COLLEGE OF ENGINEERING AND TECHNOLOGIES

ALMUSTAQBAL UNIVERSITY

Digital Signal Processing (DSP)

CTE 306

Lecture 16

- Infinite Impulse Response (IIR) Filters -

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Infinite Impulse Response (IIR) Filters

- IIR filters are recursive filters.
- Difference equation for IIR filters

$$y[n] = -\sum_{K=1}^{N} a_k\, y[n-k] + \sum_{K=0}^{M} b_k\, x[n-k]$$

a_k and b_k are the filter coefficients

Infinite Impulse Response (IIR) Filters

- IIR filters outputs depends on N past outputs and M past inputs.
- The impulse response samples getting smaller and smaller but they never settle to zero.

Example

Determine the first four samples in the impulse response for the IIR filter.

$$y\,[n] - 0.2\,y[n-1] = x[n] + x[n-1]$$

Sol:

Substituting δ[n] for $x[n]$ and h[n] for $y[n]$.

$$h\,[n] - 0.2\,h\,[n-1] = \delta\,[n] + \delta\,[n-1]$$

$$h\,[n] = 0.2\,h\,[n-1\,] + \delta\,[n] + \delta\,[n-1]$$

Solution

$$h[0] = 0.2\,h\,[0 - 1] + \delta\,[0] + \delta\,[0 - 1]$$

$$h[0] = 0.2\,h\,[-1] + \delta\,[0] + \delta\,[-1]$$

$$= 0.0 + 1.0 + 0.0 = 1.0$$

$$h[1] = 0.2\,h\,[1 - 1] + \delta\,[1] + \delta\,[1 - 1]$$

$$= 0.2\,(1) + 0.0 + 1.0 = 1.2$$

Solution

$$h[2] = 0.2\, h[2-1] + \delta[2] + \delta[2-1]$$

$$= 0.2\,(1.2) + 0.0 + 0.0 = 0.24$$

$$h[3] = 0.2\, h[3-1] + \delta[3] + \delta[3-1]$$

$$= 0.2\,(0.24) + 0.0 + 0.0 = 0.048$$

Example

Determine the first six samples in the impulse response for the IIR filter.

$$y[n] - 0.4\,y[n-1] = x[n] - x[n-1]$$

Sol:

Substituting δ[n] for $x[n]$ and h[n] for $y[n]$.

$$h[n] - 0.4\,h[n-1] = \delta[n] - \delta[n-1]$$

$$h[n] = 0.4\,h[n-1] + \delta[n] - \delta[n-1]$$

Solution

$$h\,[0] = 0.4h\,[-1] + \delta\,[0] - \delta\,[n-1]$$

$$= 0.4\;(0.0) + 1.0 - 0.0 = 1.0$$

$$h\,[1] = 0.4h\,[0] + \delta\,[1] - \delta\,[0]$$

$$= 0.4\,(1.0) + 0.0 - 1.0 = -\,0.6$$

$$h\,[2] = 0.4h\,[1] + \delta\,[2] - \delta\,[1]$$

$$= 0.4\,(-0.6) + 0.0 - 0.0 = -\,0.24$$

Solution

$$h[3] = 0.4h[2] + \delta[3] - \delta[2]$$

$$= 0.4(-0.24) + 0.0 - 0.0 = -0.096$$

$$h[4] = 0.4h[3] + \delta[4] - \delta[3]$$

$$= 0.4(-0.96) + 0.0 - 0.0 = -0.0384$$

$$h[5] = 0.4h[4] + \delta[5] - \delta[4]$$

$$= 0.4(-0.0384) + 0.0 - 0.0 = -0.01536$$

Example

Given the difference equation

$$y(n) = 0.25y(n-1) + x(n) \text{ for } n \geq 0 \text{ and } y(-1) = 0,$$

a) Determine the unit-impulse response $h(n)$.

b) Draw the system block diagram.

c) Write the output using the obtained impulse response.

d) For a step input $x(n) = u(n)$, verify and compare the output responses for the first three output samples using the difference equation

Solution

a) Let $x(n) = \delta(n)$, then

$$h(n) = 0.25h(n-1) + \delta(n).$$

To solve for $h(n)$, we evaluate

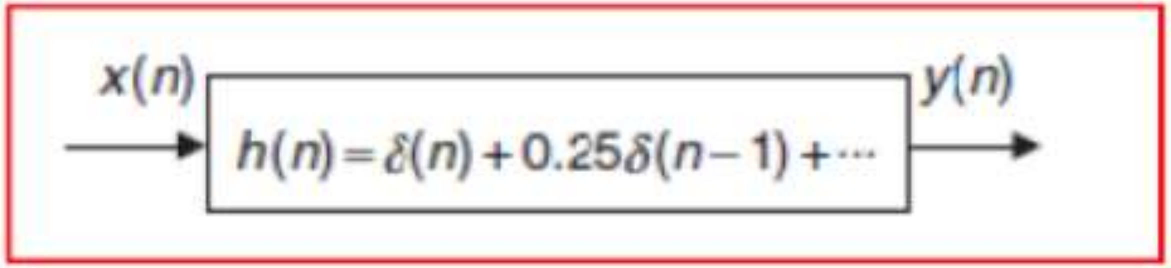
$$h(0) = 0.25h(-1) + \delta(0) = 0.25 \times 0 + 1 = 1$$
$$h(1) = 0.25h(0) + \delta(1) = 0.25 \times 1 + 0 = 0.25$$
$$h(2) = 0.25h(1) + \delta(2) = 0.25 \times 0.5 + 0 = 0.0625$$

With the calculated results, we can predict the impulse response as

$$h(n) = (0.25)^n u(n) = \delta(n) + 0.25\delta(n-1) + 0.0625\delta(n-2) + \ldots.$$

Solution

b) The system block diagram is given in Figure below.



c) The output sequence is a sum of infinite terms expressed as

$$
\begin{aligned}
y(n) &= h(0)x(n) + h(1)x(n-1) + h(2)x(n-2) + \ldots \\
&= x(n) + 0.25x(n-1) + 0.0625x(n-2) + \ldots
\end{aligned}
$$

d) From the difference equation and using the zero-initial condition, we have

$$y(n) = 0.25y(n-1) + x(n) \text{ for } n \geq 0 \text{ and } y(-1) = 0$$

$$n = 0, y(0) = 0.25y(-1) + x(0) = u(0) = 1$$

$$n = 1, y(1) = 0.25y(0) + x(1) = 0.25 \times u(0) + u(1) = 1.25$$

$$n = 2, y(2) = 0.25y(1) + x(2) = 0.25 \times 1.25 + u(2) = 1.3125$$

$$\ldots$$

Solution

$$y(n) = x(n) + 0.25x(n-1) + 0.0625x(n-2) + \ldots$$

$$n = 0,\ y(0) = x(0) + 0.25x(-1) + 0.0625x(-2) + \ldots$$
$$= u(0) + 0.25 \times u(-1) + 0.125 \times u(-2) + \ldots = 1$$

$$n = 1,\ y(1) = x(1) + 0.25x(0) + 0.0625x(-1) + \ldots$$
$$= u(1) + 0.25 \times u(0) + 0.125 \times u(-1) + \ldots = 1.25$$

$$n = 2,\ y(2) = x(2) + 0.25x(1) + 0.0625x(0) + \ldots$$
$$= u(2) + 0.25 \times u(1) + 0.0625 \times u(0) + \ldots = 1.3125$$