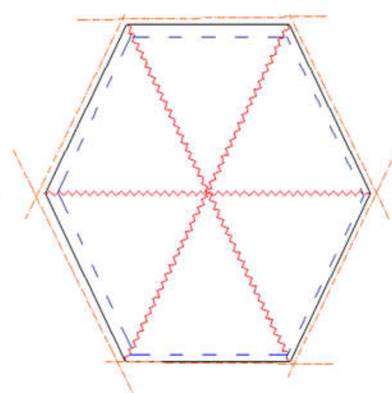
$$\therefore x = \frac{3l}{2\sqrt{3}}$$



EXAMPLE 19: For the same hexagon shaped slab and subjected to the same loading, determine the ultimate resisting moment per linear meter if the supports are fixed at all sides.

SOLUTION:





$$W_E = W_I$$

$$W_E = w \times \left(\frac{1}{2} \times l \times \frac{3}{2\sqrt{3}} \times l \times \frac{1}{3} \times 6\right) = \frac{3wl^2}{2\sqrt{3}}$$

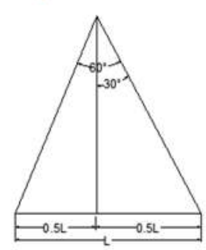
$$W_I = m \times l \times \theta$$

$$= \left(m \times l \times \frac{1}{\frac{3}{2\sqrt{3}}l} + m \times l \times \frac{1}{\frac{3}{2\sqrt{3}}l}\right) \times 6 = 8\sqrt{3}m$$

$$\therefore m = \frac{wl^2}{16}$$

$$\tan 30 = \frac{0.5l}{x}$$

$$\therefore x = \frac{3l}{2\sqrt{3}}$$



EXAMPLE 20: for the simply supported octagon shaped 2-way slab, using the yield line theory determine the resisting moment per linear meter (m) withstanding a uniformly distributed load of w.

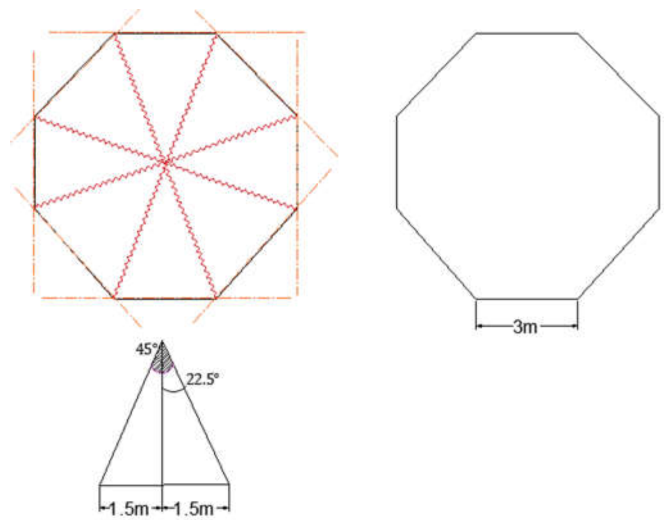
SOLUTION:

$$W_E = W_I$$

$$W_E = w \times \left(3 \times 3.621 \times \frac{1}{2}\right) \times \frac{1}{3} \times 8 = 14.485w.$$

$$W_I = m \times 3 \times \frac{1}{3.621} \times 8 = 6.627m$$

$$\therefore m = 2.185w$$



Example 21: The triangular simply supported two-way slab shown in the figure below is subjected to a concentrated load of P kN. Using the yield line method, determine the resisting moment per linear meter (m).

SOLUTION:

$$W_E = W_I$$

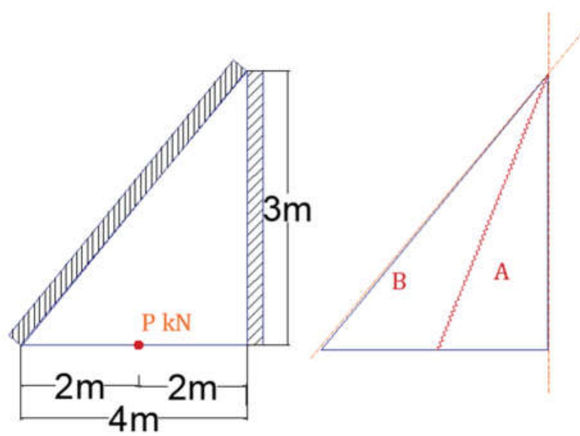
$$W_E = P \times 1 = P\ kN.m$$

$$W_I = m \times l \times \theta$$

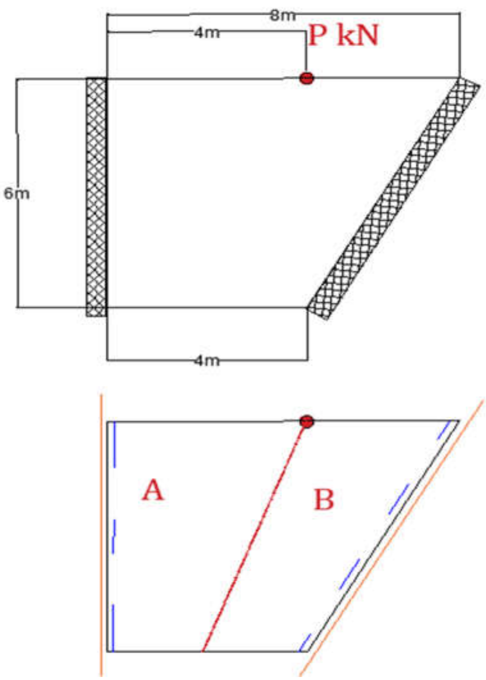
$$= \left(m \times 3 \times \tfrac{1}{2}\right)_{for\ A} + \left(m \times 3 \times \tfrac{1}{2}\right)_{y-axis} + \left(m \times 2 \times \tfrac{1}{1.5}\right)_{x-}$$

$$= 4.33m$$

$$\therefore m = \frac{P}{4.33}\ kN.m$$



EXAMPLE 22: For the fixed two-way slab shown below, using the yield line theory determine the ultimate resisting moment per linear meter (m) if the slab was subjected to a concentrated load of P kN.



SOLUTION:

$W_E = W_I$

$W_E = P \times 1 = P\ kN.m$

$W_I = m \times l \times \theta$

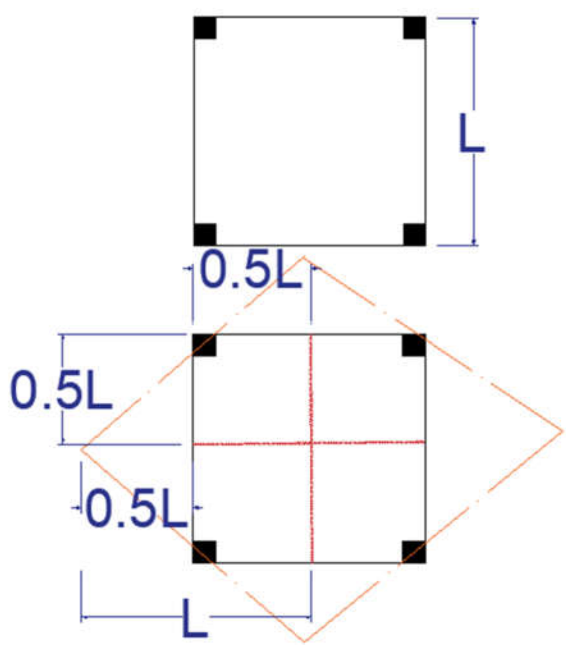
$$= \left(m_{+ve} \times 6 \times \frac{1}{4} + m_{-ve} \times 6 \times \frac{1}{4}\right) +$$

$$\left(\left(m_{+ve} \times 2 \times \frac{1}{6}\right)_{x-axis} + \left(m_{+ve} \times 6 \times \frac{1}{4}\right)_{y-axis}\right) +$$

$$\left(\left(m_{-ve} \times 4 \times \frac{1}{6}\right)_{x-axis} + \left(m_{-ve} \times 6 \times \frac{1}{4}\right)_{y-axis}\right) = 7m.$$

$$\therefore P = 7m \quad \rightarrow m = \frac{P}{7}$$

EXAMPLE 23: Using the yield line method, determine the ultimate resisting moment (m) per linear meter for the isotropic reinforced concrete two-way slab subjected to a uniformly distributed load.



SOLUTION:

$W_E = W_I$

$$W_E = w \times \left(\frac{l}{2} \times \frac{l}{2}\right) \times \frac{1}{2} \times 4 = \frac{wl^2}{2}.$$

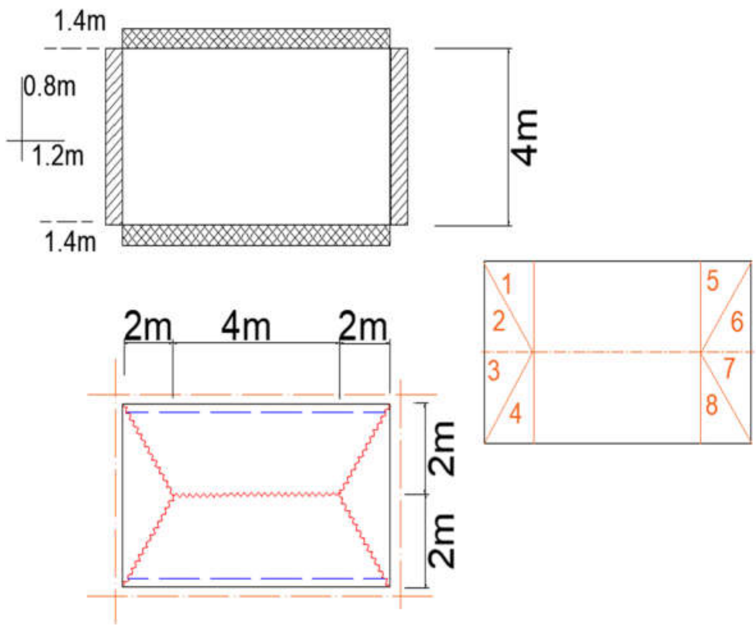
$W_I = m \times l \times \theta$

$$= \left(\left(m \times \tfrac{l}{2} \times \tfrac{1}{l}\right) + \left(m \times \tfrac{l}{2} \times \tfrac{1}{l}\right)\right) \times 4$$

$$= 4m$$

$$\therefore \quad m = \frac{wl^2}{8}.$$

EXAMPLE 24: The two-way reinforced concrete slab is supported as shown in the figure below. Using the proposed moment proportions, determine the ultimate resisting moment per linear meter (m) using the yield line theory when the slab is subjected to a load of $12kN/m^{2}$.



SOLUTION:

$W_E = W_I$

$W_E = w \times A \times \delta$

$$= 12 \times \left(\left(2 \times 2 \times \tfrac{1}{2} \times \tfrac{1}{3} \times 8\right) + \left(4 \times 2 \times \tfrac{1}{2} \times 2\right)\right)$$

$$= 160kN.m$$

$W_I = m \times l \times \theta$

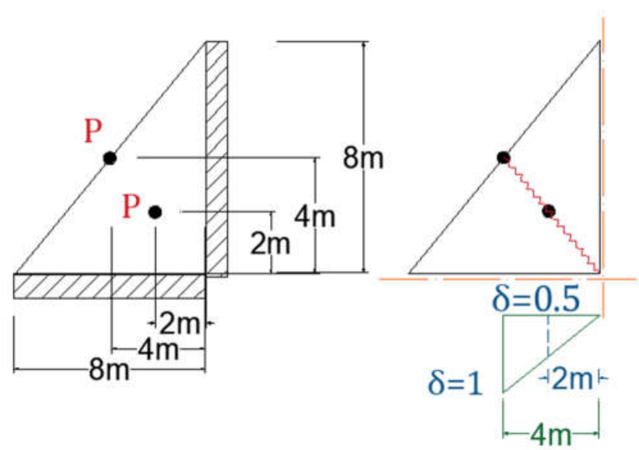
$$= \left(0.8m \times 4 \times \tfrac{1}{2}\right) \times 2 + \left(1.2m \times 8 \times \tfrac{1}{2} \times 2\right) +$$

$$\left(1.4m \times 8 \times \frac{1}{2} \times 2\right)$$

$$= 24m$$

$\therefore\ \ 160 = 24m \quad \rightarrow m = 6.7m\ kN.\dfrac{m}{m}.$

EXAMPLE 25: Using the yield line theory, determine the ultimate resisting moment per linear meter (m) for the isotropic reinforced two-way slab shown in the figure below when subjected to two concentrated loads.



SOLUTION:

$W_E = W_I$

$W_E = P \times 1 + P \times 0.5 = 1.5P$

$W_I = m \times 4 \times \dfrac{1}{4} \times 2 = 2m.$

$\therefore 1.5P = 2m \ \rightarrow \ \ m = 0.75P.$

IMPORTANT NOTE:

- To determine the internal angles for any polygon shape, use the following formula:

  $(n-2) \times 180° = sum\ of\ angles$

  $\dfrac{sum\ of\ angles}{n} = interior\ angle.,$ Where: n=number of interior angles.

EXAMPLE 26: Determine the ultimate resisting moment per linear meter (m), by using the yield line theory, for the two-way simply supported slab subjected to a concentrated load of P kN.

SOLUTION:

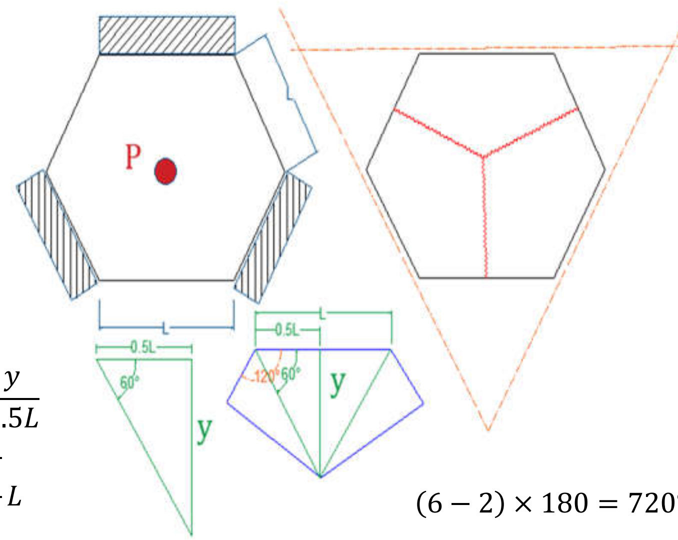
$$W_E = W_I$$

$$W_E = P \times 1 = P$$

$$W_I = \left(m \times 1.5L \times \frac{1}{\frac{\sqrt{3}}{2} \times L} \right) \times 3 = 3\sqrt{3}m$$

$$\therefore P = 3\sqrt{3}m$$

$$m = \frac{P}{3\sqrt{3}}$$



$$tan60 = \frac{y}{0.5L}$$

$$\therefore y = \frac{\sqrt{3}}{2}L$$

$$(6-2) \times 180 = 720°$$

$$\therefore \frac{720°}{6} = 120°$$

EXAMPLE 27: Using the yield line theory, determine the ultimate resisting moment per linear meter (m) for the isotropic reinforced two-way slab sustaining a uniformly distributed load of 9kN/m² and a line load of 5kN/m as shown in the figure below.

SOLUTION:7

$$W_E = W_I$$

$$W_E = 9 \times \left[\left(\frac{1}{2} \times 2 \times 2 \times \frac{1}{3} \times 8 \right) + \left(4 \times 2 \times \frac{1}{2} \times 2 \right) \right] + 5 \times (4 \times 1)$$

$$\left(2 \times \frac{1}{2} \times 2 \right) = 150kN.m.$$

$$W_I = \left[0.7m \times 4 \times \frac{1}{2} \right] \times 2 + [\left(m \times 8 \times \frac{1}{2} \right) + (1.2m \times 8 \times \frac{1}{2})] \times 2$$

$$= 20.4\, m$$

$$\therefore 150 = 20.4m \quad \rightarrow m = 7.353kN.m/m$$