Cantilever Sheet pile wall in Sand

Methods of construction

construction methods generally can be divided into two categories:-

1. Backfilled structure

   The sequence of construction for a backfilled structure is as follows (Figure 1):

Step 1. Dredge the in situ soil in front and back of the proposed structure.

Step 2. Drive the sheet piles.

Step 3. Backfill up to the level of the anchor, and place the anchor system.

Step 4. Backfill up to the top of the wall.

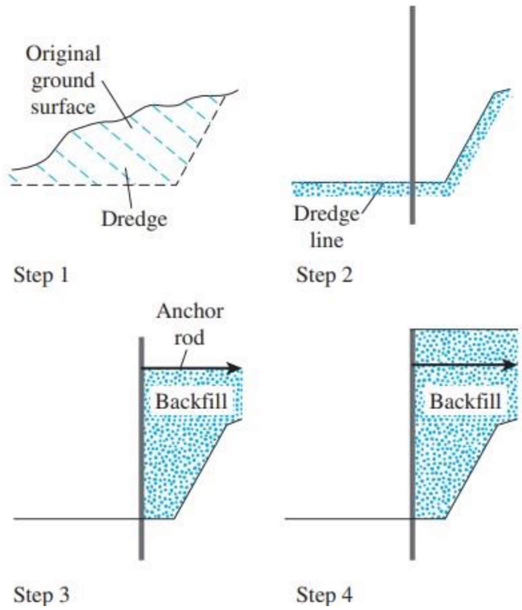


Fig.1 Sequence of construction for backfilled structures

2. Dredged structure

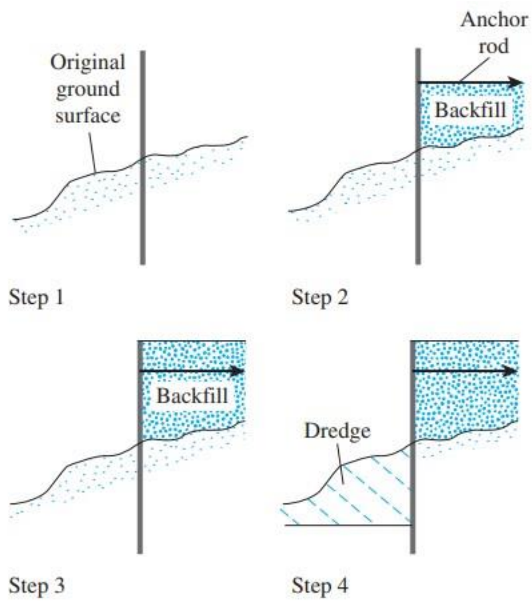


Fig.2 Sequence of construction for Dredged structures

Cantilever Sheet pile walls

Cantilever sheet pile walls are usually recommended for walls of moderate height— about 6 m or less, measured above the dredge line. In such walls, the sheet piles act as a wide cantilever beam above the dredge line. The basic principles for estimating net lateral pressure distribution on a cantilever sheet-pile wall can be explained with the aid of Fig.3. The figure shows the nature of lateral yielding of a cantilever wall penetrating a sand layer below the dredge line. The wall rotates about point O (Fig.3a). Because the hydrostatic pressures at any depth from both sides of the wall will cancel each other, we consider only the effective lateral soil pressures. In zone A, the lateral pressure is just the active pressure from the land side. In zone B, because of the nature of yielding of the wall, there will be active pressure from the land side and passive pressure from the water side. The condition is reversed in zone C—that is, below the point of rotation, O. The net actual pressure distribution on the wall is like that shown in Fig.1b. However, for design purposes, Fig.1c shows a simplified version.

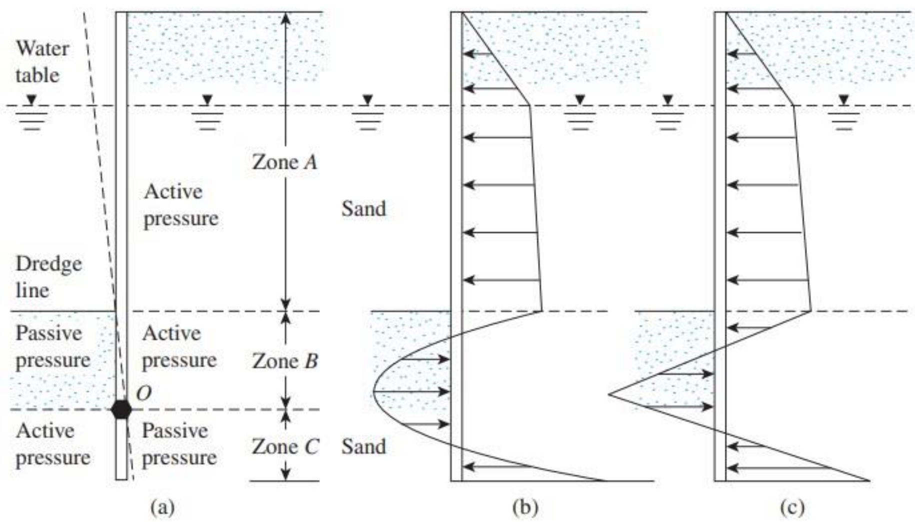


Fig.7 Cantilever sheet pile penetrating sand

Cantilever sheet piling penetrating sandy soil

To develop the relationships for the proper depth of embedment of sheet piles driven into a granular soil, examine Fig.4a The soil retained by the sheet piling above the dredge line also is sand. The water table is at a depth below the top of the wall.

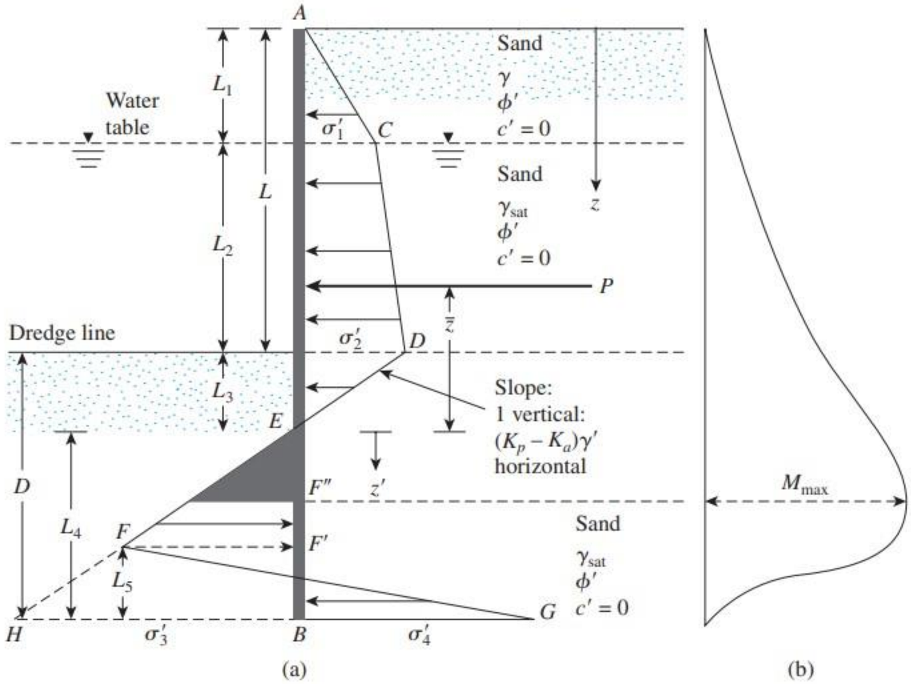


Fig.4 Cantilever sheet pile penetrating sand: (a) variation of net pressure diagram; (b) variation of moment

Let the effective angle of friction of the sand be Ø. The intensity of the active pressure at a depth $Z = L_1$ is :-

$$\sigma_1 = k_a \gamma L_1 \qquad (1)$$

Where

$k_a = tan^2(45 - \frac{\emptyset}{2})$, γ: *Unit weight of soil above water table*

Similarly, the active pressure at a depth $Z = L_1 + L_2$ (i.e., at the level of the dredge line) is :-

$$\sigma_2 = (L_1\gamma + L_2\gamma')k_a \qquad (2)$$

Note that, at the level of the dredge line, the hydrostatic pressures from both sides of the wall are the same magnitude and cancel each other. To determine the net lateral pressure below the dredge line up to the point of rotation, O, as shown in Fig.1a, an engineer has to consider the passive pressure acting from the left side (the water side) toward the right side (the land side) of the wall and also the active pressure acting from the right side toward the left side of the wall. For such cases, ignoring the hydrostatic pressure from both sides of the wall, the active pressure at depth z is

$$\sigma_a = [(L_1\gamma + L_2\gamma' + \gamma'(z - L_1 - L_2)]k_a \qquad (3)$$

Also, the passive pressure at depth z is

$$\sigma_p = \gamma'(z - L_1 + L_2)k_p \qquad (4)$$

Where k_p is Rankine passive pressure Coefficient $k_p = tan^2(45 + \frac{\emptyset}{2})$. Combining Eqs. (3) and (4) yields the net lateral pressure, namely,

$$\sigma = (L_1\gamma + L_2\gamma^{'})k_a + \gamma^{'}(z - L_1 - L_2)(k_a - k_p)$$

$$\sigma = (L_1\gamma + L_2\gamma^{'})k_a - \gamma^{'}(z - L_1 - L_2)(k_p - k_a)$$

$$\sigma = \sigma_2 - \gamma^{'}(z - L)(k_p - k_a) \qquad (5)$$

. The net pressure, equals zero at a depth L_3 below the dredge line, so

$$0 = \sigma_2 - \gamma^{'}(z - L)(k_p - k_a)$$

$$\sigma_2 = \gamma^{'}(z - L)(k_p - k_a)$$

$$(z - L) = L_3 = \frac{\sigma_2}{\gamma^{'}(k_p - k_a)} \qquad (6)$$

$$\text{And HB} = \sigma_3 = L_4\,\gamma^{'}(k_p - k_a) \qquad (7)$$

At the bottom of the sheet pile, passive pressure σ_p, acts from the right toward the left side, and active pressure acts from the left toward the right side of the sheet pile, so, at

$z = L + D$

$$\sigma_p = (\gamma L_1 + \gamma^{'} L_2 + \gamma^{'} D)k_p$$

$$\sigma_a = \gamma^{'} D k_a$$

Hence, the net lateral pressure at the bottom of the sheet pile is

$$\begin{aligned}\sigma'_p - \sigma'_a = \sigma'_4 &= (\gamma L_1 + \gamma' L_2)K_p + \gamma' D(K_p - K_a)\\ &= (\gamma L_1 + \gamma' L_2)K_p + \gamma' L_3(K_p - K_a) + \gamma' L_4(K_p - K_a)\\ &= \sigma'_5 + \gamma' L_4(K_p - K_a) \qquad (8)\end{aligned}$$

where

$$\sigma'_5 = (\gamma L_1 + \gamma' L_2)K_p + \gamma' L_3(K_p - K_a)$$
$$D = L_3 + L_4$$

For the stability of the wall, the principles of statics can now be applied:

$$\Sigma \text{ horizontal forces per unit length of wall} = 0$$

and

$$\Sigma \text{ moment of the forces per unit length of wall about point } B = 0$$

For the summation of the horizontal forces, we have

Area of the pressure diagram $ACDE$ − area of $EFHB$ + area of $FHBG = 0$

or

$$P - \tfrac{1}{2}\sigma_3' L_4 + \tfrac{1}{2} L_5(\sigma_3' + \sigma_4') = 0$$

where P = area of the pressure diagram $ACDE$.

Summing the moment of all the forces about point B yields

$$P(L_4 + \bar{z}) - \left(\frac{1}{2}L_4\sigma_3'\right)\left(\frac{L_4}{3}\right) + \frac{1}{2}L_5(\sigma_3' + \sigma_4')\left(\frac{L_5}{3}\right) = 0 \quad (9)$$

From Eq. (9.13),

$$L_5 = \frac{\sigma_3' L_4 - 2P}{\sigma_3' + \sigma_4'} \quad (10)$$

Combining Eqs. (7), (8), (9), and (10) and simplifying them further, we obtain the following fourth-degree equation in terms of L_4

$$L_4^4 + A_1 L_4^3 - A_2 L_4^2 - A_3 L_4 - A_4 = 0 \quad (11)$$

In this equation,

$$A_1 = \frac{\sigma_5'}{\gamma'(K_p - K_a)}$$

$$A_2 = \frac{8P}{\gamma'(K_p - K_a)}$$

$$A_3 = \frac{6P[2\bar{z}\gamma'(K_p - K_a) + \sigma_5']}{\gamma'^2(K_p - K_a)^2}$$

$$A_4 = \frac{P(6\bar{z}\sigma_5' + 4P)}{\gamma'^2(K_p - K_a)^2}$$

Step by step procedure for obtaining the pressure diagram

Based on the preceding theory, a step-by-step procedure for obtaining the pressure diagram for a cantilever sheet pile wall penetrating a granular soil is as follows:

Step 1. Calculate k_a and k_p

Step 2. Calculate σ_1 [Eq. 1] and σ_2 [Eq. 2]. (Note: L_1 and L_2 will be given.)

Step 3. Calculate [Eq. 6].
Step 4. Calculate P.
Step 5. Calculate Ź (i.e., the center of pressure for the area ACDE) by taking the moment about E.
Step 6. Calculate σ_5.
Step 7. Calculate A1, A2, A3, A4.
Step 8. Solve [Eq. 11) by trial and error to determine
Step 9. Calculate σ_4[Eq.8].
Step 10. Calculate σ_3 [Eq.7].
Step 11. Obtain $L_5$from [Eq.10].
Step 12. Draw a pressure distribution diagram like the one shown in Figure 4a.
Step 13. Obtain the theoretical depth of penetration as $L_3 + L_4$

The actual depth of penetration is increased by about 20 to 30%. Note that some designers prefer to use a factor of safety on the passive earth pressure coefficient at the beginning. In that case, in Step 1,

Calculations of max bending moment

The nature of the variation of the moment diagram for a cantilever sheet pile wall is shown in Fig.4b. The maximum moment will occur between points E and $F^{\acute{}}$Obtaining the maximum moment (M_{max})per unit length of the wall requires determining the point of zero shear. For a new axis (with origin at point E) for zero shear.

$$P = \tfrac{1}{2}(z')^2(K_p - K_a)\gamma'$$

or

$$z' = \sqrt{\frac{2P}{(K_p - K_a)\gamma'}}$$

Once the point of zero shear force is determined ($F^{"}$point in Fig.4a), the magnitude of the maximum moment can be obtained as

$$M_{max} = P(\bar{z} + z') - [\tfrac{1}{2}\gamma' z'^2 (K_p - K_a)](\tfrac{1}{3})z'$$

The necessary profile of the sheet piling is then sized according to the allowable flexural stress of the sheet pile material, or

$$S = \frac{M_{max}}{\sigma_{all}}$$

where

S: is section modulus of the sheet pile required per unit length of the structure.

σ_{all} : is Allowable flexural stress of steel.

Figure 9.9 shows a cantilever sheet pile wall penetrating a granular soil. Here, $L_1 = 2$ m, $L_2 = 3$ m, $\gamma = 15.9$ kN/m^3, $\gamma_{sat} = 19.33$ kN/m^3, and $\phi' = 32°$.

a. What is the theoretical depth of embedment, D?
b. For a 30% increase in D, what should be the total length of the sheet piles?
c. What should be the minimum section modulus of the sheet piles? Use $\sigma_{all} = 172$ MN/m^2.

Solution

Part a

Using Figure 9.8a for the pressure distribution diagram, one can now prepare the following table for a step-by-step calculation.

Quantity required	Eq. no.	Equation and calculation
K_a	—	$\tan^2\left(45 - \frac{\phi'}{2}\right) = \tan^2\left(45 - \frac{32}{2}\right) = 0.307$
K_p	—	$\tan^2\left(45 + \frac{\phi'}{2}\right) = \tan^2\left(45 + \frac{32}{2}\right) = 3.25$
σ_1'	9.1	$\gamma L_1 K_a = (15.9)(2)(0.307) = 9.763$ kN/m^2
σ_2'	9.2	$(\gamma L_1 + \gamma' L_2)K_a = [(15.9)(2) + (19.33 - 9.81)(3)](0.307) = 18.53$ kN/m^2
L_3	9.6	$\frac{\sigma_2'}{\gamma'(K_p - K_a)} = \frac{18.53}{(19.33 - 9.81)(3.25 - 0.307)} = 0.66$ m
P	—	$\frac{1}{2}\sigma_1' L_1 + \sigma_1' L_2 + \frac{1}{2}(\sigma_2' - \sigma_1')L_2 + \frac{1}{2}\sigma_2' L_3$ $= (\frac{1}{2})(9.763)(2) + (9.763)(3) + (\frac{1}{2})(18.53 - 9.763)(3) + (\frac{1}{2})(18.53)(0.66)$ $= 9.763 + 29.289 + 13.151 + 6.115 = 58.32$ kN/m
$\bar{z}$	—	$\frac{\Sigma M_E}{P} = \frac{1}{58.32}\left[9.763(0.66 + 3 + \frac{2}{3}) + 29.289(0.66 + \frac{3}{2}) + 13.151(0.66 + \frac{3}{3}) + 6.115(0.66 \times \frac{2}{3})\right] = 2.23$ m
σ_5'	9.11	$(\gamma L_1 + \gamma' L_2)K_p + \gamma' L_3(K_p - K_a) = [(15.9)(2) + (19.33 - 9.81)(3)](3.25) + (19.33 - 9.81)(0.66)(3.25 - 0.307) = 214.66$ kN/m^2
A_1	9.17	$\frac{\sigma_5'}{\gamma'(K_p - K_a)} = \frac{214.66}{(19.33 - 9.81)(3.25 - 0.307)} = 7.66$
A_2	9.18	$\frac{8P}{\gamma'(K_p - K_a)} = \frac{(8)(58.32)}{(19.33 - 9.81)(3.25 - 0.307)} = 16.65$

A_3	9.19	$\dfrac{6P[2\bar{z}\gamma'(K_p - K_a) + \sigma_5']}{\gamma'^2(K_p - K_a)^2}$ $= \dfrac{(6)(58.32)[(2)(2.23)(19.33 - 9.81)(3.25 - 0.307) + 214.66]}{(19.33 - 9.81)^2(3.25 - 0.307)^2}$ $= 151.93$
A_4	9.20	$\dfrac{P(6\bar{z}\sigma_5' + 4P)}{\gamma'^2(K_p - K_a)^2} = \dfrac{58.32[(6)(2.23)(214.66) + (4)(58.32)]}{(19.33 - 9.81)^2(3.25 - 0.307)^2}$ $= 230.72$
L_4	9.16	$L_4^4 + A_1L_4^3 - A_2L_4^2 - A_3L_4 - A_4 = 0$ $L_4^4 + 7.66L_4^3 - 16.65L_4^2 - 151.93L_4 - 230.72 = 0; L_4 \approx 4.8\text{ m}$

Thus,

$$D_{\text{theory}} = L_3 + L_4 = 0.66 + 4.8 = \mathbf{5.46\ m}$$

Part b

The total length of the sheet piles is

$$L_1 + L_2 + 1.3(L_3 + L_4) = 2 + 3 + 1.3(5.46) = \mathbf{12.1\ m}$$

Part c

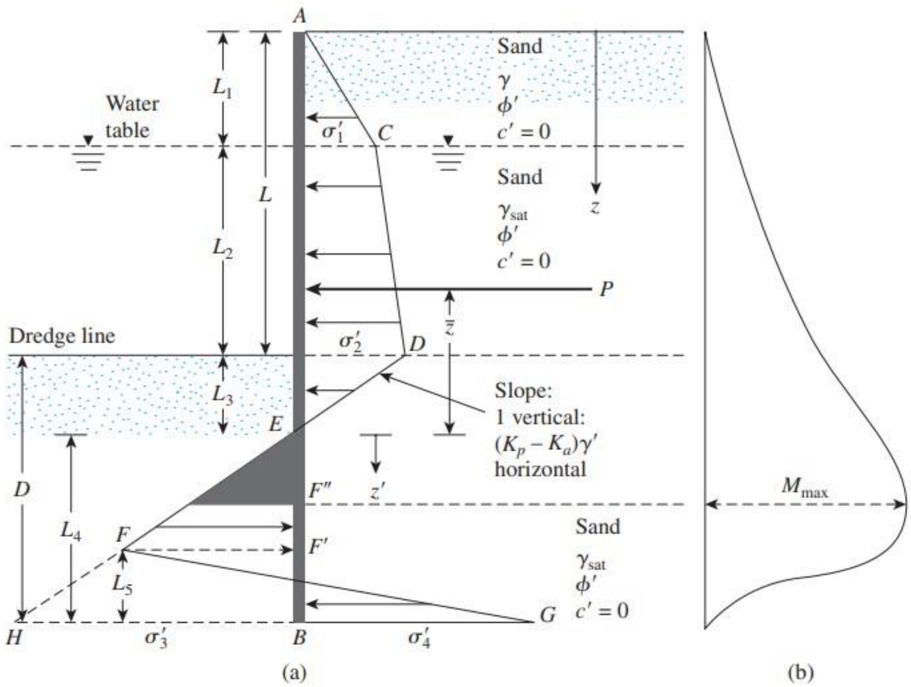
Finally, we have the following table.

Quantity required	Eq. no.	Equation and calculation
z'	9.21	$\sqrt{\dfrac{2P}{(K_p - K_a)\gamma'}} = \sqrt{\dfrac{(2)(58.32)}{(3.25 - 0.307)(19.33 - 9.81)}} = 2.04\text{ m}$
M_{max}	9.22	$P(\bar{z} + z') - \left[\dfrac{1}{2}\gamma' z'^2(K_p - K_a)\right]\dfrac{z'}{3} = (58.32)(2.23 + 2.04)$ $-\left[\left(\dfrac{1}{2}\right)(19.33 - 9.81)(2.04)^2(3.25 - 0.307)\right]\dfrac{2.04}{3}$ $= 209.39\text{ kN}\cdot\text{m/m}$
S	9.29	$\dfrac{M_{max}}{\sigma_{all}} = \dfrac{209.39\text{ kN}\cdot\text{m}}{172 \times 10^3\text{ kN/m}^2} = \mathbf{1.217 \times 10^{-3}\ m^3/m\ of\ wall}$ ■

$$S = \frac{Bt^2}{6}$$

$0.001217 = \frac{1*t^2}{6}, t = 0.08546m = 85.45mm,$ *Thickness of the sheet pile*

Let there is no water table, repeat the solution of the preceding Ex. $\gamma = 15.9, k_a = 0.307, k_p = 3.25$



$\sigma_2 = \gamma L k_a = 15.9 * 5 \quad * 0.307 = 24.4 KN/m2$

$L_3 = \frac{k_a H}{(k_p - k_a)} = \frac{0.307*5}{3.25-0.307} = 0.52m$

$P = \sigma_2 * \frac{L}{2} + \sigma_2 * \frac{L_3}{2} = 24.4\left(\frac{5}{2} + \frac{0.52}{2}\right) = 67.344$

$\acute{Z} = L_3 + \frac{L}{3} = \ 2.188m$

$\sigma_5 = \gamma L k_p + L_3\gamma(k_p - k_a)$=$15.9 * 5 * 3.25 + 0.52 * 15.9 * (3.25 - 0.307) = 282.7$

$A_1 = \frac{\sigma_5}{\gamma(k_p-k_a)} = \frac{282.7}{15.9(3.25-0.307)} = 6.04$

$A_2 = \frac{8P}{\gamma(k_p-k_a)} = \frac{8*67.344}{15.9(3.25-0.307)} = 11.52$

$$A_3 = \frac{6P[2\ \acute{Z}\gamma(k_p - k_a) + \sigma_5]}{\gamma^2(k_p - k_a)^2} = \frac{6 * 67.344[2 * 2.188 * 15.9(3.25 - 0.307) + 282.7]}{15.9^2(3.25 - 0.307)^2}$$
$$= \frac{196968.7}{744} = 90$$

$$A_4 = \frac{P(6\acute{Z}\sigma_5 + 4P)}{\gamma^2(k_p-k_a)^2} = \frac{67.344(6*2.188*282.7+4*67.344)}{15.9^2(3.25-0.307)^2} = 122$$

$L_4^4 + A_1 L_4^3 - A_2 L_4^2 - A_3 L_4 \ - A_4 = 0$

$L_4^4 + 6.04 L_4^3 - 11.52 L_4^2 - 90 L_4 \ - 122 = 0$

$L_4 = 4.062m$

Total length $= 5 + (4.062 + 0.52) * 1.3 = 10.95m$

Steel design

$$\boldsymbol{z'} = \sqrt{\frac{\boldsymbol{2P}}{(\boldsymbol{k_p} - \boldsymbol{k_a})\boldsymbol{\gamma}}} = \sqrt{\frac{\boldsymbol{2*67.344}}{(\boldsymbol{3.25 - 0.307})\boldsymbol{15.9}}} = \boldsymbol{1.7m}$$

$$M_{max} = P\left(\acute{Z} + \boldsymbol{z'}\right) - \frac{1}{2} * \gamma \boldsymbol{z'}^2 \left(k_p - k_a\right) * \frac{\boldsymbol{z'}}{3} = 67.344 * (2.188 + 1.7) - \frac{1}{6} * 15.9 * 1.7^3 (3.25 - 0.307)$$
$$= 261.8 - 38.3 = 223.5KN.m/m$$

$S = \frac{223.5}{172000} = 0.0013$

$S = \frac{1*t^2}{6}, \; 0.0013 = \frac{1*t^2}{6}, \quad t = 0.08829m = 88mm$