Subject: Information Theory and Coding
Problems#2 Information + Source Code + ECC

1-

(a) Design ternary (D=3) Huffman source code for the source shown beside.

(b) Determine the efficiency of the source and the efficiency of the source code.

(c) Is the code decodable? Why?

Symbol x_i	Probability $P(x_i)$
x_1	0.15
x_2	0.21
x_3	0.07
x_4	0.28
x_5	0.13
x_6	0.1
x_7	0.06

2.

(a) Find the efficiency of the following continues source:

$f(x) = 0.5 \quad for \quad |x| < 1$ and 0 elsewhere

(b) Consider the transmission of binary information over channel having bandwidth of 10 kHz at rate of 40 kbps:

i- Find the corresponding S/N in dB.

ii- If the AWGN power spectral density N_o is 2 $\mu\, Watt/Hz$, what is the signal power.

3. Use the channel model shown beside:

If $p(x_1) = p(x_2) = 0.4$

$$P(y/x) = \begin{array}{c|ccc} & y1 & y2 & y3 \\ x1 & 0.9 & 0 & 0.1 \\ x2 & 0 & 1 & 0 \\ x3 & 0.1 & 0 & 0.9 \end{array}$$

a) Is the channel symmetric or noiseless?

b) Find $p(x_3)$ and source efficiency

c) Find H(y), H(y|x), and I

4- Consider the following three error control codes;

i- First Code: Binary repetition code with $n_1=3$.

ii- Second Code: Linear block code with $(n_2, k_2) = (15,11)$

iii- Third Code: Odd parity check code with n3=3

Then answer the followings;

1- Find the value of k (number of input data bits) for the first and third code.

2- Give the dimension of the parity check matrix H for second code.

3- Find the error correction capability t_c of the second code.

4- What is the generator matrix for first code?

5- Consider cyclic code with generator polynomial $g(x) = x^4 + x + 1$ with codeword length n=15.

i- Find all other code parameters (k, R_c and t)

ii- Determine the first codeword polynomial C(x) for the input word [0 1 1 0 0 0 0 0 0 0 0]

iii- Draw the encoder circuit

iv- Find the syndrome polynomial for the received word $R(x) = x^4 + x$.

6- Answer TRUE or FALSE and correct the FALSE statement.

No.	Statement
1	Source code is used to reduce channel errors.
2	Cascading of more than one BSC channel reduces the composite channel capacity
3	ARQ is more efficient than FEC
4	The size of look ahead buffer is usually 10 times that of search buffer in LZ77 compressor.
5	Run Length Encoding is a universal data compression technique.

7.

(a) Find the channel capacity (in bps) if the bandwidth B=100 kHz and S/N = 30 dB.

(b) Consider colored (24 bits/pixel) image with dimension of 1200x800 pixels/frame, and equal probable pixels. Find:

i- the amount of information carried by one image frame (in bits/frame).

ii- the rate of information (in bps), if 2000 frames are sent within 100 sec.

iii- the required signal to noise power ration (dB) in (ii) if the channel bandwidth is 40 MHz.

(c) Find the efficiency of ternary source with P(x1) = P(x2) = 3.P(x3).

8. Consider 6-symbol source with given probabilities;

X_i	X_1	X_2	X_3	X_4	X_5	X_6
$P(x_i)$	0.11	0.21	A	0.17	0.08	0.15

(a) Find A

(b) Construct binary Huffman code for the source.

(c) Determine the source and code efficiency.

(d) Is the code decodable? Why?

(e) Find the probability of binary "0" and "1" at encoder output

9- LZ77 decompressor received the shown tokens; find the decoded (decompressed sequence) and the compression factor.

Distance	0	1	1	1	6
Match Length	0	1	1	1	6
Next Char.	A	B	C	A	A

10._Consider an information transmission system consisting of:

Source -- Binary Source Code – Binary Symmetric Channel – Source Decoder

The source is given

$\mathbf{X_i}$	X_1	X_2	X_3	X_4	X_5
$\mathbf{P(x_i)}$	0.3	0.2	0.22	0.17	0.11

by :

The error probability of the channel P_e=0.1

Use **any two** of the following source codes:

1- Suitable Fixed Length code 2- Fano Source Code 3- Huffman Source Code

Then for each selected code find:

a- The code efficiency b- The probabilities of "0" and "1" after the channel

11. Consider the following binary code:

1- Is the given code decodable? Why?
2- Find L of the given code
3- Construct fixed length code.
4- Which has more efficiency? (the given or fixed length code)

Symbol x_i	**Probability** P_{xi}	**Given Code**	L_i	**Fixed Length Code**
x_1	**0.25**	**00**		
x_2	**0.25**	**11**		
x_3	**0.125**	**100**		
x_4	**o.125**	**101**		
x_5	**0.25**	**01**		