



**ALMUSTAQBAL UNIVERSITY**

**DEPARTMENT OF BUILDING & CONSTRUCTION  
ENGINEERING TECHNOLOGY**

**ANALYSIS AND DESIGN OF REINFORCED CONCRETE  
STRUCTURES II**

**EXAMPLES IN THE EQUIVALENT  
FRAME METHOD**

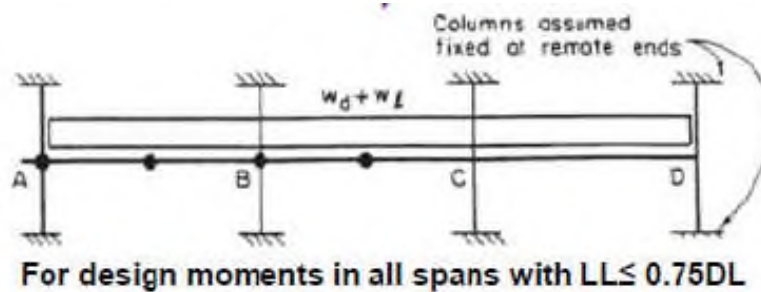
**II**

## ARRANGEMENT OF LIVE LOAD FOR MAXIMUM (+VE) AND (-VE) MOMENTS.

In the usual case where the exact loading pattern is not known, the maximum factored moments are developed with loading conditions illustrated by the three-span partial frame shown in the figures below and described as follows:

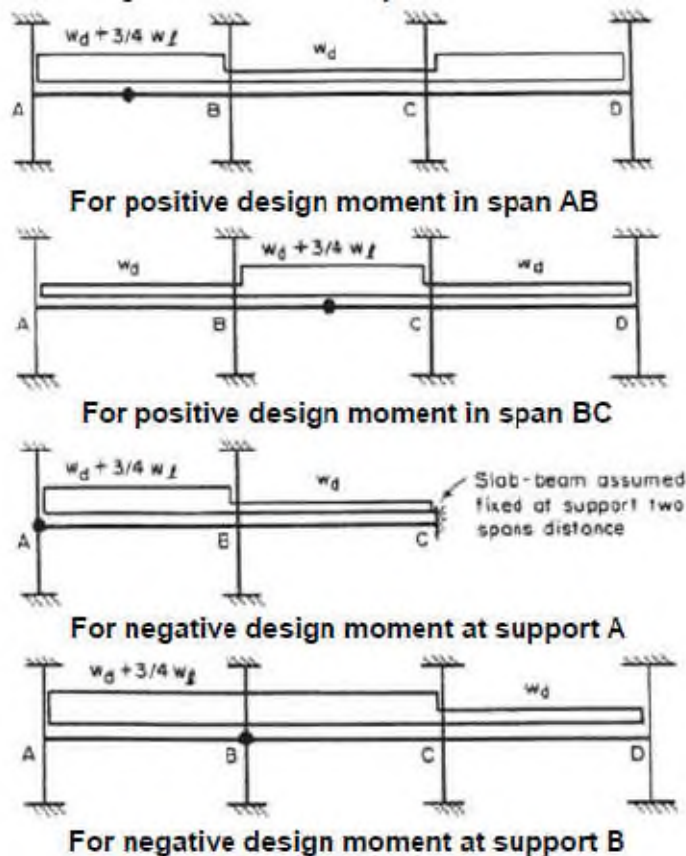
### 1. $LL \leq 0.75DL$

When the unfactored live load does not exceed 0.75 of the unfactored dead load, only loading pattern 1 with full factored live load on all spans need to be analysed for negative and positive factored moments.



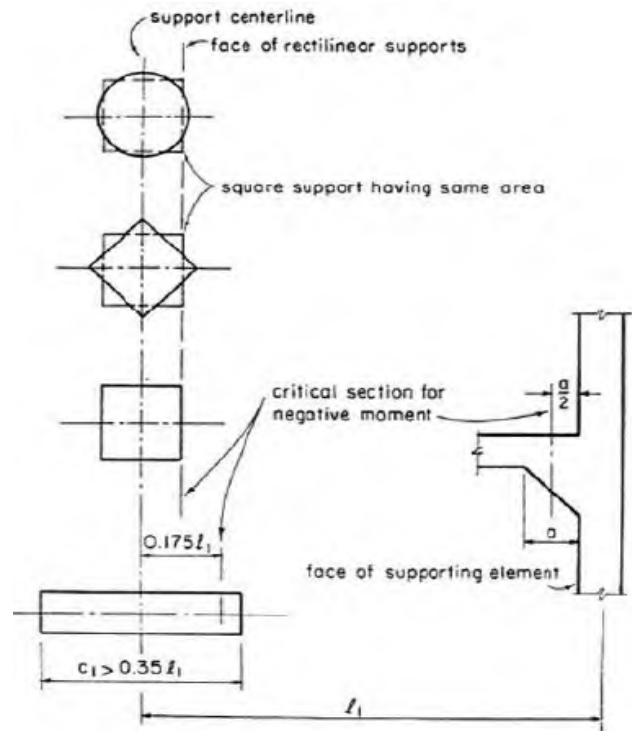
### 2. $LL > 0.75DL$

When the unfactored live load exceeds 0.75 the unfactored dead load, the four loading condition patterns shown below and the loading condition above need to be analysed to determine all factored moments in the slab-beam members.



## CRITICAL SECTION FOR (-VE) MOMENTS (ACI 13.7.7).

- Negative factored moments for design must be taken at faces of rectilinear supports, but not at a distance greater than  $0.175l_1$  from the centre of the support.
- Non-rectangular supports should be treated as square supports having the same cross-sectional area.
- At exterior supports with brackets or capitals, the critical section for (-ve) moment shall be taken at a distance from face of support element not greater than  $0.5$  projection of the bracket or capital beyond the face of the supporting element.
- Locations of the critical section for (-ve) factored moment for various support conditions are illustrated in the figure.



**EXAMPLE 7:** The intermediate flat plate floor, storey height = 2.75m,  $t = 190\text{mm}$ ,  $f'_c = 20\text{MPa}$  for the slab and  $f'_c = 35\text{MPa}$  for the columns,  $f_y = 420\text{MPa}$ , columns are of  $(400 \times 400\text{mm})$ , service live load is  $2.5\text{kN/m}^2$ , partitions  $2\text{kN/m}^2$ . Find the design moments.

### SOLUTIONS:

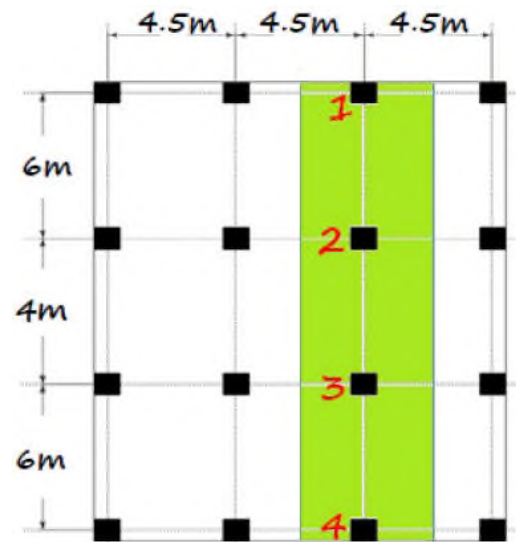
$$K_{sb} = K \times \frac{E(I_s + I_b)}{l_1}, \quad I_b = 0$$

**End Span:** use table (A) to find the factors:

$$\frac{C_{1A}}{l_1} = \frac{400}{6000} = 0.067, \quad \frac{C_{1B}}{l_1} = \frac{400}{6000} = 0.067$$

By interpolation:

$C_{1A}/l_1$	$C_{1B}/l_1$	$K_{AB}$	$K_{BA}$
0.05	0.05	4.05	4.05
0.067	0.067	4.094	4.094
0.1	0.1	4.18	4.18



$$\frac{4.18 - 4.05}{0.05} = \frac{x}{0.067 - 0.05} \rightarrow x = 0.0442 \quad \therefore K_{AB} = K_{BA} = 4.05 + 0.0442 = 4.094$$

$$K_{1-2} = 4.094, COF_{coeff.} = 0.506, FEM_{coeff.} = 0.0843$$

$$K_{2-1} = 4.094, COF_{coeff.} = 0.506, FEM_{coeff.} = 0.0843$$

**Interior Span:** use table (A) to find the factors:

$$\frac{C_{1A}}{l_1} = \frac{400}{4000} = 0.1 \quad , \quad \frac{C_{1B}}{l_1} = \frac{400}{4000} = 0.1$$

$$K_{2-3} = 4.18, COF_{coeff.} = 0.513, FEM_{coeff.} = 0.085$$

$$K_{3-2} = 4.18, COF_{coeff.} = 0.513, FEM_{coeff.} = 0.085$$

$$E_{slab} = 4700\sqrt{20} = 21019 MPa$$

$$I_s = \frac{4500 \times 190^3}{12} = 2.572 \times 10^9 mm^4 \quad , \quad I_b = 0$$

**Slab Beam Stiffness:**

$$K_{sb_{1-2}} = K_{sb_{2-1}} = K \times \frac{EI_s}{l_1} = 4.094 \times \frac{21019 \times 2.572 \times 10^9}{6000} = 3.69 \times 10^{10} N.mm$$

$$K_{sb_{2-3}} = K_{sb_{3-2}} = K \times \frac{EI_s}{l_1} = 4.18 \times \frac{21019 \times 2.572 \times 10^9}{4000} = 5.65 \times 10^{10} N.mm$$

FIND Kc

$$\frac{C_{1A}}{l_c} = \frac{190}{2750} = 0.069 \quad \text{from table (C):}$$

$$K_{bot} = 4.91 + \frac{0.069 - 0.05}{0.1 - 0.05} (6.09 - 4.91) = 5.36$$

$$K_{top} = 4.21 + \frac{0.069 - 0.05}{0.1 - 0.05} (4.44 - 4.21) = 4.29$$

$$I_{col} = \frac{bh}{12} = \frac{C_2 C_1^3}{12} = \frac{400^4}{12} = 2.133 \times 10^9 mm^4$$

$$E = 4700\sqrt{35} = 27806 MPa$$

$$K_{c_{bot}} = K \times \frac{EI_{col}}{l_{col}} = 5.36 \times \frac{27806 \times 2.133 \times 10^9}{2750} = 11.56 \times 10^{10} N.mm$$

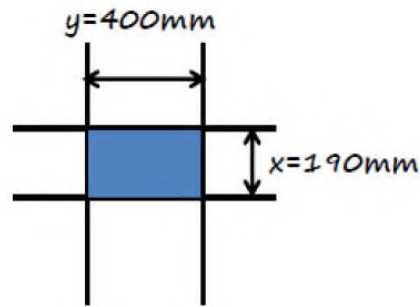
$$K_{c_{top}} = K \times \frac{EI_{col}}{l_{col}} = 4.29 \times \frac{27806 \times 2.133 \times 10^9}{2750} = 9.25 \times 10^{10} N.mm$$

FIND  $K_t$  (Interior & Exterior)

$$K_t = \sum \frac{9E_{cs}C}{l_2(1-\frac{c_2}{l_2})^3}, \quad C = \sum (1-0.63\frac{x}{y})\frac{x^3y}{3}, \quad E = 4700\sqrt{20} = 21019 MPa$$

$$C = (1-0.63 \times \frac{190}{400}) \times \frac{190^3 \times 400}{3} = 6.409 \times 10^8 mm^4$$

$$K_t = \frac{9 \times 21019 \times 6.409 \times 10^8}{4500(1-\frac{400}{4500})^3} = 3.56 \times 10^{10} N.mm$$



FIND  $K_{ec}$ :

$$\frac{1}{K_{ec}} = \frac{1}{\sum K_c} + \frac{1}{\sum K_t} = \frac{1}{(9.25+11.56) \times 10^{10}} + \frac{1}{2 \times 3.56 \times 10^{10}} \rightarrow K_{ec} = 5.3 \times 10^{10} N.mm$$

Joints 1 and 4 (ext.):

$$DF_{sb_{1-2}} = \frac{K_{sb_{1-2}}}{K_{sb_{1-2}} + K_{ec}} = \frac{3.69}{3.69 + 5.3} = 0.41$$

$$(DF_{ec})_1 = \frac{K_{ec}}{K_{sb_{1-2}} + K_{ec}} = \frac{5.3}{3.69 + 5.3} = 0.59$$

Joints 2 and 3 (int.):

$$DF_{sb_{2-1}} = \frac{K_{sb_{2-1}}}{K_{sb_{2-1}} + K_{sb_{2-3}} + K_{ec}} = \frac{3.69}{3.69 + 5.65 + 5.3} = 0.252$$

$$DF_{sb_{2-3}} = \frac{K_{sb_{2-3}}}{K_{sb_{2-1}} + K_{sb_{2-3}} + K_{ec}} = \frac{5.65}{3.69 + 5.65 + 5.3} = 0.386$$

$$(DF_{ec})_2 = \frac{K_{ec}}{K_{sb_{2-1}} + K_{sb_{2-3}} + K_{ec}} = \frac{5.3}{3.69 + 5.65 + 5.3} = 0.362$$

Slab beam	D.F	FEMcoeff.	C.O.F
Sb 1-2	0.41	0.0843	0.506
Sb 2-1	0.252	0.0843	0.506
Sb 2-3	0.386	0.085	0.513
Sb 3-2	0.386	0.085	0.513

Frame analysis by moment distribution method:

$$\text{Service D.L} = 2 + 0.19 \times 24 = 6.56 \text{ kN/m}^2$$

$$\text{Service L.L} = 2.5 \text{ kN/m}^2$$

$$\text{L.L} < 0.75 \times 6.56 = 4.92 \text{ kN/m}^2$$

∴ Design moments occur at all critical sections with full factored load

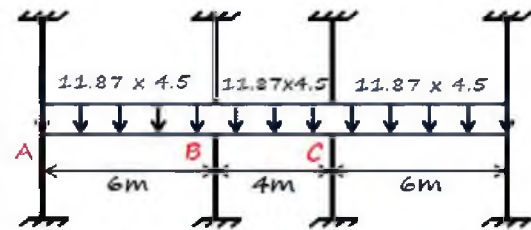
$$W_u = 1.2D.L + 1.6L.L$$

$$W_u = 1.2 \times 6.56 + 1.6 \times 2.5 = 11.87 \text{ kN/m}^2$$

$$FEMsb_{1-2} = FEMsb_{2-1} = 0.0843(W_u l_2 l_1^2) = 0.0843 \times 11.87 \times 4.5 \times 6^2 = 162.1 \text{ kN.m}$$

$$FEMsb_{2-3} = FEMsb_{3-2} = 0.085(W_u l_2 l_1^2) = 0.085 \times 11.87 \times 4.5 \times 4^2 = 72.7 \text{ kN.m}$$

C.O.F	0.506		0.513	
Joint	A	B	B	C
Member	AB	BA	BC	CB
D.F	0.41	0.252	0.386	0.386
FEM	162.1	-162.1	72.7	-72.7
Bal. M	-66.46	22.53	34.5	-34.5
C.O.M	11.4	-33.63	-17.7	17.7
Bal. M	-4.674	12.9	19.8	-19.8
C.O.M	6.527	-2.36	-10.157	10.157
Bal. M	-2.67	3.15	4.83	-4.83
-ve M @ support	106.223	-159.51	103.97	-103.97



$$M_o = M^+ + \frac{1}{2} |M_1^- + M_2^-|$$

$$+ve M_{C.L-AB} = \frac{W_u l_2 l_1^2}{8} - \frac{1}{2} (|M_A| + |M_B|) = \frac{11.87 \times 4.5 \times 6^2}{8} - \frac{1}{2} (106.223 + 159.51) = 107.501 \text{ kN.m}$$

$$+ve M_{C.L-BC} = \frac{W_u l_2 l_1^2}{8} - \frac{1}{2} (|M_B| + |M_C|) = \frac{11.87 \times 4.5 \times 4^2}{8} - \frac{1}{2} (103.97 + 103.97) = 2.86 \text{ kN.m}$$

Transform the negative moments from centers of columns to faces of columns.

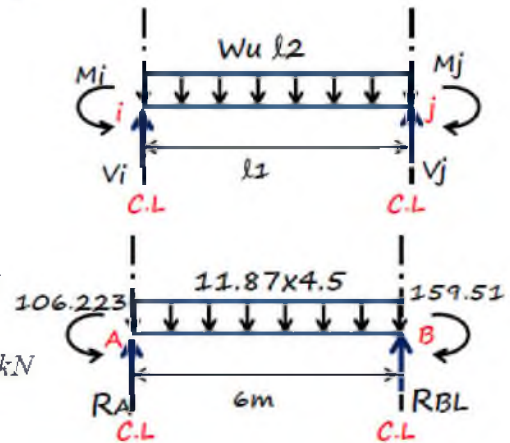
1-Finding shear forces at centers of columns:

$$V_i = \frac{W_u \times l_1 \times l_2}{2} - \left( \frac{M_j - M_i}{l_1} \right)$$

$$V_j = \frac{W_u \times l_1 \times l_2}{2} - \left( \frac{M_i - M_j}{l_1} \right)$$

$$V_i = R_A = \frac{11.87 \times 6 \times 4.5}{2} - \left( \frac{159.51 - 106.223}{6} \right) = 151.36 \text{ kN}$$

$$V_j = R_{BL} = \frac{11.87 \times 6 \times 4.5}{2} - \left( \frac{106.223 - 159.51}{6} \right) = 169.126 \text{ kN}$$



2-Finding bending moments at faces of columns:

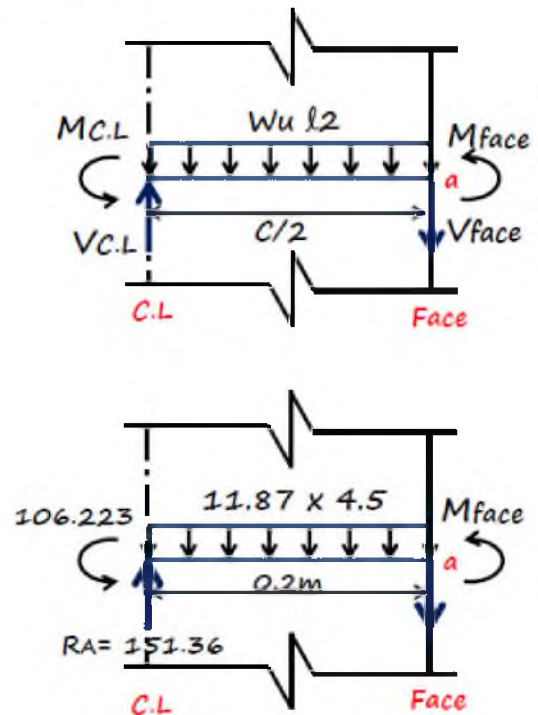
$$\sum M_a = 0$$

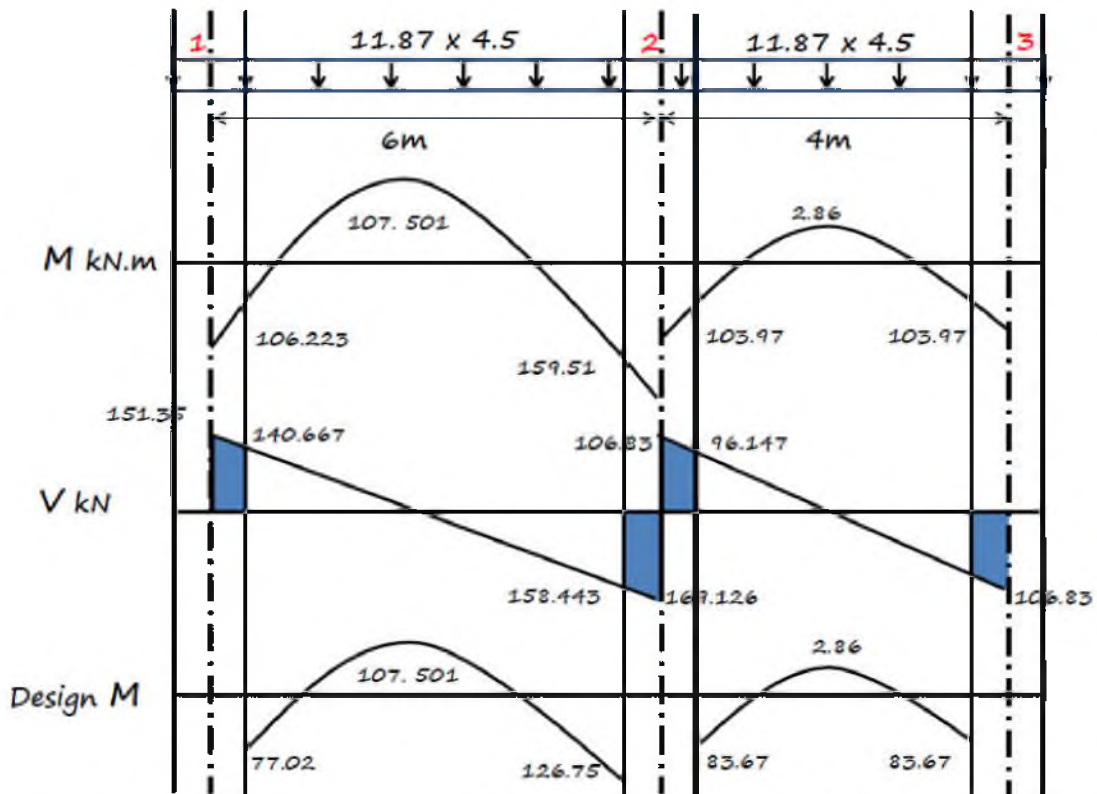
$$M_{C.L} + M_{face} + W_u l_2 \times \frac{(C/2)^2}{2} = V_{C.L} \times \frac{C}{2}$$

For example at Joint (1):

$$106.223 + M_f + 11.87 \times 4.5 \times \frac{0.2^2}{2} = 151.36 \times \frac{0.4}{2}$$

$$M_f = -77.02 \text{ kN.m}$$





Distribute the design moments into the column and middle strips:

Width of column strip =  $2(4.5/4) = 2.25$  m (span 1-2)

Width of column strip =  $2(4/4) = 2$  m (span 2-3)

	position	Moment	% C.S	Moment c.s	Moment m.s
End Span	-ve M. ext.	77.02	100	77.02	0
	+ve M.	107.501	60	64.5	43
	-ve M. int.	126.75	75	95.062	31.68
Int. Span	-ve M. int	83.67	75	62.75	20.9
	+ve M	286	60	1.716	1.144