Subject: Information Theory and Coding
Problems #1 Probability and Information

1- A biased die with P(odd) = 2.P(even) is considered as the random experiment. Let x_i be the top face number as the outcome of the experiment.

(a) Find the sample space (values of x_i) and their probabilities.

(b) If the above die is thrown for 8 times, find the probability of ;

1-No Odd numbers occurred 2-Even number occurred at most 6 times

2- Let x has the following pdf ; $f(x) = \begin{cases} A & for \quad -1 < x < 0 \\ 0.5A & for \quad 1 < x < 2 \\ 0 & elsewhere \end{cases}$

(a) Find A and sketch the f(x) and F(x).

(b) Find $\bar{x}$, $\overline{x^2}$, and the variance σ^2

3- Consider joint random variables x and y, with the following joint pdf;

$$f(x,y) = \begin{cases} K\,x & for \quad 0 < x < 2 \\ & and \quad 0 < y < 4 \\ 0 & elsewhere \end{cases}$$

(a) Find the constant K, f(x) and f(y).

(b) Find $\bar{x}$, and $\overline{y^2}$

(c) From the results of (a) could you decide whether x and y are independent or not.

4- Three balls are drawn successively at random from a box containing 6 red balls, 4 white balls, and 5 green balls. Find the probability that they are drawn in the order red, white, and green, if each ball is (a) Replaced (b) Not replaced.

5- Let the random variable x_i is defined as the sum of the two numbers occurring in the experiment of the throwing two dice. Then:

(a) Find all possible values of x_i with their probabilities

(b) Find P(x > 9) , P(2 < x < 8), P(x =odd) , P(x < 10)

(c) Find : $\bar{x}$, $\overline{x^2}$, σ^2 , E[$4x^2$] , E[$(x+2)^2$]

6- Consider **four-digit Octal number,** then:

i-How many possible numbers if the 1st digit is non-zero

ii- As in (i) but considering even numbers greater than 6000

7- For CRV defined by the pdf beside;

$$f(x) = \begin{cases} = 0.25 & for \quad -1 < x < 0 \\ = 0.5 & for \quad 1 < x < A \\ = 0 & elsewhere \end{cases}$$

(a) Find the value of A

(b) If the above is considered as an information source find its entropy

8. The joint probability matrix of ternary channel is given by;

$$P(x_i, y_j) = \begin{bmatrix} 0.32 & 0.04 & 0.04 \\ 0.02 & 0.16 & 0.02 \\ 0.04 & 0.04 & 0.32 \end{bmatrix}$$

(a) Find all $P(x_i)$ and $P(y_j)$

(b) Find the channel matrix $P(y_j|x_i)$ and channel model.

(c) Find both the source and channel efficiencies

9. Find the efficiency of the continues source described by the following pdf:

$$f(x) = 0.25 \quad for \quad -2 < x < 2$$
$$= 0 \qquad\qquad Elsewhere$$

10. Transmission system over continues channel with bandwidth of 20MHz needs signal to noise power ratio (S/N) of 30dB:

a- Determine the resultant bit rate in bps.

b- If it is required to double the above rate while keeping the bandwidth unchanged, what is the new value of the signal to noise power ratio (S/N) in dB

11. Find the amount of information provided by each of the following systems;

a- Computer file storage for 30 sec. recording of source having an entropy of 3 bits/symbol and an average symbol rate R_x of 10 k symbol/s.

b- Ten frames of gray scale digital image with the following specifications:
Image frame dimension = 800x600 pixels/frame, Gray scale image having 256 different levels, The pixels are equal probable to have any level.

12. Consider binary channel with:

$P(0_T) = 0.5$, $P(0_R \mid 1_T) = 0.1$ & $P(0_R \mid 0_T) = 0.9$, $P(1_R \mid 0_T) = 0$ & $P(1_R \mid 1_T) = 1$

Find: $P(1_T)$, $P(0_R, 0_T)$, $P(0_R)$ and $P(1_R)$

13- Consider the following ternary channel;

$$P(y_j|x_i) = \begin{bmatrix} 0.7 & 0.3 & 0 \\ 0 & 0.7 & 0.3 \\ 0.3 & 0 & 0.7 \end{bmatrix}$$

If p(x1) = p(x2) = p(x3), then

(a) Specify whether the channel is symmetric or not

(b) Find the source entropy

(c) Find the average mutual information I (in bits/symbol).

(d) Find both the source and channel efficiencies

14- Use the channel model shown beside:

If $p(x_1) = p(x_2) = 0.4$

$$P(\ y/x\ ) = \begin{array}{c|ccc} & y1 & y2 & y3 \\ x1 & 0.9 & 0 & 0.1 \\ x2 & 0 & 1 & 0 \\ x3 & 0.1 & 0 & 0.9 \end{array}$$

a) Is the channel symmetric or noiseless?

b) Find $p(x_3)$ and source efficiency

c) Find H(y), H(y|x), and I