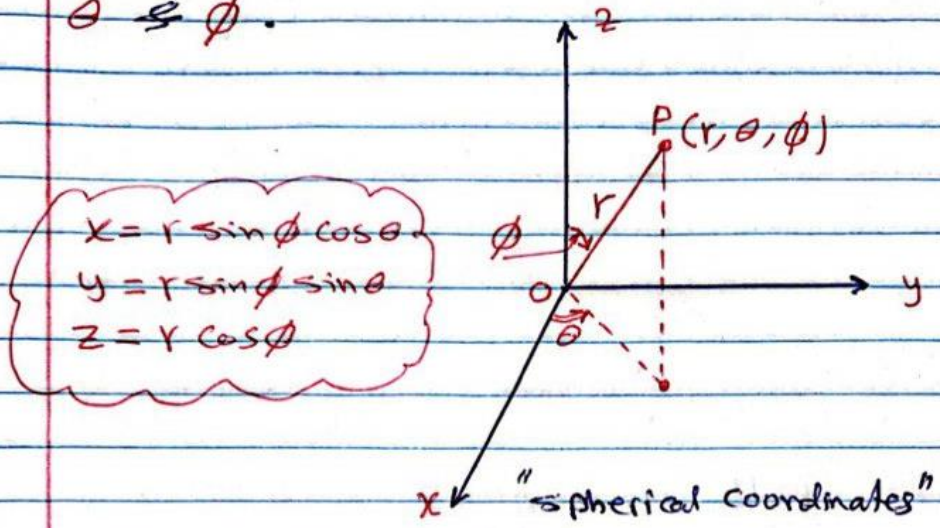
عنوان المحاضرة:

الأحداثيات الكروية Spherical Coordinates

Spherical Coordinates :- الاحداثيات الكروية

Spherical coordinates determine the position of a point in 3-dimensional space based on the distance "r" from the origin and two angles θ & ϕ.



"Spherical Coordinates"

$$x = r\sin\phi\cos\theta$$
$$y = r\sin\phi\sin\theta$$
$$z = r\cos\phi$$

θ is essentially the same as the angle θ from Polar Coordinates.

$$r = \sqrt{x^2+y^2+z^2} \qquad r \to [0, \infty)$$

$$\phi = \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{z}\right) \qquad \phi \to [0, 2\pi]$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) \qquad \theta \to [0, 2\pi]$$

Range

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Ex ③ convert the point $(1, \sqrt{3}, 2)$ to spherical coordinates (r, θ, ϕ)

Sol.

$r = \sqrt{x^2+y^2+z^2}$, $\theta = \tan^{-1}\frac{y}{x}$, $\phi = \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{z}\right)$

$(1, \sqrt{3}, 2) \longrightarrow x=1 , y=\sqrt{3} , z=2$

$\therefore r^2 = 1^2 + (\sqrt{3})^2 + (2)^2 = 8 \longrightarrow r = \sqrt{8} = \boxed{2\sqrt{2}}$

$\theta = \tan^{-1}\frac{y}{x} = \tan^{-1}\frac{\sqrt{3}}{1} = \boxed{\frac{\pi}{3}}$

$\phi = \tan^{-1}\left(\frac{\sqrt{1^2+(\sqrt{3})^2}}{2}\right) = \tan^{-1}\left(\frac{\sqrt{4}}{2}\right) = \tan^{-1}\frac{2}{2} = \boxed{\frac{\pi}{4}}$

$\therefore (1, \sqrt{3}, 2)$ in Cartesian equals to $(2\sqrt{2}, \frac{\pi}{3}, \frac{\pi}{4})$ in spherical coordinates

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Ex ④ Convert the point $(3, \frac{\pi}{4}, \frac{3\pi}{4})$ to cartesian coordinates.

Sol.

$x = r\sin\phi\cos\theta$
$y = r\sin\phi\sin\theta$
$z = r\cos\phi$

$(r, \theta, \phi) \rightarrow (x, y, z)$
$(3), (\pi/4), (3\pi/4)$

$r = 3$
$\theta = \pi/4$
$\phi = 3\pi/4$

$\therefore x = 3\cdot\sin\frac{3\pi}{4}\cdot\cos\frac{\pi}{4} = 3\cdot\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}} = \boxed{\frac{3}{2}}$

$y = 3\cdot\sin\frac{3\pi}{4}\cdot\sin\frac{\pi}{4} = 3\cdot\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}} = \boxed{\frac{3}{2}}$

$z = 3\cos\frac{3\pi}{4} = 3\cdot\frac{-1}{\sqrt{2}} = \boxed{-\frac{3}{\sqrt{2}}}$

$\rightarrow (\frac{3}{2}, \frac{3}{2}, \frac{-3}{\sqrt{2}})$

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----- نهاية محاضرة " الأحداثيات الكروية Spherical Coordinates" -----

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