CHAPTER 3 Questions to Guide Your Review

1. What is the derivative of a function f? How is its domain related to the domain of f? Give examples.
2. What role does the derivative play in defining slopes, tangent lines, and rates of change?
3. How can you sometimes graph the derivative of a function when all you have is a table of the function's values?
4. What does it mean for a function to be differentiable on an open interval? On a closed interval?
5. How are derivatives and one-sided derivatives related?
6. Describe geometrically when a function typically does *not* have a derivative at a point.
7. How is a function's differentiability at a point related to its continuity there, if at all?
8. What rules do you know for calculating derivatives? Give some examples.
9. Explain how the three formulas

   a. $\frac{d}{dx}(x^n) = nx^{n-1}$ **b.** $\frac{d}{dx}(cu) = c\frac{du}{dx}$

   c. $\frac{d}{dx}(u_1 + u_2 + \cdots + u_n) = \frac{du_1}{dx} + \frac{du_2}{dx} + \cdots + \frac{du_n}{dx}$

   enable us to differentiate any polynomial.
10. What formula do we need, in addition to the three listed in Question 9, to differentiate rational functions?
11. What is a second derivative? A third derivative? How many derivatives do the functions you know have? Give examples.
12. What is the relationship between a function's average and instantaneous rates of change? Give an example.
13. How do derivatives arise in the study of motion? What can you learn about an object's motion along a line by examining the derivatives of the object's position function? Give examples.
14. How can derivatives arise in economics?
15. Give examples of still other applications of derivatives.
16. What do the limits $\lim_{h\to 0}((\sin h)/h)$ and $\lim_{h\to 0}((\cos h - 1)/h)$ have to do with the derivatives of the sine and cosine functions? What *are* the derivatives of these functions?
17. Once you know the derivatives of $\sin x$ and $\cos x$, how can you find the derivatives of $\tan x$, $\cot x$, $\sec x$, and $\csc x$? What *are* the derivatives of these functions?
18. At what points are the six basic trigonometric functions continuous? How do you know?
19. What is the rule for calculating the derivative of a composition of two differentiable functions? How is such a derivative evaluated? Give examples.
20. If u is a differentiable function of x, how do you find $(d/dx)(u^n)$ if n is an integer? If n is a real number? Give examples.
21. What is implicit differentiation? When do you need it? Give examples.
22. How do related rates problems arise? Give examples.
23. Outline a strategy for solving related rates problems. Illustrate with an example.
24. What is the linearization $L(x)$ of a function $f(x)$ at a point $x = a$? What is required of f at a for the linearization to exist? How are linearizations used? Give examples.
25. If x moves from a to a nearby value $a + dx$, how do you estimate the corresponding change in the value of a differentiable function $f(x)$? How do you estimate the relative change? The percentage change? Give an example.

CHAPTER 3 Practice Exercises

Derivatives of Functions

Find the derivatives of the functions in Exercises 1–40.

1. $y = x^5 - 0.125x^2 + 0.25x$
2. $y = 3 - 0.7x^3 + 0.3x^7$
3. $y = x^3 - 3(x^2 + \pi^2)$
4. $y = x^7 + \sqrt{7}x - \frac{1}{\pi + 1}$
5. $y = (x + 1)^2(x^2 + 2x)$
6. $y = (2x - 5)(4 - x)^{-1}$
7. $y = (\theta^2 + \sec\theta + 1)^3$
8. $y = \left(-1 - \frac{\csc\theta}{2} - \frac{\theta^2}{4}\right)^2$
9. $s = \frac{\sqrt{t}}{1 + \sqrt{t}}$
10. $s = \frac{1}{\sqrt{t} - 1}$
11. $y = 2\tan^2 x - \sec^2 x$
12. $y = \frac{1}{\sin^2 x} - \frac{2}{\sin x}$
13. $s = \cos^4(1 - 2t)$
14. $s = \cot^3\left(\frac{2}{t}\right)$
15. $s = (\sec t + \tan t)^5$
16. $s = \csc^5(1 - t + 3t^2)$
17. $r = \sqrt{2\theta\sin\theta}$
18. $r = 2\theta\sqrt{\cos\theta}$
19. $r = \sin\sqrt{2\theta}$
20. $r = \sin\left(\theta + \sqrt{\theta + 1}\right)$
21. $y = \frac{1}{2}x^2\csc\frac{2}{x}$
22. $y = 2\sqrt{x}\sin\sqrt{x}$
23. $y = x^{-1/2}\sec(2x)^2$
24. $y = \sqrt{x}\csc(x + 1)^3$
25. $y = 5\cot x^2$
26. $y = x^2\cot 5x$
27. $y = x^2\sin^2(2x^2)$
28. $y = x^{-2}\sin^2(x^3)$
29. $s = \left(\frac{4t}{t + 1}\right)^{-2}$
30. $s = \frac{-1}{15(15t - 1)^3}$
31. $y = \left(\frac{\sqrt{x}}{1 + x}\right)^2$
32. $y = \left(\frac{2\sqrt{x}}{2\sqrt{x} + 1}\right)^2$
33. $y = \sqrt{\frac{x^2 + x}{x^2}}$
34. $y = 4x\sqrt{x + \sqrt{x}}$

35. $r = \left(\frac{\sin\theta}{\cos\theta - 1}\right)^2$

36. $r = \left(\frac{1 + \sin\theta}{1 - \cos\theta}\right)^2$

37. $y = (2x + 1)\sqrt{2x + 1}$

38. $y = 20(3x - 4)^{1/4}(3x - 4)^{-1/5}$

39. $y = \frac{3}{(5x^2 + \sin 2x)^{3/2}}$

40. $y = (3 + \cos^3 3x)^{-1/3}$

Implicit Differentiation

In Exercises 41–48, find dy/dx by implicit differentiation.

41. $xy + 2x + 3y = 1$

42. $x^2 + xy + y^2 - 5x = 2$

43. $x^3 + 4xy - 3y^{4/3} = 2x$

44. $5x^{4/5} + 10y^{6/5} = 15$

45. $\sqrt{xy} = 1$

46. $x^2y^2 = 1$

47. $y^2 = \frac{x}{x + 1}$

48. $y^2 = \sqrt{\frac{1 + x}{1 - x}}$

In Exercises 49 and 50, find dp/dq.

49. $p^3 + 4pq - 3q^2 = 2$

50. $q = (5p^2 + 2p)^{-3/2}$

In Exercises 51 and 52, find dr/ds.

51. $r\cos 2s + \sin^2 s = \pi$

52. $2rs - r - s + s^2 = -3$

53. Find d^2y/dx^2 by implicit differentiation:

a. $x^3 + y^3 = 1$ **b.** $y^2 = 1 - \frac{2}{x}$

54. a. By differentiating $x^2 - y^2 = 1$ implicitly, show that $dy/dx = x/y$.

b. Then show that $d^2y/dx^2 = -1/y^3$.

Numerical Values of Derivatives

55. Suppose that functions $f(x)$ and $g(x)$ and their first derivatives have the following values at $x = 0$ and $x = 1$.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
0	1	1	−3	1/2
1	3	5	1/2	−4

Find the first derivatives of the following combinations at the given value of x.

a. $6f(x) - g(x)$, $x = 1$ **b.** $f(x)g^2(x)$, $x = 0$

c. $\frac{f(x)}{g(x) + 1}$, $x = 1$ **d.** $f(g(x))$, $x = 0$

e. $g(f(x))$, $x = 0$ **f.** $(x + f(x))^{3/2}$, $x = 1$

g. $f(x + g(x))$, $x = 0$

56. Suppose that the function $f(x)$ and its first derivative have the following values at $x = 0$ and $x = 1$.

x	$f(x)$	$f'(x)$
0	9	−2
1	−3	1/5

Find the first derivatives of the following combinations at the given value of x.

a. $\sqrt{x}\,f(x)$, $x = 1$ **b.** $\sqrt{f(x)}$, $x = 0$

c. $f(\sqrt{x})$, $x = 1$ **d.** $f(1 - 5\tan x)$, $x = 0$

e. $\frac{f(x)}{2 + \cos x}$, $x = 0$ **f.** $10\sin\left(\frac{\pi x}{2}\right)f^2(x)$, $x = 1$

57. Find the value of dy/dt at $t = 0$ if $y = 3\sin 2x$ and $x = t^2 + \pi$.

58. Find the value of ds/du at $u = 2$ if $s = t^2 + 5t$ and $t = (u^2 + 2u)^{1/3}$.

59. Find the value of dw/ds at $s = 0$ if $w = \sin\left(\sqrt{r} - 2\right)$ and $r = 8\sin(s + \pi/6)$.

60. Find the value of dr/dt at $t = 0$ if $r = (\theta^2 + 7)^{1/3}$ and $\theta^2 t + \theta = 1$.

61. If $y^3 + y = 2\cos x$, find the value of d^2y/dx^2 at the point (0, 1).

62. If $x^{1/3} + y^{1/3} = 4$, find d^2y/dx^2 at the point (8, 8).

Applying the Derivative Definition

In Exercises 63 and 64, find the derivative using the definition.

63. $f(t) = \frac{1}{2t + 1}$

64. $g(x) = 2x^2 + 1$

65. a. Graph the function

$$f(x) = \begin{cases} x^2, & -1 \le x < 0 \\ -x^2, & 0 \le x \le 1. \end{cases}$$

b. Is f continuous at $x = 0$?

c. Is f differentiable at $x = 0$?

Give reasons for your answers.

66. a. Graph the function

$$f(x) = \begin{cases} x, & -1 \le x < 0 \\ \tan x, & 0 \le x \le \pi/4. \end{cases}$$

b. Is f continuous at $x = 0$?

c. Is f differentiable at $x = 0$?

Give reasons for your answers.

67. a. Graph the function

$$f(x) = \begin{cases} x, & 0 \le x \le 1 \\ 2 - x, & 1 < x \le 2. \end{cases}$$

b. Is f continuous at $x = 1$?

c. Is f differentiable at $x = 1$?

Give reasons for your answers.

68. For what value or values of the constant m, if any, is

$$f(x) = \begin{cases} \sin 2x, & x \le 0 \\ mx, & x > 0 \end{cases}$$

a. continuous at $x = 0$?

b. differentiable at $x = 0$?

Give reasons for your answers.

Slopes, Tangents, and Normals

69. Tangent lines with specified slope Are there any points on the curve $y = (x/2) + 1/(2x - 4)$ where the slope is $-3/2$? If so, find them.

70. Tangent lines with specified slope Are there any points on the curve $y = x - 1/2x$ where the slope is 2? If so, find them.