COLLEGE OF ENGINEERING AND TECHNOLOGIES
ALMUSTAQBAL UNIVERSITY

Digital Signal Processing (DSP)

CTE 306

Lecture 11

- System Properties -

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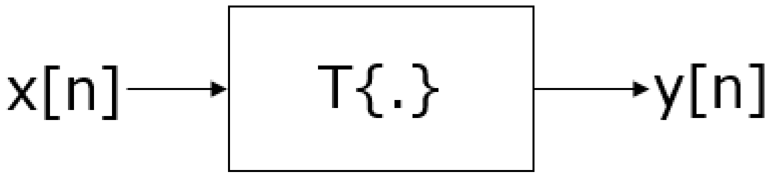
Dr. Zaidoon AL-Shammari
Lecturer / Researcher
zaidoon.waleed@mustaqbal-college.edu.iq

Discrete-Time Systems

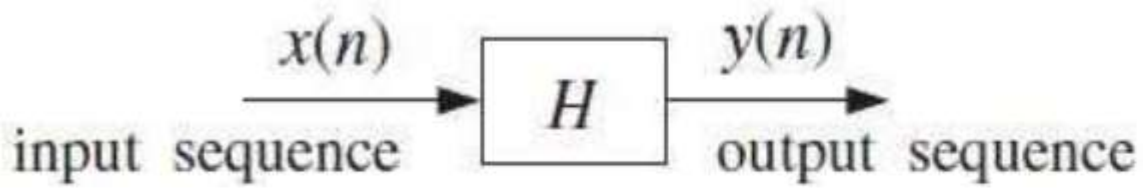
- A system is a physical or mathematical entity, typically realized in hardware or software, which performs operations on signals to extract or modify information.

- Maps a given input sequence x[n] into an output sequence y[n]

$$y[n] = T\{x[n]\}$$



- The general block diagram is shown in the figure below.



- Therefore, the system H represent the mathematical relation between the input x(n) and the output y(n).

- The system can be presented by a difference equation.

Basic System Properties

- Memoryless systems
- Causality
- Linearity
- Time Invariance
- Linear Time Invariant (LTI)

- Memoryless systems & Not Memoryless systems

A system is memoryless if the output y[n] at every value of n depends only on the input x[n] at the same value of n.

- Static and Dynamic Systems

Static system is memoryless whereas dynamic system is a not memoryless system.

Examples

- Memoryless Systems

Square

$$y[n] = (x[n])^2$$

Sign

$$y[n] = \text{sign}\{x[n]\}$$

- Not Memoryless Systems

$$y[n] = x[n - n_o]$$

Examples

Ex 1: $y(t) = 2\,x(t)$

For present value t = 0, the system output is $y(0) = 2\,x(0)$.

Here, the output is only dependent upon present input. Hence the system is memoryless or static.

Ex 2: $y(t) = 2\,x(t) + 3\,x(t-3)$

For present value t=0, the system output is $y(0) = 2\,x(0) + 3\,x(-3)$.

Here $x(-3)$ is past value for the present input for which the system requires memory to get this output. Hence, the system is not memoryless or dynamic system.

Causality

- A causal system is one in which the output y(n) at time n depends only on the current input x(n) at time n, its past input sample values such as x(n -1), x(n - 2), . . . Otherwise, if a system output depends on the future input values, such as x(n + 1), x(n + 2), . . . , the system is noncausal.

- A system is causal it's output is a function of only the current and previous samples.

Backward Difference (Causal).

Forward Difference (Non-Causal).

Examples

Ex 1: $y(n) = 2\,x(n) + 3\,x(n-3)$

For present value n = 1, the system output is $y(1) = 2\,x(1) + 3\,x(-2)$.

Here, the system output only depends upon present and past inputs. Hence, the system is causal.

Ex 2: $y(n) = 2\,x(t) + 3\,x(n-3) + 6\,x(n+3)$

For present value n = 1, the system output is $y(1) = 2\,x(1) + 3\,x(-2) + 6\,x(4)$. Here, the system output depends upon future input. Hence the system is non-causal system.

Time Invariant

A system is said to be time variant if its input and output characteristics vary with time. Otherwise, the system is considered as time invariant.

- The condition for time invariant system is:

  $$y(n, t) = y(n - t)$$

- The condition for time variant system is:

  $$y(n, t) \neq y(n - t)$$

Where $y(n, t) = T[x(n-t)]$ = input change

$y(n-t)$ = output change

Example

Ex: $y(n) = x(-n)$

$y(n, t) = T[x(n-t)] = x(-n - t)$

$y(n-t) = x(-(n-t)) = x(-n + t)$

$y(n, t) \neq y(n-t)$.

Hence, the system is time variant.

Liner and Non-liner Systems

- A system is said to be linear when it satisfies the superposition principle (i.e additivity and homogeneity).
- Then, according to the superposition and homogenate principles,

$$T\{x_1[n]+x_2[n]\}=T\{x_1[n]\}+T\{x_2[n]\} \quad \text{(additivity)}$$

and

$$T\{ax[n]\}=aT\{x[n]\} \quad \text{(scaling)}$$

$$T\,[a_1\,x_1(t) + a_2\,x_2(t)] = a_1\,T[x_1(t)] + a_2\,T[x_2(t)]$$

$$T\,[a_1\,x_1(t) + a_2\,x_2(t)] = a_1\,y_1(t) + a_2\,y_2(t)$$

Liner and Non-liner Systems

- From the above expression, is clear that response of overall system is equal to response of individual system.
- For a certain system, let the outputs $y1(n)$and $y2(n)$ correspond to the inputs $x1(n)$ and $x2(n)$, respectively.

Now the system is said to be Linear if:-

$$H[a_1x_1(n) + a_2x_2(n)] = a_1H[x_1(n)] + a_2H[x_2(n)]$$

- Where $a1$, $a2$ are arbitrary constants.
- H [.] is the system function.
- Linearity is a very nice property which makes analysis much simpler.

Example

Ex: $(t) = x^2(t)$

Solution:

$y_1(t) = T[x_1(t)] = x_1^2(t)$

$y_2(t) = T[x_2(t)] = x_2^2(t)$

$T[a_1 x_1(t) + a_2 x_2(t)] = [a_1 x_1(t) + a_2 x_2(t)]^2$

Which is not equal to $a_1 y_1(t) + a_2 y_2(t)$. Hence the system is said to be non linear.