COLLEGE OF ENGINEERING AND TECHNOLOGIES
ALMUSTAQBAL UNIVERSITY

Digital Signal Processing (DSP)

CTE 306

Lecture 13

- Impulse Response -

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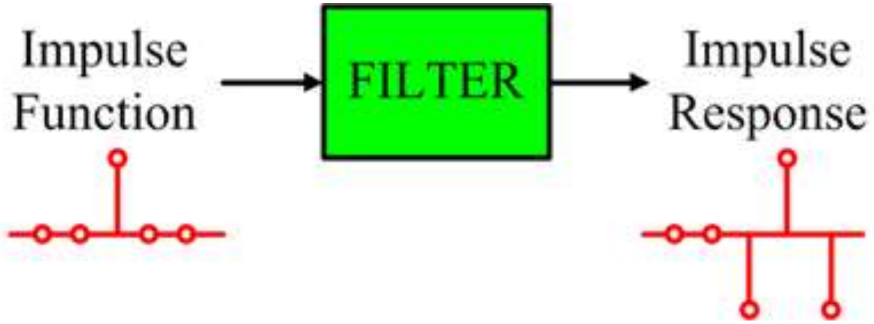
Impulse Response

- An impulse response for a filter is a the response of the filter to an impulse.



- The impulse response for a filter is designated as h[n].

- The impulse response can be calculated from the difference equation by replacing the input x[n] and output of the filter by δ[n] and h[n] respectively.

Transfer Function

- Transfer function, H(z) of digital filters is the ratio of output to input in the z domain.

$$H\,[z] = \frac{Y\,[Z]}{X\,[Z]}$$

Term-by-term transformation of a general difference equation.

$$\sum_{K=0}^{N} a_k\, y[n-k] \; + \; \sum_{K=0}^{M} b_k\, x[n-k]$$

$$H\,[Z] = \frac{\sum_{K=0}^{M} b_k z^{-1}}{\sum_{L=0}^{M} a_k z^{-1}}$$

Example

Determine the transfer function of a digital filter described by the difference equation.

$$2y[n] + y[n-1] + 0.9y[n-2] = x[n-1] + x[n-4]$$

Taking z transforms term by term

$$2Y[z] + z^{-1}Y[z] + 0.9z^{-2}Y[z] = z^{-1}X[z] + z^{-4}X[z]$$

$$H[z] = \frac{z^{-1} + z^{-4}}{2 + z^{-1} + 0.9z^{-2}}$$

Difference equation and impulse responses

A causal, linear, time-invariant system can be described by a difference equation having the following general form:

$$y(n) + a_1 y(n-1) + \ldots + a_N y(n-N) = b_0 x(n) + b_1 x(n-1) + \ldots + b_M x(n-M), \quad (1)$$

where $a_1, \ldots, a_N$ and $b_0, b_1, \ldots, b$ are the coefficients of the difference equation. Equation (1) can further be written as

$$y(n) = -a_1 y(n-1) - \ldots - a_N y(n-N) + b_0 x(n) + b_1 x(n-1) + \ldots + b_M x(n-M) \quad (2)$$

Or

$$y(n) = -\sum_{i=1}^{N} a_i y(n-i) + \sum_{j=0}^{M} b_j x(n-j). \qquad (3)$$

Notice that y(n) is the current output, which depends on the past output samples y(n _ 1), . . . , y(n _ N), the current input sample x(n), and the past input samples, x(n _ 1), . . . , x(n _ N).
We will examine the specific difference equations in the following examples.

Recursive Digital Filters

- Recursive digital filters are filters which rely on both inputs and past outputs.
- Difference equation for recursive digital filters:

$$y[n] = -\sum_{K=1}^{N} a_k \, y[n-k] \quad + \sum_{K=0}^{M} b_k \, x[n-k]$$

a_k and b_k are the filter coefficients

Example

A digital filter has the difference equation:

$$y[n] = 0.5\, y[n-1] + x[n]$$

(a) Determine the type of filter (recursive or Nonrecursive).

(b) Determine the filter coefficients.

Example (Sol)

a) Since the output, $y\,[n]$ depends on both the inputs, x[n] and past output $y[\,n-1]$, the digital filter is recursive.

b) Rewrite the difference equation:

$$y\,[n] - 0.5\,y\,[n-1] = x\,[n]$$

$$a_0 = 1,\ a_1 = -\,0.5,\ and\ b_0 = 1$$

Nonrecursive Digital Filters

- Nonrecursive digital filters are filters which rely only on inputs and not on past outputs
- Difference equation for nonrecursive digital filters:

$$y[n] = \sum_{K=0}^{M} b_k \, x[n-k]$$

b_k *are the filter coefficients*

Example

A digital filter has the difference equation:

$$y[n] = 0.5\, x[n] - 0.3\, x[n-1]$$

(a) Determine type of filter (recursive or Nonrecursive).

(b) Determine the filter coefficients.

Sol:

(a) Since the output, y[n] does not depend on the past output, y[n − k], the digital filter is nonrecursive.

(b) The filter coefficients:

$$a_0 = 1,\ b_0 = 0.5,\ and\ b_1 = -0.3$$

A linear time-invariant system can be completely described by its unit-impulse response, which is defined as the system response due to the impulse input δ(n) with zero initial conditions, depicted in Figure (1).

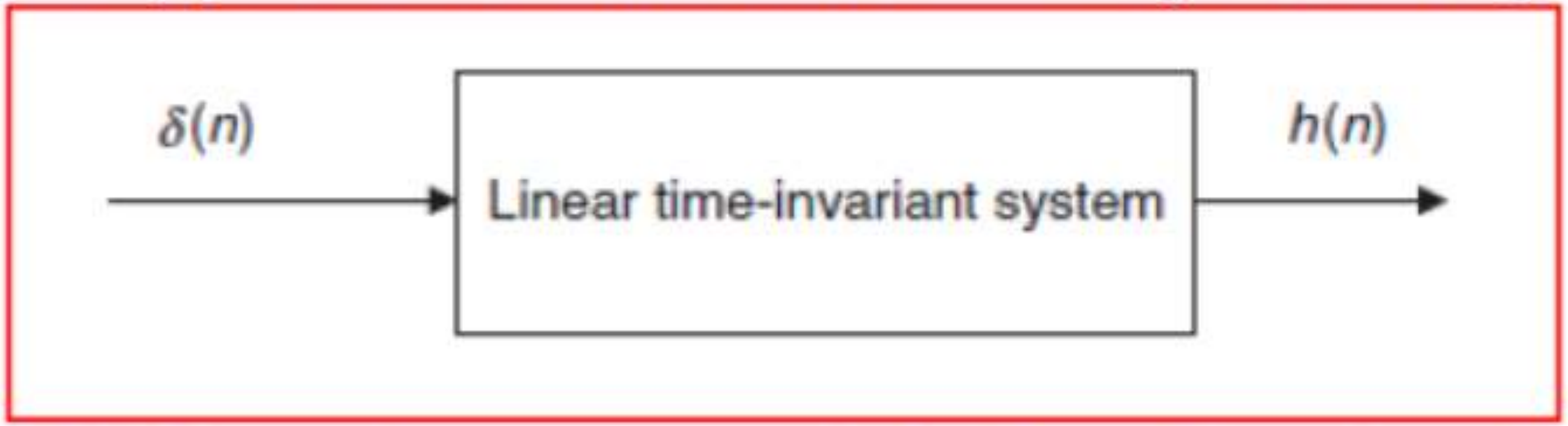


Figure (1): Unit-impulse response of the linear time-invariant system.

System representation

With the obtained unit-impulse response h(n), we can represent the linear time-invariant system in Figure (2).

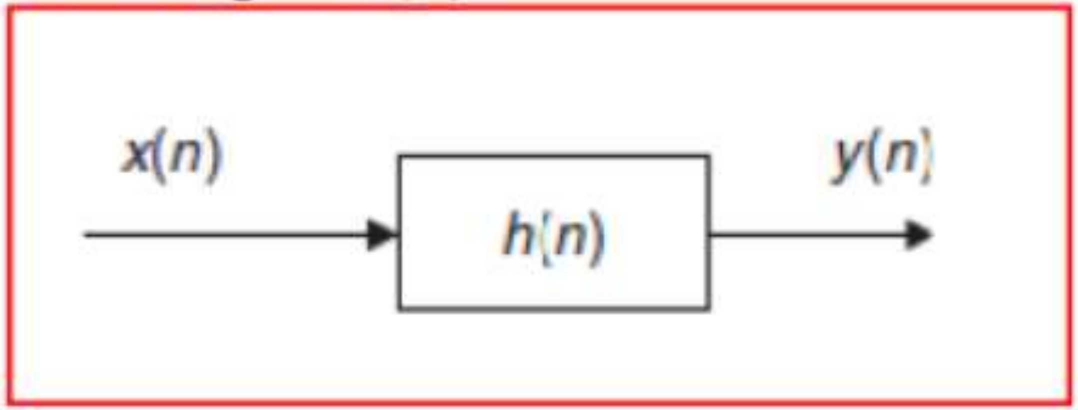


Figure (2): Representation of a linear time-invariant system using the impulse response.

System representation

In general, we can express the output sequence of a linear time-invariant system from its impulse response and inputs as

$$y(n) = \ldots + h(-1)x(n+1) + h(0)x(n) + h(1)x(n-1) + h(2)x(n-2) + \ldots.$$

Equation (3) is called the digital convolution sum, which will be explored in a later section. We can verify Equation (3) by substituting the impulse sequence $x(n) = \delta(n)$ to get the impulse response

$$h(n) = \ldots + h(-1)\delta(n+1) + h(0)\delta(n) + h(1)\delta(n-1) + h(2)\delta(n-2) + \ldots,$$