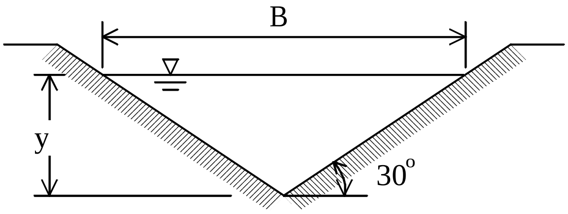
Worked examples:

1. A triangular channel with an angel of 120° made by 2 equal slopes. For a flow rate of 3 m^3/s, determine the critical depth and hence the maximum depth of the flow.



Answer

For critical flow,

$$v^2 = g*y_{ave}$$

$$Q^2 = g*y_{ave}*A^2$$

$$= \frac{gA^3}{B} \qquad (y_{ave} = \frac{A}{B})$$

For critical flow,

$$B = 2*y*\cot 30°$$

$$\& \quad A = y^2*\cot 30°$$

$$\therefore \quad Q^2 = \frac{3g}{2}y^5$$

Hence

$$y = (\frac{2Q^2}{3g})^{1/5}$$

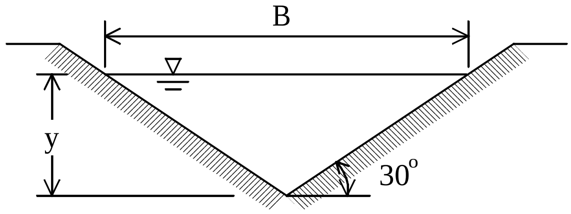
$$= (\frac{2*3^2}{3*9.81})^{1/5} \text{ m}$$

$$= \underline{0.906 \text{ m}}$$

The maximum depth is 0.906 m.

~~The critical depth, y_c =~~ $y_{ave} = \frac{A}{B} = \frac{(B*y)/2}{B} = \frac{y}{2} = \frac{0906}{2} = 0.453m$.

2. In the last example, the channel Manning roughness coefficient is 0.012 and the flow rate is 3 m^3/s. What is the value of the channel slope if the flow is critical, subcritical or supercritical?



Answer

$$B = 2\sqrt{3} * y$$
$$A = \frac{1}{2} * B * y$$
$$P = 4 * y$$

Using Manning equation,

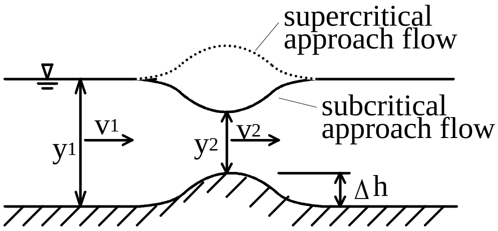
$$Q = \frac{A}{n} * R^{2/3} * S^{1/2}$$
$$= \frac{1}{n} * (\frac{1}{2} * B * y) * (\frac{B}{8})^{2/3} * S^{1/2}$$
$$S_c^{1/2} = \frac{2nQ}{By} (\frac{B}{8})^{-2/3}$$
$$= \frac{2nQ}{2\sqrt{3} * y^2} (\frac{\sqrt{3} * y}{4})^{-2/3}$$

For critical flow, y = y

$$\therefore \qquad S_c^{1/2} = \frac{2 * 0.012 * 3}{2\sqrt{3} * (0.906)^2} (\frac{\sqrt{3} * 0.906}{4})^{-2/3}$$
$$S_c = 0.0472$$

For flow is critical, S = 0.0472 - critical slope
subcritical, S < 0.0472
supercritical, S > 0.0472

8.4 Frictionless Flow over a Bump



- When fluid is flowing over a bump, the behaviour of the free surface is sharply different according to whether the approach flow is subcritical or supercritical.
- The height of the bump can change the character of the results.
- Applying Continuity and Bernoulli's equations to sections 1 and 2,

$$v_1 * y_1 = v_2 * y_2$$

$$\& \qquad \frac{v_1^2}{2g} + y_1 = \frac{v_2^2}{2g} + y_2 + \Delta h$$

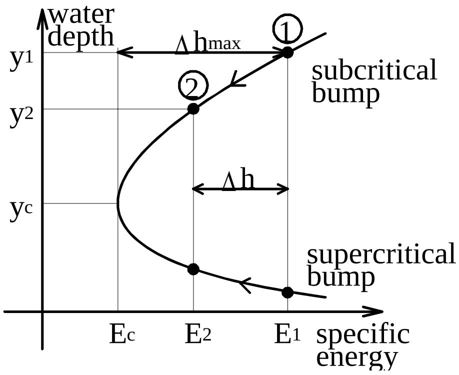
- Eliminating v_2 between these two gives a cubic polynomial equation for the water depth y_2 over the bump,

$$y_2^3 - E_2 * y_2^2 + \frac{v_1^2 * y_1^2}{2g} = 0 \qquad (8.19)$$

where $$E_2 = \frac{v_1^2}{2g} + y_1 - \Delta h \qquad (8.20)$$

This equation has one negative and two positive solutions if Δh is not too large.

◆ The free surface's behaviour depends upon whether condition 1 is in subcritical or supercritical flow.



The specific energy E_2 is exactly Δh less than the approach energy, E_1, and point 2 will lie on the same leg of the curve as E_1.

◆ A subcritical approach, $Fr_1 < 1$, will cause the water level to decrease at the bump.

◆ Supercritical approach flow, $Fr_1 > 1$, causes a water level increase over the bump.

◆ If the bump height reaches $\Delta h_{max} = E_1 - E_c$, the flow at the crest will be exactly critical ($Fr = 1$).

◆ If the bump > Δh_{max}, there are no physical correct solution. That is, a bump to large will choke the channel and cause frictional effects, typically a hydraulic jump.

Worked example:

Water flow in a wide channel approaches a 10 cm high bump at 1.5 m/s and a depth of 1 m. Estimate

(a) the water depth y_2 over the bump, and
(b) the bump height which will cause the crest flow to be critical.

Answer

(a) For the approaching flow,

$$Fr = \frac{v_1}{\sqrt{gy_1}} = \frac{1.5}{\sqrt{9.81*1}}$$
$$= 0.479 \Rightarrow \text{subcritical}$$

For subcritical approach flow, if Δh is not too large, the water level over the bump will depress and a higher subcritical Fr at the crest.

$$E_1 = \frac{v_1^2}{2g} + y_1 = \frac{1.5^2}{2*9.81} + 1.0 \text{ m}$$
$$= 1.115 \text{ m}$$

Hence $E_2 = E_1 - \Delta h = 1.115 - 0.1$ m
$= 1.015$ m

Substitute E_2 into (8.24),

$$y_2^3 - 1.015*y_2^2 + 0.115 = 0$$

By trial and error,

$y_2 = 0.859$ m, 0.451 m and -0.296 m (inadmissible)

The second (smaller) solution is the supercritical condition for E_2 and is not possible for this subcritical bump.

Hence y_2 = <u>0.859 m</u>

Checking: $v_2 = 1.745$ m/s (By continuity)

$Fr_2 = 0.601$ ($> Fr_1$ and < 1) (OK)

(b) By considering per m width of the channel,

$$q = v*y = 1.5*1 \ m^2/s$$

For critical flow,

$$E_2 = E_{min} = \frac{3}{2}y_c$$

$$y_c = (\frac{q^2}{g})^{1/3}$$

$$= (\frac{1.5^2}{9.81})^{1/3}$$

$$= 0.612 \ m$$

$$E_2 = \frac{3}{2}*0.612 \quad m$$

$$= 0.918 \ m$$

$$\Delta h_{max} = E_1 - E_{min}$$

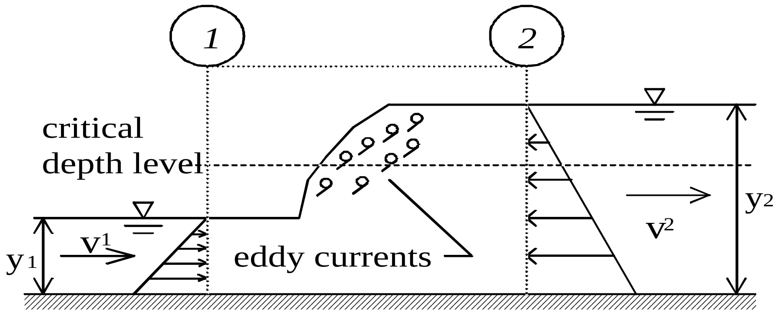
$$= 1.115 - 0.918 \ m$$

$$= \underline{0.197 \ m}$$

8.5 Hydraulic Jump in Rectangular Channel

- A hydraulic jump is a **sudden** change from a supercritical flow to subcritical flow.

- Assumptions:
  - the bed is horizontal.
  - the velocity over each cross-section is uniform.
  - the depth is uniform across the width.
  - frictionless boundaries.
  - surface tension effects are neglect.



- Considering the control volume between 1 and 2, the forces are

$$F_{31} = \rho g y_1 * \frac{b}{2} * y_1 = \rho g b * \frac{y_1^2}{2} \quad (8.21a)$$

Similarly $$F_{32} = \rho g b * \frac{y_2^2}{2} \quad (8.21b)$$

- By continuity equation,

$$Q = b*y_1*v_1 = b*y_2*v_2 \quad (8.22)$$

- By the momentum equation,

$$F_1 = F_2 = 0$$

hence $$F_{31} - F_{32} = \rho*Q*(v_2 - v_1) \quad (8.23)$$

- Sub. (8.21a, b) and (8.22) into (8.23), then

$$\frac{\rho g b}{2}(y_1^2 - y_2^2) = \rho Q\left(\frac{Q}{y_2 b} - \frac{Q}{y_1 b}\right)$$

$$= \frac{\rho Q^2}{b}\left(\frac{y_1 - y_2}{y_1 y_2}\right) \quad (8.24)$$

- In a hydraulic jump, $y_1 \neq y_2$,

$$y_1 * y_2 * (y_1 + y_2) = \frac{2Q^2}{gb^2}$$

$$\therefore \quad y_1^2 y_2 + y_1 y_2^2 = \frac{2Q^2}{gb^2}$$

i.e. $$\left(\frac{y_2}{y_1}\right)^2 + \left(\frac{y_2}{y_1}\right) - \frac{2Q^2}{gb^2 y_1^3} = 0 \qquad (8.25)$$

Solving (8.25),

$$\frac{y_2}{y_1} = \frac{1}{2}\left[-1 + \sqrt{1 + \frac{8Q^2}{gb^2 y_1^3}}\right] \qquad (8.26)$$

This is the hydraulic jump equation.

- Using Froude number,

$$Fr_1^2 = \frac{v_1^2}{gy_1} = \frac{Q^2}{gy_1^3 b^2} \qquad (8.27)$$

then, $$\frac{y_2}{y_1} = \frac{1}{2}\left[-1 + \sqrt{1 + 8Fr_1^2}\right] \qquad (8.28a)$$

or $$\frac{y_1}{y_2} = \frac{1}{2}\left[-1 + \sqrt{1 + 8Fr_2^2}\right] \qquad (8.28b)$$

- (y_1, y_2) are called **conjugate depths.**

- The energy loss in a jump is given by

$$\frac{v_1^2}{2g} + y_1 = \frac{v_2^2}{2g} + y_2 + h_f$$

i.e. $$h_f = \left(\frac{v_1^2 - v_2^2}{2g}\right) + (y_1 - y_2) \qquad (8.29)$$

- Sub. (8.22) into above,

$$h_f = \left[-\frac{Q^2}{2gb^2}\left(\frac{y_1 + y_2}{y_1^2 y_2^2}\right) + 1\right](y_1 - y_2) \qquad (8.29)$$

- Using (8.25), (8.29) becomes

$$h_f = \frac{(y_2 - y_1)^3}{4y_1y_2} \qquad (8.30)$$

This is the energy loss equation for the hydraulic jump ($y_2>y_1$, $h_f>0$).

- The power loss in a jump is

$$P = \rho g h_f * Q$$

- This energy loss is useful for getting away with the unwanted energy of a flow. The energy loss is due to the frictional forces amount the eddy currents in the pump. It will increase the temperature of the fluid.

Worked example:

Water flows in a wide channel at $q = 10\ m^2/s$ and $y_1 = 1.25$ m. If the flow undergoes a hydraulic jump, calculate

(a) y_2,
(b) v_2,
(c) Fr_2,
(d) h_f, and
(e) the percentage dissipation of the energy.

Answer

(a)

$$v_1 = \frac{q}{y_1}$$

$$= \frac{10}{1.25}\ \text{m/s} \quad = 8\ \text{m/s}$$

$$Fr_1 = \frac{v_1}{\sqrt{gy_1}}$$

$$= \frac{8}{\sqrt{9.81*1.25}} \quad = 2.285$$

Since

$$\frac{y_2}{y_1} = \frac{1}{2}[-1+\sqrt{1+8Fr_1^2}]$$

$$= \frac{1}{2}[-1+\sqrt{1+8*(2.285)^2}]$$

$$= 2.77$$

or

$$y_2 = 2.77*1.25\ \text{m}$$

$$= \underline{3.46\ \text{m}}$$

(b) By Continuity equation,

$$v_2 = v_1*(\frac{y_1}{y_2})$$

$$= 8*\frac{1.25}{3.46}\ \text{m/s}$$

$$= \underline{2.89\ \text{m/s}}$$

(c) $$Fr_2 = \frac{v_2}{\sqrt{gy_2}} = \frac{2.89}{\sqrt{9.81 * 3.46}} = \underline{0.496}$$

(d) $$h_f = \frac{(y_2 - y_1)^3}{4y_1y_2} = \frac{(3.46 - 1.25)^3}{4 * 3.46 * 1.25} = \underline{0.625\ \text{m}}$$

(e) $$E_1 = \frac{v_1^{\ 2}}{2g} + y_1 = \frac{8^2}{2 * 9.81} + 1.25 \quad \text{m} = \underline{4.51\ \text{m}}$$

percentage loss $$= \frac{h_f}{E_1} * 100\% = \frac{0.625}{4.51} * 100\% = \underline{14\ \%}$$