# Power Electronics

## 3rd Year

## 2023 / 2024

## Chapter 4

## Single-phase & Three-phase Rectifier


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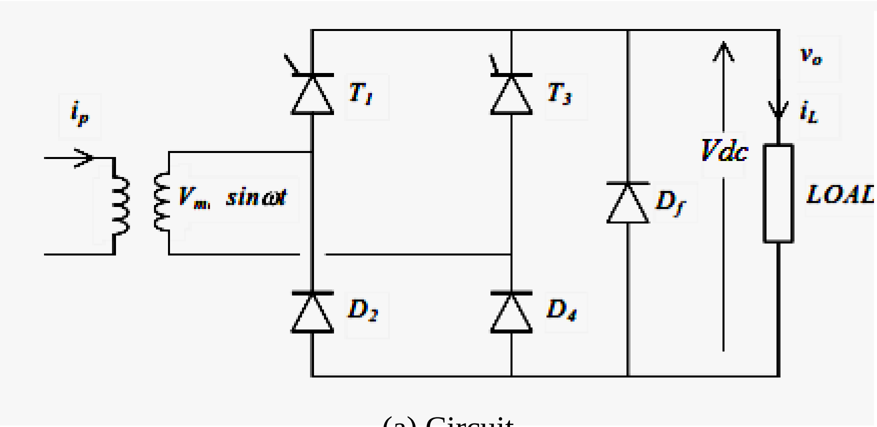
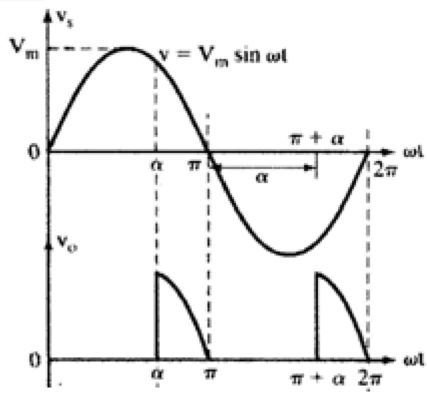
# 7. Single-phase Half – Controlled ( Semiconverter) Rectifier

Fig. 7.1 (a) shows a single-phase half-controlled (semiconverter) rectifier. This configuration consists of a combination of thyristors and diodes and used to eliminate any negative voltage occurrence at the load terminals. This is because the diode $D_f$ is always activated (forward biased) whenever the load voltage tends to be negative. For one total period of operation of this circuit, the corresponding waveforms are shown in Fig. 7.1(b).




(a) Circuit




(b) Waveform

Fig.7.1

The average value of the load voltage $V_{dc}$ can be calculated as follows,

$$V_{dc} = \frac{1}{\pi}\int_{\alpha}^{\pi} v_s(\omega t)\,d\omega t = \frac{1}{\pi}\int_{\alpha}^{\pi} V_m \sin(\omega t)\,d\omega t$$

$$\therefore V_{dc} = \frac{V_m}{\pi}\left(1+\cos(\alpha)\right)$$

Therefore, the average output voltage can vary from 0 to $V_m/\pi$ when varying $\alpha$ from $\pi$ to 0 respectively.

The average value of the load current $I_{dc}$ as follows,

$$I_{dc} = I_a = \frac{V_{dc}}{R}$$

The rms value of the load voltage $V_{rms}$ can be calculated as follows

$$V_{rms} = \sqrt{\frac{1}{\pi}\int_{\alpha}^{\pi}\{v_s(\omega t)\}^2\,d\omega t} = \sqrt{\frac{1}{\pi}\int_{\alpha}^{\pi}\{V_m \sin(\omega t)\}^2\,d\omega t}$$

$$\therefore V_{rms} = \frac{V_m}{\sqrt{2}}\sqrt{\frac{1}{\pi}\left(\pi - \alpha + \frac{\sin(2\alpha)}{2}\right)}$$

# 7.2 Three-phase controlled rectifier

## 1. Three- phase half – wave controlled rectifier ( *p = 3*)

The Three- phase half – wave controlled rectifier is shown in Fig. 7.2 .As for the half – wave 3-phase uncontrolled diode rectifier, the load is connected between the converter positive terminal (cathodes of all thyristors) and the supply neutral. The diode with the highest voltage w.r.t. the neutral conducts. As the voltage of another diode becomes the highest, the load current is transferred to that device, and the previously conducting device is reverse-biased and naturally commutated. The waveforms for the supply voltage, output voltage, and load current are shown in Fig.7.3 .

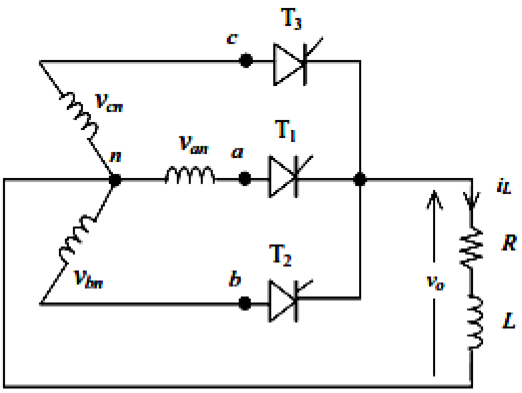



Fig.7.2 Circuit.

Note:
α is usually measured from point P in Fig.7.3.

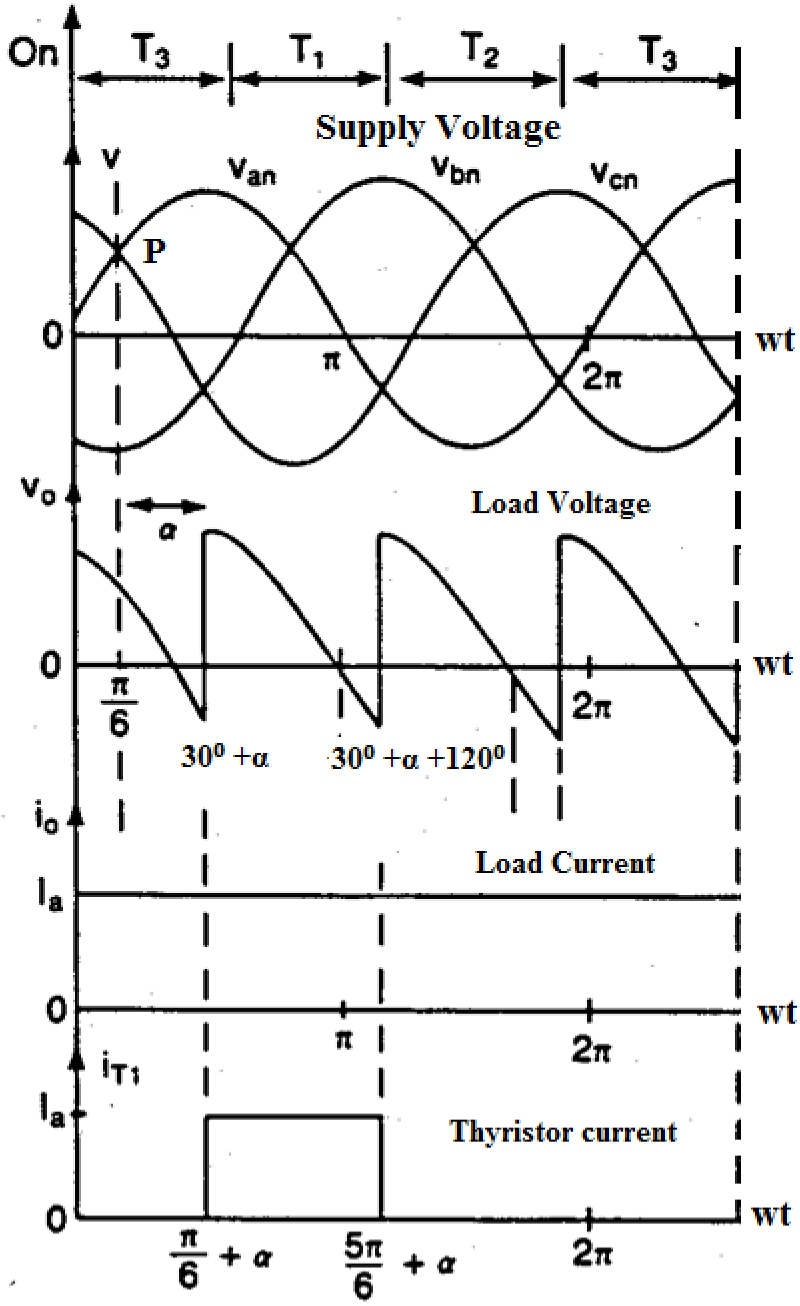



Fig.7.3 Waveforms.

The average value of the output voltage $V_{dc}$ can be found as:

Let $v_{an} = V_m \sin \omega t$

$V_{bn} = V_m \sin(\omega t - 2\pi/3)$

$V_{cn} = V_m \sin(\omega t - 4\pi/3)$

The average value of the load voltage wave is

$$V_{dc} = \frac{1}{\frac{2\pi}{3}} \int_{30°+\alpha}^{30°+\alpha+120°} V_m \ \sin\omega t \ d\omega t = \frac{3V_m}{2\pi} \left[-\cos\omega t\right]_{30° + \alpha}^{150° + \alpha}$$

$$= \frac{3V_m}{2\pi}\left[-(\cos(150° + \alpha) - \cos(30° + \alpha)\right] = \frac{3V_m}{2\pi}\left[-\left(-\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}\right) \cos\alpha\right]$$

$$= \frac{3\sqrt{3}V_m}{2\pi} \cos\alpha$$

The load current $I_{dc}$ is:

$$I_{dc} = \frac{3\sqrt{3}.Vm}{2\pi R} \cos\alpha$$

The operation of the 3 – phase half – wave rectifier with different values of α is illustrated in Fig.7.4.It can be seen that this converter can operate either as a rectifier or as an inverter as

For $0° < \alpha < 90°$ $\longrightarrow$ Rectifier

$90° < \alpha < 180°$ $\longrightarrow$ Inversion

Notes:

1. The mean output voltage is zero for $\alpha = \pi/2$. The converter is idle (no output)
2. Negative average output voltage occurs when $\alpha > \pi/2$.
3. Power inversion is possible, if a load with an e.m.f. to assist the current flow.

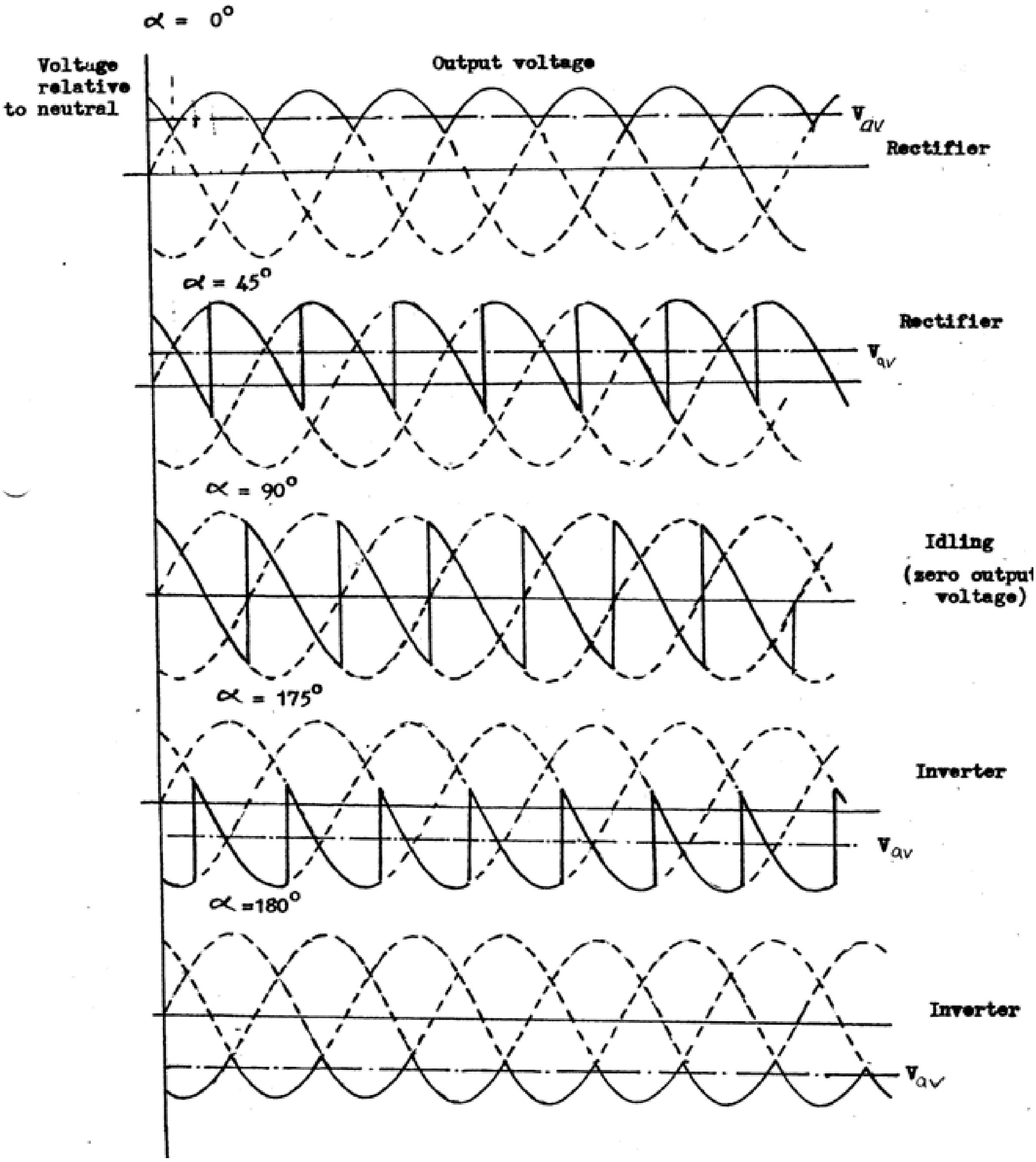



Fig.7.4 Output voltage waveform of the 3-phase half-wave rectifier for different values of firing angle α. Case of R-L load.

Example: The load in Fig.1 consists of a resistance and a very large inductance. The inductance is so large that the output current $I_L$ can be assumed to be continuous and ripple-free. For $\alpha = 60°$:

(a) Draw the waveforms of $v_L$ and $I_L$.

(b) Determine the average value of the output voltage, if phase voltage $V_{an} = 120V$.

Solution:

(a) The waveforms of $v_L$ and $I_L$ are as shown in Fig.3

(b) $$V_{av} = \frac{3\sqrt{3}\,V_m}{2\pi}\cos\alpha$$

$$\alpha = 60°$$

$$V_m = \sqrt{2}\,V_{an} = \sqrt{2} \times 120 \text{ V}$$

$$\therefore V_{av} = \frac{3\sqrt{6}\,V_{an}}{2\pi}\cos 60$$

$$= \frac{3\sqrt{6} \times 120}{2\pi} \times \frac{1}{2} = \underline{\underline{70.2\text{ V}}}.$$

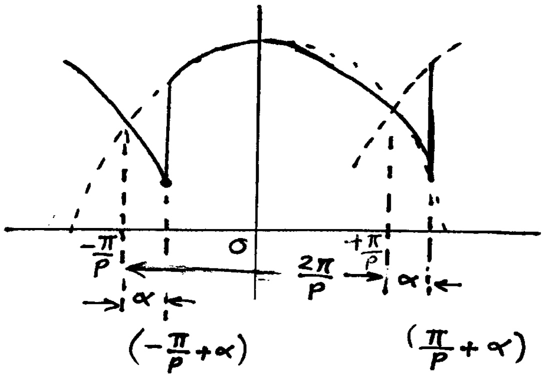
H.W: Find the average output voltage if a freewheel diode is connected across the load.

## General Solution of P-pulse Half-wave Controlled Rectifier

For p-pulse (or p-phase, i.e 3-phase, 6-phase, 12-phase .. etc) half-wave rectifier circuit, a general formula can be obtained as follows:

It has been found usefull for calculation to express the a.c. voltages on the thyristor side by cosine functions to avoide the mistake in the polarity sign. Hence:

$$v_{an} = V_m \cos(\omega t + 120^\circ)$$
$$v_{bn} = V_m \cos\omega t$$
$$v_{cn} = V_m \cos(\omega t - 120^\circ)$$




$$V_{dc} = \frac{1}{\frac{2\pi}{p}} \int_{(-\frac{\pi}{p}+\alpha)}^{(\frac{\pi}{p}+\alpha)} V_m \cos\omega t \, d\omega t$$

$$= V_m \left(\frac{p}{2\pi}\right) \left\{ \sin\left(\frac{\pi}{p}+\alpha\right) - \sin\left(-\frac{\pi}{p}+\alpha\right) \right\}$$

$$= V_m \left(\frac{p}{2\pi}\right) \left[ \sin\frac{\pi}{p}\cos\alpha + \cancel{\sin\alpha\cos\frac{\pi}{p}} - \sin\left(-\frac{\pi}{p}\right)\cos\alpha - \cancel{\cos\left(-\frac{\pi}{p}\right)\sin\alpha} \right]$$

$$= \frac{p V_m}{\cancel{2}\pi} \left[ \cancel{2} \sin\frac{\pi}{p} \cos\alpha \right]$$

$$V_{dc} = \frac{p}{\pi} V_m \sin\frac{\pi}{p} \cos\alpha$$

8/30

Or

$$V_{dc} = \frac{V_m}{2\pi / P} \int_{-\frac{\pi}{P}+\alpha}^{\frac{\pi}{P}+\alpha} \cos \omega t \, d\omega t = V_m \frac{\sin \frac{\pi}{P}}{\frac{\pi}{P}} \cos \alpha$$

For a 3-phase, half-wave circuit, p= 3, hence

$$V_{dc} = V_m \frac{\sin \frac{\pi}{3}}{\pi / 3} \cos \alpha = \frac{V_m \frac{\sqrt{3}}{2}}{\pi / 3} \cos \alpha = \frac{3\sqrt{3}\, V_m}{2\pi} \cos \alpha$$

The diode conduction angle is $\frac{2\pi}{3} - \alpha$ .

## 2. Three- phase full – wave fully - controlled rectifier ( *p = 6*)

The circuit configuration of the three- phase full – wave controlled rectifier is shown in Fig.7.5. In this circuit, the thyristor which has the most positive voltage at its anode conducts when triggered, and the thyristor with the most negative voltage at its cathode returns the load current, if triggered. The waveforms are shown in Fig. 7.6.

- Commutation of the load current from one thyristor to the next occurs at the firing instant, when the incoming thyristor reverse biases the previously conducting thyristor.
- The output dc voltage waveform is determined by the difference of potentials of the positive and negative rails.

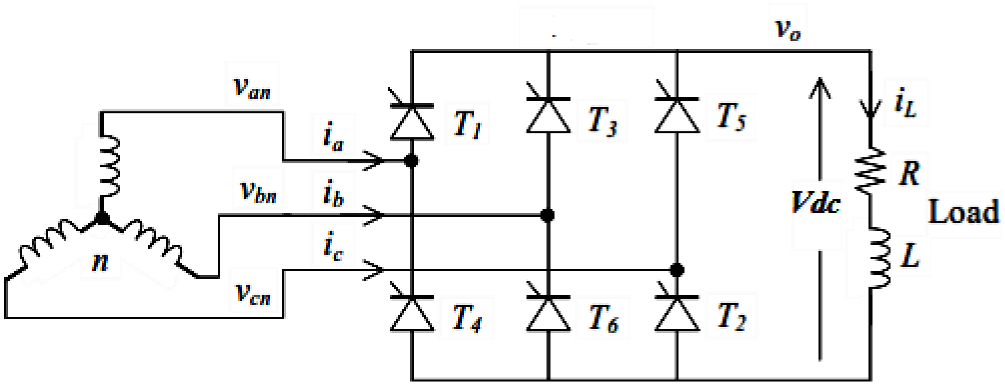



Fig.7.5 Three-phase, fully- controlled bridge converter circuit.

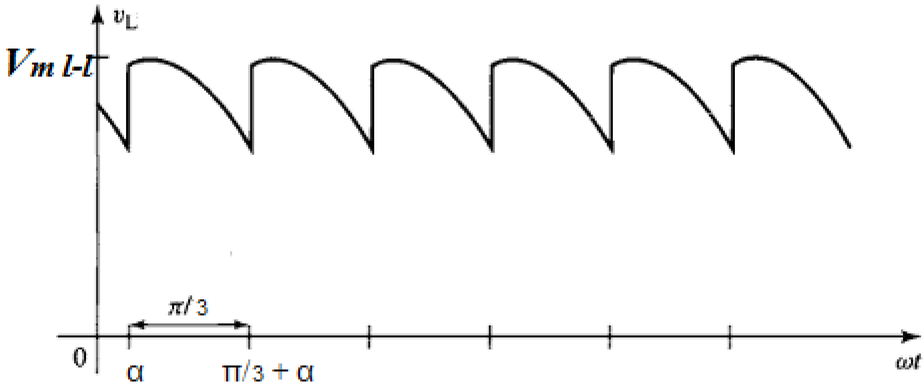



Fig. 7.6 Waveforms

Assuming continuous conduction, the average dc output voltage can be evaluated from the general p-phase formula:

$$V_{dc} = \frac{V_{ml-l}}{2\pi / P}\int_{-\frac{\pi}{P}+\alpha}^{\frac{\pi}{P}+\alpha} \cos \omega t \, d\omega t = V_{ml-l}\,\frac{\sin\frac{\pi}{P}}{\frac{\pi}{P}}\cos\alpha$$

Here p = 6 , $V_{m\,l\text{-}l} = \sqrt{3}V_m$ where $V_{m\,l\text{-}l}$ = maximum line – to –line voltage,

$V_m$ = maximum line –to-neutral voltage. Hence

$$V_{dc} = \sqrt{3}\,\frac{V_m}{2\pi / 6}\int_{-\frac{\pi}{6}+\alpha}^{\frac{\pi}{6}+\alpha} \cos \omega t \, d\omega t = \sqrt{3}V_m\,\frac{\sin\frac{\pi}{6}}{\frac{\pi}{6}}\cos\alpha$$

$$V_{dc} = \frac{3\sqrt{3}}{\pi}V_m \cos\alpha$$

This converter operates in quadrants 1 and 4, developing both positive and negative polarity dc output voltage. For firing angles, $0^o \le \alpha \le 90^o$ the converter operates in quadrant 1 (giving positive output power, i.e., rectifier operation) and for $90^o \le \alpha \le 180^o$ , the operation is in quadrant 4 (giving negative output power, i.e., inverter operation). Operation in quadrant 4 is of course possible only when the load includes an active dc source, able to source power into the ac supply circuit. See Fig.7.7.

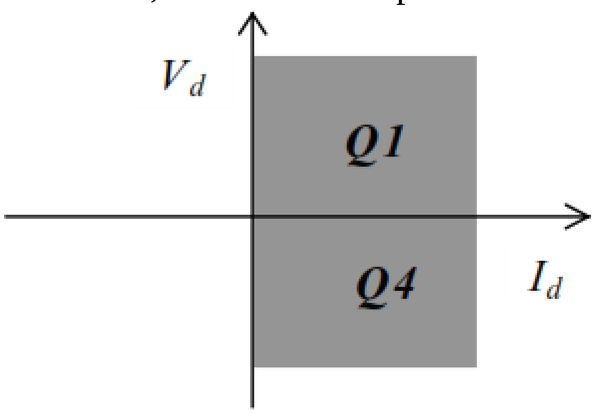



Fig. 7.7 Operating quadrants of the 3-phase Fully-C controlled converter

## 3. Three- phase full – wave , Half -controlled rectifier

This converter is shown in Fig.7.8. It consists of three thyristors and three diodes with freewheeling diode across the load. It gives positive voltage and positive current only (not regenerative converter) i.e, it operates in the first quadrant only (Fig.7.9).

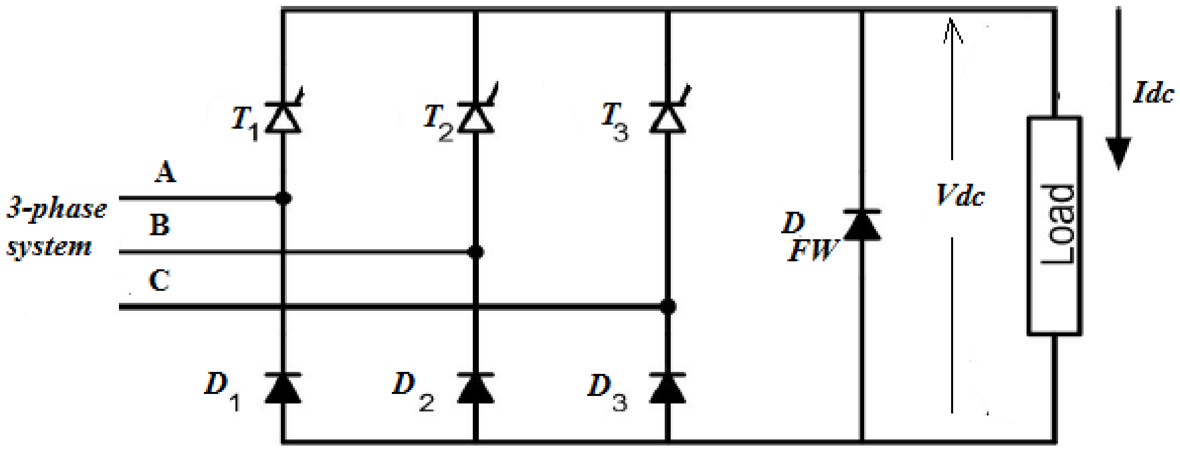



Fig.7.8

The output voltage is given by:

$$V_{dc} = \frac{3\sqrt{3}}{2\pi} V_m (1 + \cos\alpha)$$

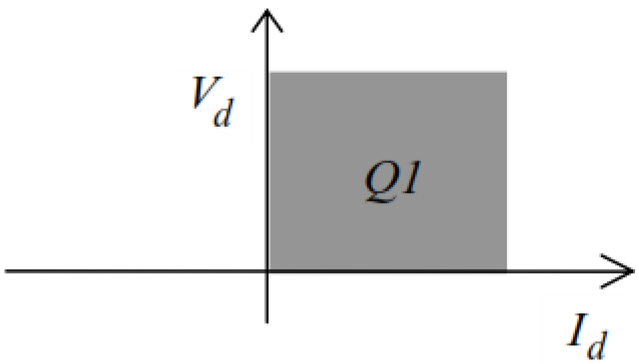



Fig.7.9

2

Example 1: A 3-phase Full-wave Fully controlled rectifier supply a highly inductive load with $R = 10\,\Omega$. The supply is a 3-phase Y-connected with 208 r.m.s.

Calculate :

(a) The load current when the firing angle $\alpha = 40^\circ$.

(b) The power drawn from the supply.

(c) If the current value kept at (a) and $\alpha$ changed to $135^\circ$, calculate the power returned to the supply.

Solution

(a) For $\alpha = 40^\circ$

$$V_m = \frac{\sqrt{2} \times 208}{\sqrt{3}} = 169.83 \text{ V} \qquad \text{(per phase)}$$

$$V_{dc} = \frac{3\sqrt{3}\,V_m}{\pi}\cos\alpha = \frac{3 \times \sqrt{3} \times 169.83}{\pi}\cos 40^\circ = 215.18 \text{ V}$$

$$I_{dc} = \frac{V_{dc}}{R} = \frac{215.18}{10} = 21.518 \text{ A}.$$

(b) The power drawn from the source = the power dissipated at the resistance of the load

$$P_s = P_{Load} = V_{dc}\,I_{dc} = 215.18 \times 21.518 = 4630.25 \text{ W}$$

(c) For $\alpha = 135^\circ$ , $i_o = I_{dc} = 21.518$ A

$$V'_{dc} = \frac{3\sqrt{3}\,V_m}{\pi}\cos 135^\circ = \frac{3\sqrt{3} \times 169.83}{\pi}\cos(135)$$

$$= -198.625 .$$

$\therefore$ power return to the source :

$$P_s = V'_{dc}\,I_{dc} = -198.625 \times 21.518$$
$$= -3945.189 \text{ W}.$$

3

Example 2 : If the converter in example 1 is replaced by Full-wave half-controlled converter, Calculate :

(a) $V_{dc}$ when $\alpha = 45°$
(b) $V_{dc}$ when $\alpha = 75°$
(c) $V_{dc}$ when $\alpha = 135°$
(d) Maximum value of $V_{dc}$.
(e) The value of $\alpha$ to obtain $I_{dc} = 6A$.

Solution :

(a)

$$V_{dc} = \frac{3\sqrt{3}V_m}{2\pi}(1+\cos\alpha)$$

For $\alpha = 45°$

$$V_{dc} = \frac{3\sqrt{3} \times 169.83}{2\pi}(1+\cos 45°) = 239.76\ V.$$

(b) For $\alpha = 75°$

$$V_{dc} = \frac{3\sqrt{3} \times 169.83}{2\pi}(1+\cos 75°) = 176.8\ V.$$

(c) For $\alpha = 135°$

$$V_{dc} = \frac{3\sqrt{3} \times 169.83}{2\pi}(1+\cos 135°) = 41.36V.$$

(d) Max. Voltage output is when $\alpha = 0°$ :

$$V_{dc} = \frac{3\sqrt{3} \times 169.83}{2\pi}(1+\cos 0°) = 280.8\ V.$$

(e) $I_{dc} = \frac{V_{dc}}{R} = 6\ A$ or $V_{dc} = 6R = 6 \times 10 = 60V.$

$$60 = \frac{3\sqrt{3} \times 169.83}{2\pi}(1+\cos\alpha)$$

From which $\underline{\alpha = 52.9°}$.