Example (5.1) Steam at a velocity of 500 m/s enters an impulse wheel having a nozzle angle (α1) of 25°. The exit angle of the moving blade (β2) is 20°. The relative velocity of the steam may assume constant (*Vr1=Vr2*) over the moving blades. If the blade speed is 200 m/s, find the moving blade angle at inlet (β1), the exit velocity and direction of steam (*V2*) and the work done/kg of steam. If the turbine is supplied with 2 kg/s of steam, find the axial thrust.

Solution:

From velocity triangle [figure(7.6)]:

$V_{W1} = V_1 \times \cos\alpha 1 = 500\cos(25) = V_{r1} \times \cos\beta 1 + V_b$

$V_{r1} \times \cos\beta 1 = 253$ …. (a)

$V_1 \times \sin\alpha 1 = V_{r1} \times \sin\beta 1 = 211.3$ …. (b)

$\beta 1 = \tan^{-1}\left[\frac{211.3}{253}\right] \rightarrow \beta 1 = 40^o$ substitute into (a).

$V_{r1} = 330\ m/s$

$V_{r2} = V_{r1} = 330\ m/s$

$V_{r2} \times \cos\beta 2 - V_b = V_2 \times \cos\alpha 2$ …. (c)

$V_{r2} \times \sin\beta 2 = V_2 \times \sin\alpha 2$ …. (d)

$\alpha 2 = \tan^{-1}\left[\frac{V_{r2}\times\sin\beta 2}{V_{r2}\cos\beta 2 - V_b}\right] \rightarrow \alpha 2 = 45.71^o$

From equation (d):

$V_2 = \frac{V_{r2}\times\sin\beta 2}{\sin\alpha 2} \rightarrow V_2 = 157.7\ m/s$

$\Delta V_w = (V_1 \times \cos\alpha 1 + V_2 \times \cos\alpha 2) = 500 \times \cos(25) + 157.7 \times \cos(45.71)$

$\Delta V_w = 563.27\ m/s$

$work = V_b \times \Delta V_w = 200 \times 563.27 \rightarrow work = 112.655\ kj$

$power = m^o \times Work = m^o \times (V_b \times \Delta V_w) = 2 \times 112.655 = 225.3\ kW$

$$\Delta V_f = \left(V_{f1} - V_{2f}\right) = (V_{r1} \times \sin\alpha 1 - V_{r2} \times \sin\alpha 2)$$

$$\Delta V_f = 500 \sin 25 - 157.7 \sin 45.71 = 98.43\ m/s$$

$$thrust = m^o \times \Delta V_f = 2 \times 98.43 \rightarrow thrust = 197\ kN$$

Example (5.2) The velocity of the steam leaving the nozzles of an impulse turbine is 900 m/s and the nozzle angle is 20^o. The blade velocity is 300 m/s and the blade velocity coefficient is 0.7. Calculate for a mass of 1 kg/s, and symmetrical blading: (1) the blade inlet angle. (2) The driving force on the wheel. (3) The axial thrust. (4) The diagram power. (5) The diagram efficiency. (6) If the isentropic enthalpy drop across the nozzle was 475 kj/kg, also calculate the stage efficiency of the impulse turbine.

Solution:

Given: V_1=900 m/s, α1=20^o, V_b=300m/s, K=0.7, m^o=1 kg/s, β1=β2

From velocity triangle

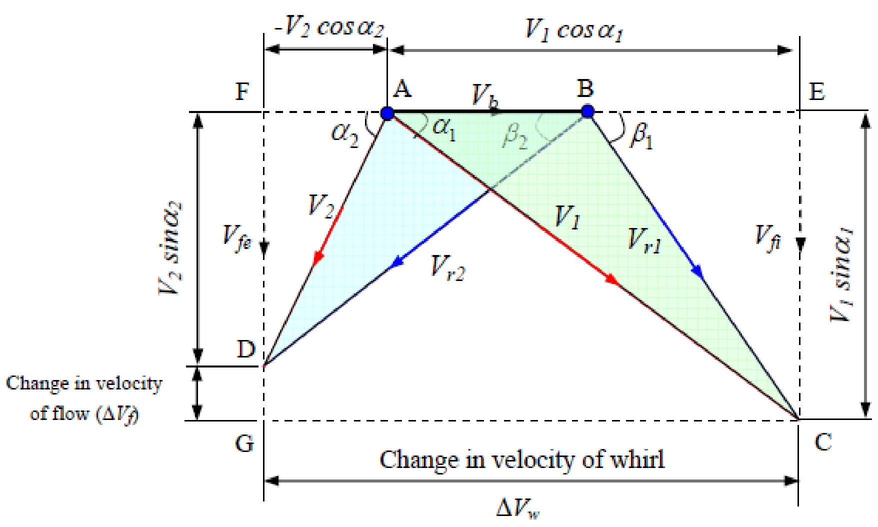
$$V_1 \times \cos \alpha 1 - V_b = V_{r1} \times \cos \beta 1$$

$$V_1 \times \sin \alpha 1 = V_{r1} \times \sin \beta 1$$

$$\beta 1 = \tan^{-1}\left[\frac{V_1 \times \sin \alpha 1}{V_1 \times \cos \alpha 1 - V_b}\right] = \tan^{-1}\left[\frac{900 \times \sin 20}{900 \times \cos 20 - 300}\right]$$

$$\beta 1 = 29.5^o$$

$$\beta 1 = \beta 2 = 29.5^o$$



Combined Inlet and Outlet Velocity Diagram for Impulse Turbine

$$V_{r1} = \frac{V_1 \times \cos \alpha 1 - V_b}{\cos \beta 1} = \frac{900 \times \cos 20 - 300}{\cos 29.5} = 627\ m/s$$

$$K = \frac{V_{r2}}{V_{r1}} \rightarrow V_{r2} = K \times V_{r1} = 0.7 \times 627 = 439\ m/s$$

$$V_{r2} \times \sin\beta 2 = V_2 \times \sin\alpha 2 \qquad \text{.... (a)}$$

$$V_{r2} \times \cos\beta 2 - V_b = V_2 \times \cos\alpha 2 \qquad \text{.... (b)}$$

$$\tan\alpha 2 = \frac{V_2 \times \sin\alpha 1}{V_2 \times \cos\alpha 2} = \left[\frac{V_{r2} \times \sin\beta 2}{V_{r2} \times \cos\beta 2 - V_b}\right]$$

$$\alpha 2 = \tan^{-1}\left[\frac{V_{r2} \times \sin\beta 2}{V_{r2} \times \cos\beta 2 - V_b}\right] = \tan^{-1}\left[\frac{439 \times \sin 29.5}{439 \times \cos 29.5 - 300}\right]$$

$$\alpha 2 = 69.4^o$$

From equation (a);

$$V_2 = \frac{V_{r2} \times \sin\beta 2}{\sin\alpha 1} = \frac{439 \times \sin 29.5}{\sin 69.4} = 231\ m/s$$

$$\Delta V_w = V_{w1} + V_{w2}$$

$$\Delta V_w = (V_1 \times \cos\alpha 1 + V_2 \times \cos\alpha 2)$$

$$\Delta V_w = 900 \times \cos(20) + 231 \times \cos(69.4)$$

$$\Delta V_w = 927\ m/s$$

$$Driving\ Force = m^o \times \Delta V_w = 1 \times 927$$

$$Driving\ Force = 927\ N$$

$$\Delta V_f = V_{f1} - V_{f2} = V_1 \times \sin\alpha 1 - V_2 \times \sin\alpha 2$$

$$\Delta V_f = 900 \times \sin 20 - 231 \times \sin 69.4$$

$$\Delta V_f = 92\ m/s$$

$$thrust = m^o \times \Delta V_f = 1 \times 92$$

$$thrust = 92\ N$$

$$power = m^o \times V_b \times \Delta V_w = 1 \times 300 \times 927$$

$$power = 278.1\ kW$$

$$\eta_{blade} = \frac{2V_b \times \Delta V_w}{V_1^2} = \frac{2 \times 300 \times 927}{900^2}$$

$$\eta_{blade} = 68.7\%$$

$$\eta_{stage} = \frac{V_b \times \Delta V_w}{h_0 - h_1} = \frac{300 \times 927}{475 \times 10^3}$$

$$\eta_{stage} = 58.54\%$$