عنوان المحاضرة:

# Derivative Rule, Algebraic & Trigonometric Derivatives, Chain Rule, Velo city and Acceleration قاعدة المشتقة، مشتقات جبرية ودوال مثلثية، قاعدة السلسلة، السرعة والتعجيل

## Derivative Rules:- قواعد المشتقة

what is a derivative? ماهي المشتقة؟

The derivative is finding a slope at any point.
المشتقة هي ايجاد الميل في أي نقطة.

Before we go over the derivative Rules, lets introduce the definition of the derivative formula.

### Definition of the Derivative Formula:-

By using the limit process as

$$\frac{df}{dx} = f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} \quad \text{---} \circledast$$

Ex①/ Find $\frac{df}{dx}$ of the following equation by using the definition of the derivative

$f(x) = 5x - 2$

Sol.

- $f(x+\Delta x) = 5(x+\Delta x) - 2$
- $f(x) = 5x - 2$

plug the above two eqns into eqn $\circledast$

$$\frac{df}{dx} = f'(x) = \lim_{\Delta x \to 0} \frac{5(x+\Delta x) - 2 - (5x-2)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{5x + 5\Delta x - 2 - 5x + 2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{5\Delta x}{\Delta x} = \lim_{\Delta x \to 0} 5 = \boxed{5}$$

**Ex ②** By using the definition of the derivative find $\frac{df}{dx}$ for the following eqs, $F(x) = x^2$

**Sol.**

$$\frac{df}{dx} = f'(x) = \lim_{\Delta x \to 0} \frac{F(x+\Delta x) - F(x)}{\Delta x}$$

$* \; F(x+\Delta x) = (x+\Delta x)^2$

$F(x) = x^2$

$$\therefore \frac{df}{dx} = \lim_{\Delta x \to 0} \frac{(x+\Delta x)^2 - x^2}{\Delta x} = \lim_{\Delta x \to 0} \frac{(x+\Delta x)(x+\Delta x) - x^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{x^2 + \Delta x \cdot x + \Delta x \cdot x + \Delta x^2 - x^2}{\Delta x} = \lim_{\Delta x \to 0} \frac{2x\Delta x + \Delta x^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\Delta x(2x + \Delta x)}{\Delta x} = \lim_{\Delta x \to 0} (2x + \Delta x)$$

$$\frac{df}{dx} = 2x + 0 = \boxed{2x}$$

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By your own exp. try to find $\frac{df}{dx}$ for the following eqs using the def.

1- $F(x) = \frac{1}{x}$

2- $F(x) = \frac{1}{\sqrt{x}}$

3- $F(x) = \frac{5}{\sqrt{x}}$

4- $F(x) = x^2 - 2x + 4$

# The Derivative Rules :- قواعد المشتقة

## (1) Constant derivative مشتقة الثابت

$$F(x) = a \Longrightarrow \frac{df}{dx} = f'(x) = zero \quad , \; a = constant$$

## (2) variable derivative مشتقة المتغير

$$f(x) = x^n \Longrightarrow \frac{df}{dx} = f'(x) = n\, x^{n-1} \quad , \; n = \text{any no.}$$

## (3) Multi-variable Funs مشتقة الدوال كثيرة الحدود

$$f(x) = h(x) \mp g(x) \Longrightarrow \frac{df}{dx} = f'(x) = h'(x) \mp g'(x)$$

## (4) Quotient Funs مشتقة قسمة دالتين

$$f(x) = \frac{h(x)}{g(x)} \Longrightarrow \frac{df}{dx} = f'(x) = \frac{g(x) \cdot h'(x) - h(x) \cdot g'(x)}{(g(x))^2}$$

## (5) Product Funs مشتقة ضرب دالتين

$$F(x) = h(x) \cdot g(x) \Longrightarrow \frac{df}{dx} = f'(x) = h(x) \cdot g'(x) + g(x) \cdot h'(x)$$

## (6) Power raised Funs: مشتقة دالة مرفوعة لأس

$$f(x) = [h(x)]^n \Longrightarrow \frac{df}{dx} = f'(x) = n\,[h(x)]^{n-1} \cdot h'(x)$$

Examples③:-

1- $F(x) = 4 \Rightarrow F'(x) = \text{zero}$

2- $F(x) = x \Rightarrow F'(x) = 1$

3- $F(x) = x^4 \Rightarrow F'(x) = 4x^3$

4- $F(x) = 5x^3 \Rightarrow F'(x) = 5*3x^2 = 15x^2$

5- $F(x) = x^{-3} \Rightarrow F'(x) = -3x^{-3-1} = -3x^{-4} = \frac{-3}{x^4}$

6- $F(x) = \sqrt{x} \Rightarrow F(x) = x^{\frac{1}{2}} \Rightarrow F'(x) = \frac{1}{2}x^{\frac{1}{2}-1} = \frac{1}{2\sqrt{x}}$

7- $F(x) = \sqrt[5]{x^2} \Rightarrow F(x) = x^{\frac{2}{5}} \Rightarrow F'(x) = \frac{2}{5}x^{\frac{2}{5}-1} = \frac{2}{5}x^{-\frac{3}{5}} = \frac{2}{5\sqrt[5]{x^3}}$

8- $F(x) = 3x^5 + 7x \Rightarrow F'(x) = 3*5x^{5-1} + 7 = 15x^4 + 7$

9- $F(x) = (x^4 - x^2 + 1)(5x^6 - 3x) \Rightarrow F'(x) = (x^4 - x^2 + 1)(30x^5 - 3) + (5x^6 - 3x)(4x^3 - 2x)$

10- $F(x) = \frac{x^3+1}{x^4+1} \Rightarrow F'(x) = \frac{(x^4+1)(3x^2) - (x^3+1)(4x^3)}{(x^4+1)^2}$

11- $F(x) = (x^3 + x^2 + x + 1)^5 \Rightarrow F'(x) = 5(x^3 + x^2 + x + 1)^4 * (3x^2 + 2x + 1)$

12- $F(x) = \sqrt{x^2 - 2x + 1} \Rightarrow F'(x) = \frac{2x - 2}{2\sqrt{x^2 - 2x + 1}}$

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EX ④/ Find the derivative of the quotient Fn at $x = 1$, $F(x) = \frac{x^3+1}{x^4+1}$

Sol/

From Ex3 # 10 $\Rightarrow F'(x)\Big|_{x=1} = \frac{(1^4+1)(3*1^2) - (1^3+1)(4*1^3)}{(1^4+1)^2}$

$= \frac{2*3 - 2*4}{2^2} = \frac{6-8}{4}$

$= \boxed{\frac{-1}{2}}$

# Trigonometric Derivatives مشتقات الدوال المثلثية

1- $F(x) = \sin x \implies F'(x) = \cos x$

2- $F(x) = \cos x \implies F'(x) = -\sin x$

3- $F(x) = \tan x \implies F'(x) = \sec^2 x$

4- $F(x) = \cot x \implies F'(x) = -\csc^2 x$

5- $F(x) = \sec x \implies F'(x) = \sec x \tan x$

6- $F(x) = \csc x \implies F'(x) = -\csc x \cot x$

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**Ex (5)** Find the derivative of the eqs

$F(x) = 5\sin x - 4\tan x$

Sol.

$F'(x) = \frac{dF}{dx} = 5\cos x - 4\sec^2 x$

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**Ex (6)** Find $\frac{d}{dx}[8\sec x - 5\cos x]$

Sol.

$F'(x) = 8\sec x \tan x - 5(-\sin x)$

$F'(x) = 8\sec x \tan x + 5\sin x$

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**Ex (7)** Find $\frac{d}{dx}[2\cot x - 7\csc x]$

Sol.

$F'(x) = 2(-\csc^2 x) - 7(-\csc x \cot x)$

$F'(x) = -2\csc^2 x + 7\csc x \cot x$

## Chain Rule قاعدة السلسلة

- If $Z = F(x)$ & $x = g(u)$, then

$$\frac{dz}{du} = \frac{dz}{dx} * \frac{dx}{du}$$

- And If $z = f(x)$ & $x = g(u)$ & $u = h(w)$ then,

$$\frac{dz}{dw} = \frac{dz}{dx} * \frac{dx}{du} * \frac{du}{dw}$$

So, the chain rule is using whenever we have a nested (متداخلة) Functions, i.e one Function inside of another function

تستخدم قاعدة السلسلة عندما يكون لدينا دوال متداخلة، يعني يكون لدينا دالة داخل دالة اخرى.

Ex(8) $y = f(g(x)) \Longrightarrow y'(x) = f'(g(x)) * g'(x)$

Ex(9) $y = f(g(h(x))) \Longrightarrow y'(x) = f'(g(h(x))) * g'(h(x)) * h'(x)$

And So Forth ...

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Ex(10) Derive $y = (\sin(x^3+6))^2$

Sol.)

$$y'(x) = 2(\sin(x^3+6)) * \cos(x^3+6) * 3x^2$$

هنا تم اشتقاق القوس ومن ثم ما داخل القوس وهو دالة الجيب وبعدها الزاوية تحت دالة الجيب وبهذا نكون قد [illegible] دوال متداخلة باستخدام قاعدة السلسلة.

## Derivative Applications تطبيقات المشتقات

IF the time is denoted by $t$, and $s(t)$ is a location or a displacement Function (دالة ازاحة او موقع)
Then,

- Velocity $= V(t) = s'(t)$ (السرعة)
- Acceleration $= a(t) = V'(t)$ (التعجيل)

Velocity is gonna have a sign associated with it. either positive or nagative, i.e either moving to the right or to the left or it maybe moving away or back in

السرعة يكون لها اشارة واضحة، يعني اما موجبة أو سالبة، بمعنى موجبة اذا كان الجسم المتحرك يسير الى جهة اليمين وسالبة اذا عكس ذلك أو موجبة اذا كان الجسم يتحرك مبتعداً وسالبة اذا عكس ذلك.

We have anothe term, named by speed, which is always positive, so we need to take the absolute value velocity to get the speed

$$speed = |V(t)|$$

المصطلح الثاني الذي نود ذكره هو speed، والذي يكون موجباً دائماً ويكون مساوياً الى مطلق الـ Velocity.

**Ex)** The following equation of motion descripes the displacement (in meter) of a particle moving in a straight line

$$S = 5t^3 + 3t + 8$$

wher $t$ is measured in seconds.

a- find the velocity after $t=2$ seconds?

b- find the acceleration after $t=2$ seconds?

**Solution**

a- $V(t) = S'(t) = 5(3t^2) + 3$

$= 15t^2 + 3$

after 2 seconds $\Rightarrow t=2$

$V(2) = 15(2)^2 + 3 = 15 \times 4 + 3$

$V(2) = \boxed{63 \ m/sec}$ Ans (a)

b- $a(t) = V'(t) = S''(t) = 15(2t)$

$= 30t$

$\therefore a(2) = 30(2) = \boxed{60 \ m/sec^2}$ Ans (b)

**نهاية محاضرة " Derivative Rule, Algebraic & Trigonometric Derivatives, Chain Rule, Velo city and Acceleration قاعدة المشتقة، مشتقات جبرية ودوال مثلثية، قاعدة السلسلة، السرعة والتعجيل"**