COLLEGE OF ENGINEERING AND TECHNOLOGIES
ALMUSTAQBAL UNIVERSITY

Digital Signal Processing (DSP)

CTE 306

Lecture 15

- Finite Impulse Response (FIR) Filters -

Dr. Zaidoon AL-Shammari
Lecturer / Researcher
zaidoon.waleed@mustaqbal-college.edu.iq

Finite Impulse Response (FIR) Filters

- FIR filters are nonrecursive filters.

- The input-output relation of the FIR filters in time domain:

$$y[n] = \sum_{K=0}^{M} b_k \, x[n-k]$$

b_k are the filter coefficients

Finite Impulse Response (FIR) Filters

- FIR filters have a finite-duration impulse response.
- FIR filters take the number of samples equals to the number of past inputs for the impulse response to become zero.

Finite Impulse Response (FIR) Filters

- This FIR filter has the effect of averaging every N samples in the input signal.
- Any filter with this type of impulse response is called as a moving average filter.

Example

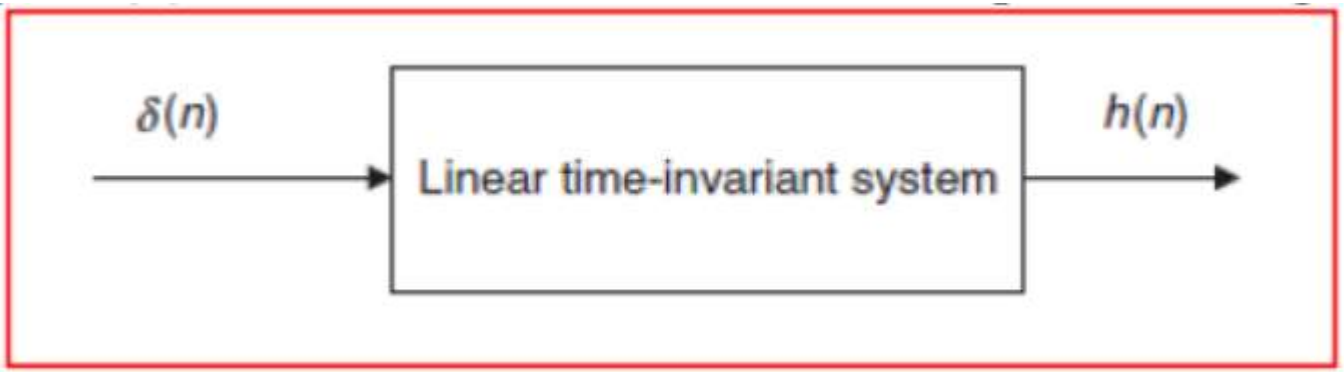
A FIR filter has a set of filter coefficients {bk} = {3, -1, 2, 1}. Determine the difference equation for the filter.

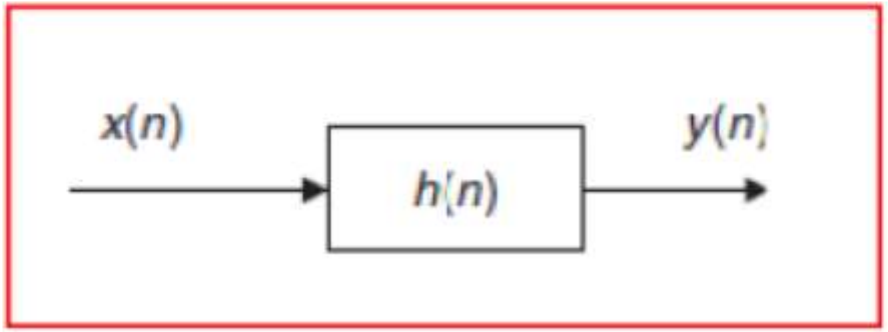
Sol:

The length of the filter is 4.

$$y\,[n] = 3x[n] - x[n-1] + 2x[n-2] + x[n-3]$$

Linear time invariant system





Example

Given the linear time-invariant system
$y(n) = 0.5x(n) + 0.25x(n - 1)$ with an initial condition

a) Determine the unit-impulse response $h(n)$.

b) Draw the system block diagram.

c) Write the output using the obtained impulse response.

Sol:

a) According to Figure 1, let $x(n) = \delta(n)$, then

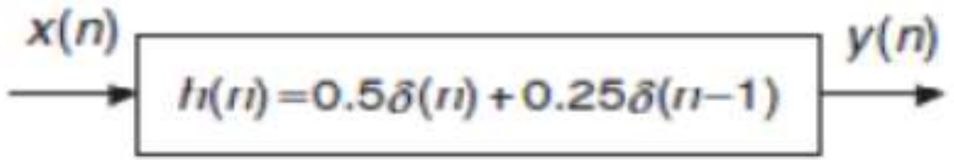
$$h(n) = y(n) = 0.5x(n) + 0.25x(n-1) = 0.5\delta(n) + 0.25\delta(n-1).$$

Solution

Thus, for this particular linear system, we have

$$h(n) = \begin{cases} 0.5 & n = 0 \\ 0.25 & n = 1 \\ 0 & elsewhere \end{cases}$$

b) The block diagram of the linear time-invariant system is shown as



c) The system output can be rewritten as

$$y(n) = h(0)x(n) + h(1)x(n-1).$$

Impulse response

In general, we can express the output sequence of a linear time-invariant system from its impulse response and inputs as

$$y(n) = \ldots + h(-1)x(n+1) + h(0)x(n) + h(1)x(n-1) + h(2)x(n-2) + \ldots \quad (3)$$

Equation (3) is called the digital convolution sum, which will be explored in a later section. We can verify Equation (3) by substituting the impulse sequence $x(n) = \delta(n)$ to get the impulse response

$$h(n) = \ldots + h(-1)\delta(n+1) + h(0)\delta(n) + h(1)\delta(n-1) + h(2)\delta(n-2) + \ldots,$$

Determine the first four samples in the impulse response for the FIR filter.

$$y\,[n] = 0.5(x[n]\ + x[n-1]\ + x[n-2])$$

Sol:

Substituting δ[n] for $x\ [n]$ and h[n] for $y\ [n]$.

$$h\,[n] = 0.5(\delta\,[n] + \delta\,[n-1]\ + \delta\,[n-2])$$

$$h\,[0] = 0.5(\delta\,[0] + \delta\,[-1]\ + \delta\,[-2])$$
$$= 0.5(1.0 + 0.0 + 0.0) = 0.5$$

Solution

$$h[1] = 0.5(\delta[1] + \delta[0] + \delta[-1])$$
$$= 0.5(0.0 + 1.0 + 0.0) = 0.5$$

$$h[2] = 0.5(\delta[2] + \delta[1] + \delta[0])$$
$$= 0.5(0.0 + 0.0 + 1.0) = 0.5$$

$$h[3] = 0.5(\delta[3] + \delta[2] + \delta[1])$$
$$= 0.5(0.0 + 0.0 + 0.0) = 0$$

Example

Determine the first six samples in the impulse response for the FIR filter.

$$y\ [n] = 0.25(x[n]\ + x[n-1]\ + x[n-2]\ + x[n-3])$$

Sol:

Substituting δ[n] for $x\ [n]$ and h[n] for $y\ [n]$.

$$h\ [n] = 0.25(\delta\ [n] + \delta\ [n-1]\ + \delta\ [n-2] + \delta\ [n-3])$$

$$h\ [0] = 0.25(\delta\ [0] + \delta\ [-1]\ + \delta\ [-2] + \delta\ [-3])$$
$$= 0.25(1.0 + 0.0 + 0.0 + 0.0) = 0.25$$

Solution

$$h\,[1] = 0.25(\delta\,[1] + \delta\,[0] + \delta\,[-1] + \delta\,[-2]\,)$$
$$= 0.25(0.0 + 1.0 + 0.0 + 0.0) = 0.25$$

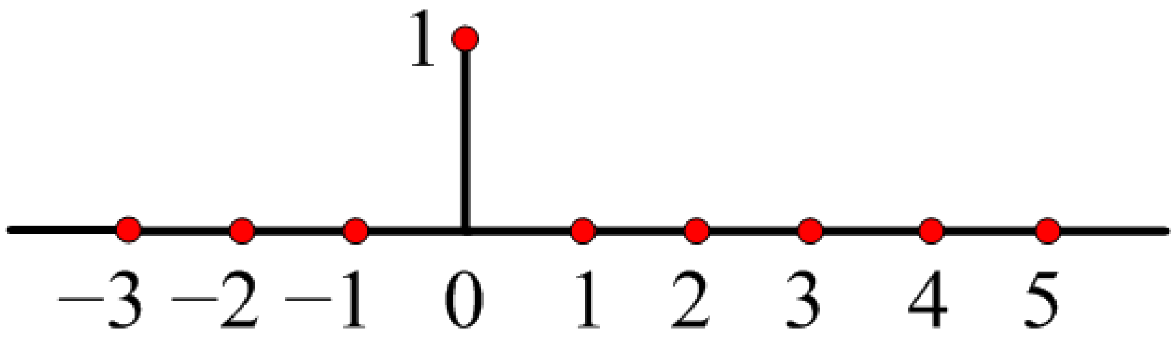
$$h\,[2] = 0.25(\delta\,[2] + \delta\,[1] + \delta\,[0] + \delta\,[-1]\,)$$
$$= 0.25(0.0 + 0.0 + 1.0 + 0.0) = 0.25$$

$$h\,[3] = 0.25(\delta\,[3] + \delta\,[2] + \delta\,[1] + \delta\,[0]\,)$$
$$= 0.25(0.0 + 0.0 + 0.0 + 1.0) = 0.25$$

$$h\,[4] = 0.25(\delta\,[4] + \delta\,[3] + \delta\,[2] + \delta\,[1]\,)$$
$$= 0.25(0.0 + 0.0 + 0.0 + 0.0) = 0.0$$

Solution

$$h[5] = 0.25(\delta[5] + \delta[4] + \delta[3] + \delta[2])$$
$$= 0.25(0.0 + 0.0 + 0.0 + 0.0) = 0.0$$



Impulse Function