ALMUSTAQBAL UNIVERSITY

DEPARTMENT OF BUILDING & CONSTRUCTION ENGINEERING TECHNOLOGY

ANALYSIS AND DESIGN OF REINFORCED CONCRETE STRUCTURES II

EQUIVALENT FRAME METHOD

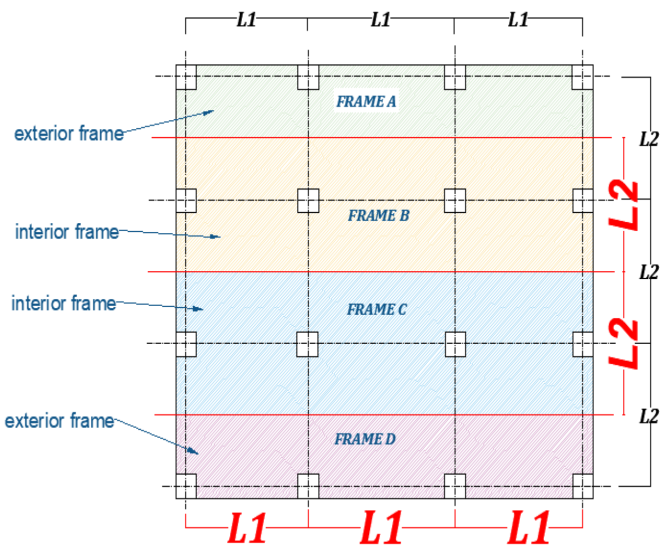
(EFM)

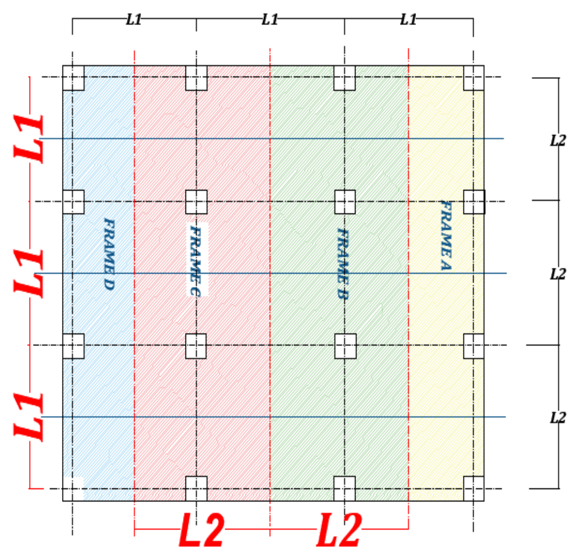
EQUIVALENT FRAME METHOD (EFM)

A more general method for the analysis of two-way slabs that have no limitations. It assumes that the analysis is to be conducted using the moment distribution method.

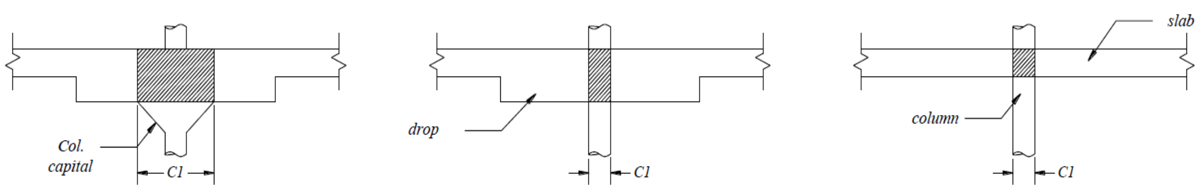
ASSUMPTIONS OF EFM (ACI 13.7.2):

1. The structure is divided into equivalent frames on column lines taken longitudinally and transversely.
2. Each frame consists of a row of columns and slab beam strips, bounded by the centreline of a panel on each side of column centrelines.

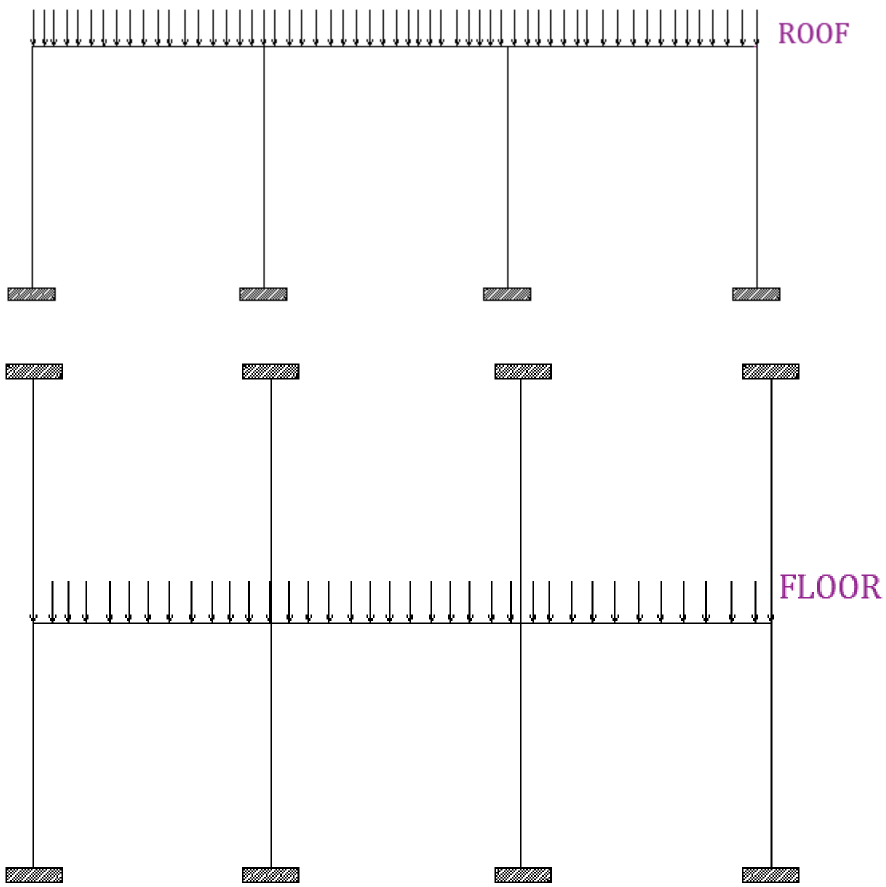




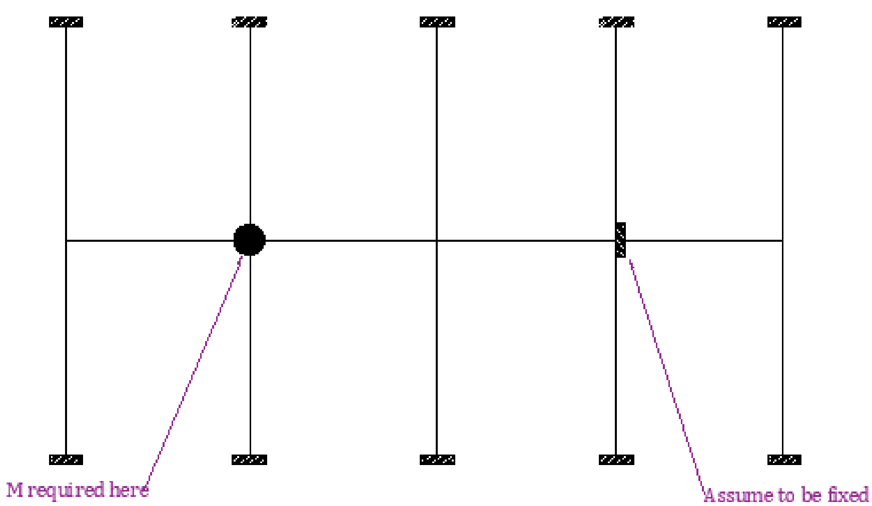
3. Columns are attached to the sub-beam strips by torsional members perpendicular to l_1.



4. Exterior frame is bounded by the edge and centreline of the adjacent panel.
5. For vertical loading, each floor and roof may be analysed separately with far ends of columns are fixed.



6. The slab beam is assumed to be fixed at the support of two panels from the support which is required to determine M at, provided that the slab continues beyond that support.



MEMBERS OF EFM:

1. SLAB-BEAM
2. EQUIVALENT COLUMNS
   2.1. COLUMNS ABOVE & BELOW THE SLAB
   2.2. TORSIOAL MEMBER ON EACH SIDE OF THE COLUMNS.

PROCEDURE:

1. The first step is to determine the flexural stiffness of equivalent frame members.
2. Determine the distribution factors, carry over factors and fixed end moments.
3. Analyse the frame using the moment distribution method.

I- SLAB-BEAM MEMBERS (ACI 13.7.3):

The stiffness factor $\boldsymbol{K_{sb} = k.\frac{E_c(I_s+I_b)}{L_1}}$

$E_c = 4700\sqrt{f_c{}'}$

$I_b = k.\frac{b_w h^3}{12}$, if there was no beam in the direction of the frame, $I_b = 0$.

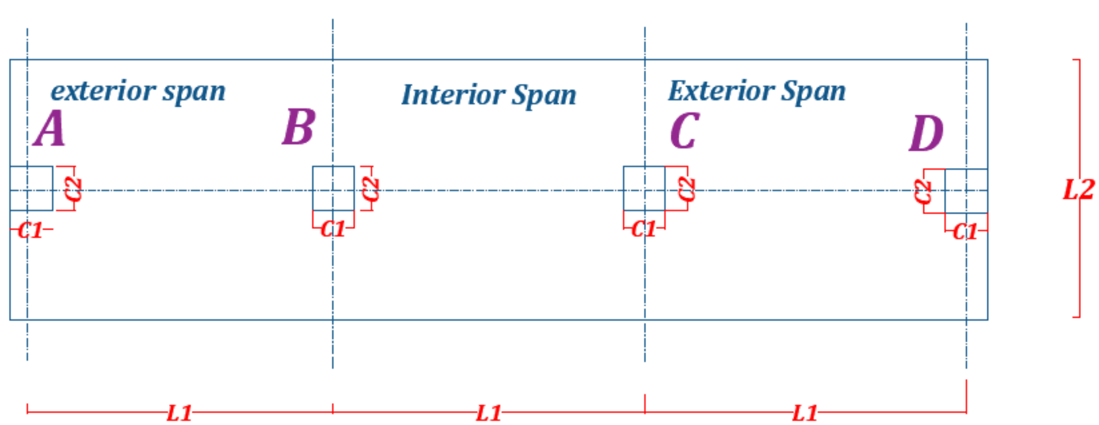
$I_s = \frac{L_2 h_s{}^3}{12}$

I_{SB} =is based on the gross concrete area. The variation of I along the slab-beam between the supports is considered.

$$I_{SB}\ within\ the\ column = \frac{I}{\left(1-\frac{c_2}{L_2}\right)^2}$$

Notes when calculating K_{sb}:

- Joint means the column.
- Check the longitudinal frame shown below:



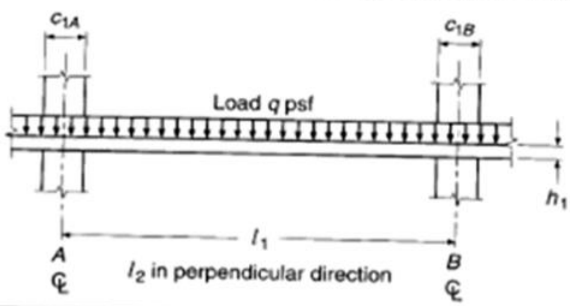
- An exterior joint has only one $\boldsymbol{K_{sb}}$.
- An interior joint has two $\boldsymbol{K_{sb}}$
- if the stiffness was required in an panel or an interior panel, there will be two stiffnesses to be calculated one from each column.

The stiffness factors (k), carry over factors and fixed end moment coefficients ca be determined from

TABLE A13.A: for slabs without drop panels.

TABLE A.13A

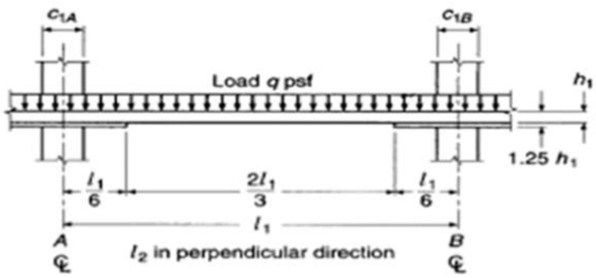
Coefficients for slabs with variable moment of inertia[a]



Column Dimension		Uniform Load FEM = Coeff.$(ql_2l_1^2)$		Stiffness Factor[b]		Carryover Factor	
c_{1A}/l_1	c_{1B}/l_1	M_{AB}	M_{BA}	k_{AB}	k_{BA}	COF_{AB}	COF_{BA}
0.00	0.00	0.083	0.083	4.00	4.00	0.500	0.500
	0.05	0.083	0.084	4.01	4.04	0.504	0.500
	0.10	0.082	0.086	4.03	4.15	0.513	0.499
	0.15	0.081	0.089	4.07	4.32	0.528	0.498
	0.20	0.079	0.093	4.12	4.56	0.548	0.495
	0.25	0.077	0.097	4.18	4.88	0.573	0.491
0.05	0.05	0.084	0.084	4.05	4.05	0.503	0.503
	0.10	0.083	0.086	4.07	4.15	0.513	0.503
	0.15	0.081	0.089	4.11	4.33	0.528	0.501
	0.20	0.080	0.092	4.16	4.58	0.548	0.499
	0.25	0.078	0.096	4.22	4.89	0.573	0.494
0.10	0.10	0.085	0.085	4.18	4.18	0.513	0.513
	0.15	0.083	0.088	4.22	4.36	0.528	0.511
	0.20	0.082	0.091	4.27	4.61	0.548	0.508
	0.25	0.080	0.095	4.34	4.93	0.573	0.504
0.15	0.15	0.086	0.086	4.40	4.40	0.526	0.526
	0.20	0.084	0.090	4.46	4.65	0.546	0.523
	0.25	0.083	0.094	4.53	4.98	0.571	0.519
0.20	0.20	0.088	0.088	4.72	4.72	0.543	0.543
	0.25	0.086	0.092	4.79	5.05	0.568	0.539
0.25	0.25	0.090	0.090	5.14	5.14	0.563	0.563

TABLE A13.B: for slabs with drop panels with depths equal to $(1.25h_{slab})$ and a length of $\left(\frac{l}{3}\right)$

TABLE A.13B
Coefficients for slabs with variable moment of inertia[a]

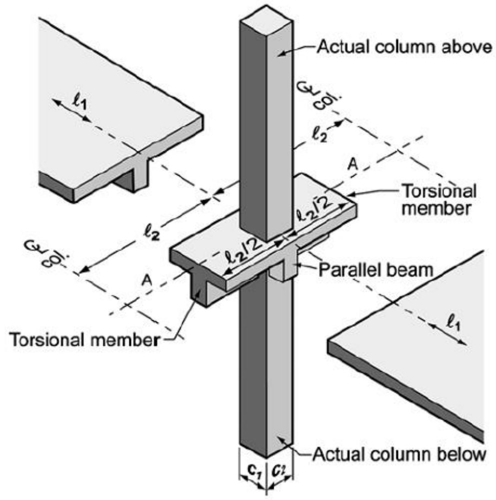


Column Dimension		Uniform Load FEM = Coeff.$(ql_2l_1^2)$		Stiffness Factor[b]		Carryover Factor	
c_{1A}/l_1	c_{1B}/l_1	M_{AB}	M_{BA}	k_{AB}	k_{BA}	COF_{AB}	COF_{BA}
0.00	0.00	0.088	0.088	4.78	4.78	0.541	0.541
	0.05	0.087	0.089	4.80	4.82	0.545	0.541
	0.10	0.087	0.090	4.83	4.94	0.553	0.541
	0.15	0.085	0.093	4.87	5.12	0.567	0.540
	0.20	0.084	0.096	4.93	5.36	0.585	0.537
	0.25	0.082	0.100	5.00	5.68	0.606	0.534
0.05	0.05	0.088	0.088	4.84	4.84	0.545	0.545
	0.10	0.087	0.090	4.87	4.95	0.553	0.544
	0.15	0.085	0.093	4.91	5.13	0.567	0.543
	0.20	0.084	0.096	4.97	5.38	0.584	0.541
	0.25	0.082	0.100	5.05	5.70	0.606	0.537
0.10	0.10	0.089	0.089	4.98	4.98	0.553	0.553
	0.15	0.088	0.092	5.03	5.16	0.566	0.551
	0.20	0.086	0.094	5.09	5.42	0.584	0.549
	0.25	0.084	0.099	5.17	5.74	0.606	0.546
0.15	0.15	0.090	0.090	5.22	5.22	0.565	0.565
	0.20	0.089	0.094	5.28	5.47	0.583	0.563
	0.25	0.087	0.097	5.37	5.80	0.604	0.559
0.20	0.20	0.092	0.092	5.55	5.55	0.580	0.580
	0.25	0.090	0.096	5.64	5.88	0.602	0.577
0.25	0.25	0.094	0.094	5.98	5.98	0.598	0.598

II- <u>EQUIVALENT COLUMN MEMBERS</u> (The Stiffness factor k_{ec}):

- The stiffness factor consists of the actual columns above and below the slab plus the attached torsional members on each side of the columns extending to the centre line of the adjacent panel.
- The assumption is that the flexibility (inverse of stiffness) of an equivalent column is taken as the sum of flexibilities of the actual columns above and below the slab and of the torsional member.

$$\frac{1}{k_{ec}} = \frac{1}{\sum k_c} + \frac{1}{\sum k_t}$$



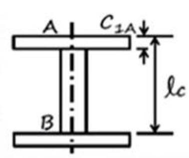
II-1: COLUMN STIFFNESS (K_C)

$$K_c = k.\frac{EI_{col}}{l_{col}}$$

Where $I_C = \frac{{C_1}^3.C_2}{12}$ (For rectangular or square column), $I_c = \frac{\pi D^4}{64}$ (For circular column).

C_{1A} = *FROM THE EDGE OF THE SLAB TO THE BEGINNING OF THE COLUMN*.

The stiffness factor (k) and carry over factors are given in the table below.



Coefficients for column stiffness.

Slab* depth	Uniform load FEM=Coeff.$(wl_2l_1^2)$		Stiffness factor Bottom	Stiffness factor Top	Carry over factor	
C_{1A}/ℓ_c	M_{AB}	M_{BA}	k_{AB}	k_{BA}	COF_{AB}	COF_{BA}
0.00	0.083	0.083	4.00	4.00	0.500	0.500
0.05	0.100	0.075	4.91	4.21	0.496	0.579
0.10	0.118	0.068	6.09	4.44	0.486	0.667
0.15	0.135	0.060	7.64	4.71	0.471	0.765
0.20	0.153	0.053	9.69	5.00	0.452	0.875
0.25	0.172	0.047	12.44	5.33	0.429	1.000

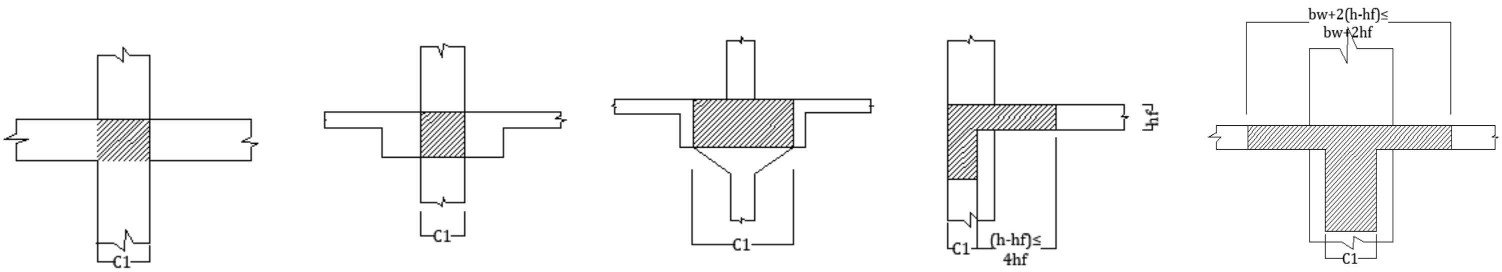
II-2: TORSIONAL MEMBERS STIFFNESS (K_t)

$$K_t = \frac{9\,E_{cs}C}{l_2(1-\frac{c_2}{l_2})^3}\ ,\quad C = \sum\left(1-0.63\frac{x}{y}\right)\frac{x^3y}{3}$$

If a parallel beam to direction l_1 exists, then increase K_t to K_{ta}.

$$K_{ta} = K_t \times \left(\frac{I_{sb}}{I_s}\right)$$

Where: $I_{sb} = \frac{bh^3}{12} \times 2\ (interior\ beams)$.



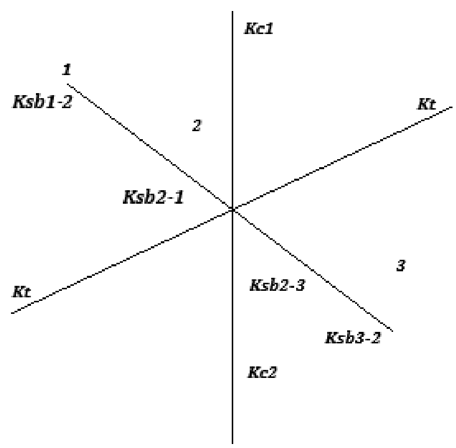
DISTRIBUTION FACTORS (DF):

@ joint 2:

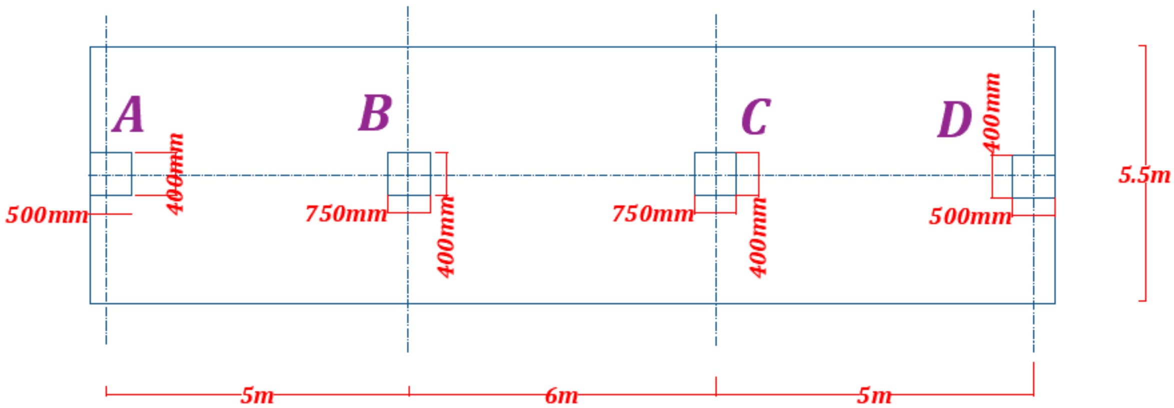
$$DF_{2-1} = \frac{K_{sb\,2-1}}{K_{sb2-1}+K_{sb2-3}+K_{ec}}$$

$$DF_{2-3} = \frac{K_{sb2-3}}{K_{sb2-1}+K_{sb2-3}+K_{ec}}$$

$$DF_{eq.col.} = \frac{K_{ec}}{K_{sb2-1}+K_{sb2-3}+K_{ec}}$$



EXAMPLE 1: For the flat plate shown below, calculate:



- Calculate $K_{sb}, COF, and\ FEM$ for interior and exterior panels.
- Calculate K_c for joint A.

Slab thickness= 250mm, $l_c = 3m$.

SOLUTION:

1. Calculate $K_{sb}, COF, and\ FEM$ for interior and exterior panels.

$$K_{sb} = k.\frac{E\ (I_s + I_b)}{L_1}$$

$$I_s = \frac{L_2 \times {h_s}^3}{12} = \frac{5500 \times 250^3}{12} = 7.161 \times 10^9 mm^4$$

- FOR EXTERIOR SPAN:

$\frac{C_1 A}{L_1} = \frac{500}{5000} = 0.1$ and $\frac{C_1 B}{L_1} = \frac{750}{5000} = 0.15$

$k_{AB} = 4.22,\ \ k_{BA} = 4.36$

$COF_{AB} = 0.528,\ \ COF_{BA} = 0.511,\ \ M_{AB} = 0.083,\ \ M_{BA} = 0.088$

$$K_{AB} = 4.22 \times \frac{E \times (7.161 \times 10^9 + 0)}{5000} = 6.0438 \times 10^6 E\ \ mm^4.$$

$$K_{BA} = 4.36\ \times \frac{E \times (7.161 \times 10^9 + 0)}{5000} = 6.24 \times 10^6 E\ mm^4.$$

- FOR INTERIOR SPAN:

$$\frac{C_1 B}{L_1} = \frac{C_1 C}{L_1} = \frac{750}{6000} = 0.125$$

$$k_{BC} = k_{CB} = \frac{4.4 + 4.18}{2} = 4.29$$

$$COF_{BC} = COF_{CB} = \frac{0.526 + 0.513}{2} = 0.52$$

$$M_{BC} = M_{CB} = \frac{0.086 + 0.085}{2} = 0.0855$$

$$K_{BC} = K_{CB} = 4.29 \times \frac{E(7.161 \times 10^9 + 0)}{6000} = 5.12 \times 10^6 E \; mm^4 \text{ N.mm}$$

2. Calculate K_c for joint A.

Slab* depth	Uniform load FEM=Coeff.($wl_2 l_1^2$)		Stiffness factor		Carry over factor	
			Bottom	Top		
C_{1A}/ℓ_c	M_{AB}	M_{BA}	k_{AB}	k_{BA}	COF_{AB}	COF_{BA}
0.00	0.083	0.083	4.00	4.00	0.500	0.500
0.05	0.100	0.075	4.91	4.21	0.496	0.579
0.10	0.118	0.068	6.09	4.44	0.486	0.667
0.15	0.135	0.060	7.64	4.71	0.471	0.765
0.20	0.153	0.053	9.69	5.00	0.452	0.875
0.25	0.172	0.047	12.44	5.33	0.429	1.000

$$\frac{C_1 A}{l_C} = \frac{250}{3000} = 0.0833$$

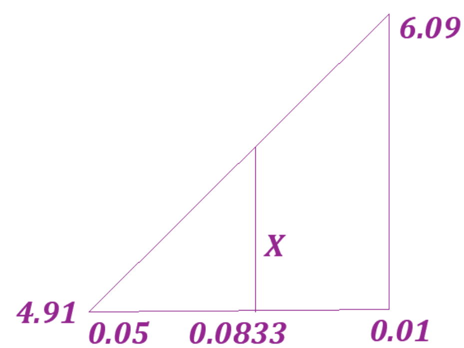
$$k_{AB} = 5.95 \; (BY\ INTERPOLATION)$$

$$k_{BA} = 4.36$$

$$K_{AB} = k.\frac{EI}{l_C}$$

$$K_{AB(BOTTOM)} = 5.95 \times \frac{E \times \frac{500^3 \times 400}{12}}{3000} = 8.263 \times 10^6 E \;\; N.mm$$

$$K_{AB\,(TOP)} = 4.36 \times \frac{E \times \frac{500^3 \times 400}{12}}{3000} = 6.055 \times 10^6 E \;\; N.mm$$



$$\frac{6.09 - 4.91}{0.01 - 0.05} = \frac{X}{0.083 - 0.05}$$

$$\therefore X = 0.785$$

$$\therefore k_{AB} = 0.785 + 4.91 = 5.95$$