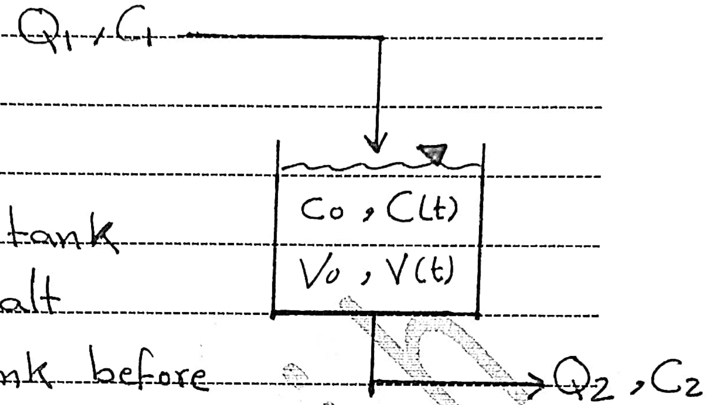
Application of F.O.D.E in chemical engineering:

① Mixing tank



Where:-

Q_1: Flowrate enters the tank

C_1: Concentration of Salt

C_o: Conc. of salt in the tank before enters Q_1, C_1

V_o: Volume of water in the tank before enters Q_1, C_1

$V(t)$: Volume of water in the tank at any time.

$C(t)$: Conc. of Salt in the tank at any time.

Q_2: Flow rate out of the tank.

C_2: Conc. of salt out of the tank.

$$\frac{dw}{dt} = Q_1 C_1 - Q_2 C_2 = Q_1 C_1 - \frac{w(t)}{V(t)} Q_2$$

$$\frac{dv}{dt} = Q_1 - Q_2 \Rightarrow \int dv = \int (Q_1 - Q_2)\, dt$$

$$V(t) = (Q_1 - Q_2)t + C \Rightarrow \text{at } t = 0, \ V = V_o$$

$$V_o = 0 + C \longrightarrow C = V_o$$

$\therefore V(t) = (Q_1 - Q_2)t + V_o$ (sub in dw/dt)

$$\Rightarrow \boxed{\frac{dw}{dt} = Q_1 C_1 - Q_2 \frac{w(t)}{(Q_1 - Q_2)t + V_o}}$$

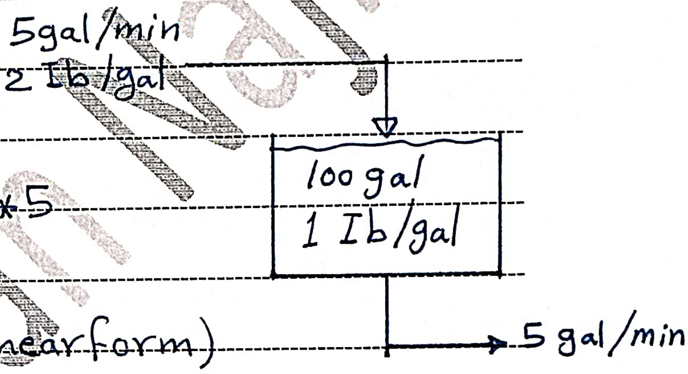
EX.1/ A tank is initially filled with 100 gal of brine containig 1 Ib/gal of salt; Find:

① amount of salt at any time?

② How long it will take the salt to reach 150 Ib?

If the flow rate enters the tank is 5 gal/min and the salt concentration is 2 Ib/gal; the out flow rate of the tank is the same of the enters.

$$\frac{dw}{dt} = Q_1 C_1 - Q_2 \frac{w(t)}{(Q_1 - Q_2)t + V_0}$$



$$\frac{dw}{dt} = (5 * 2) - \frac{w}{(5-5)t + 100} * 5$$

$$\frac{dw}{dt} + \frac{w}{20} = 10 \quad \text{(linear form)}$$

$$I.f = e^{\int \frac{1}{20} dt} \longrightarrow I.f = e^{t/20}$$

$$w(t) = \frac{1}{e^{t/20}} \int e^{t/20} * 10\, dt + \frac{c}{e^{t/20}}$$

$$= \frac{200}{\cancel{e^{t/20}}} * \cancel{e^{t/20}} + \frac{c}{e^{t/20}}$$

$$\therefore w(t) = 200 + \frac{c}{e^{t/20}} \text{ ------ } ①$$

B.C./ at $t=0$; $w_0 = C_0 * V_0$
$= 1 * 100$ } sub in eq.1

$100 = 200 + C e^{0} \longrightarrow C = -100$

① $w(t) = 200 - \frac{100}{e^{t/20}}$

② $150 = 200 - \frac{100}{e^{t/20}}$

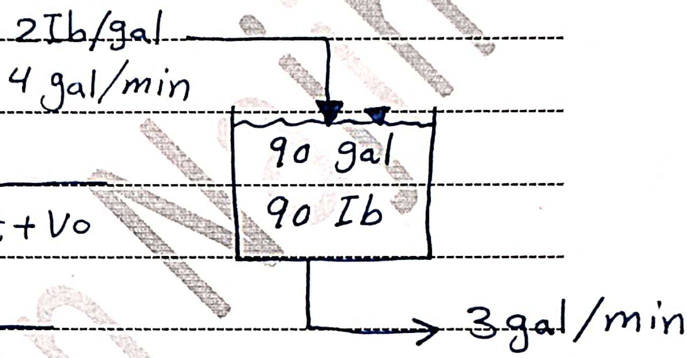
$\cancel{-}50 = \frac{\cancel{-}100}{e^{t/20}} \longrightarrow \frac{0.5}{1} = \frac{1}{e^{t/20}}$

$e^{t/20} = \frac{1}{0.5} \longrightarrow e^{t/20} = 2$

$\frac{t}{20} = \ln 2$

$\therefore t = 20 \ln 2$
$t = 13.86$ min

EX.2/ A tank initially contains 90 Ib of salt, the salt is in 90 gal of water. Brine containing 2 Ib/gal of salt flow in to the tank at the rate 4 gal/min, the mixture flow out of the tank at the rate 3 gal/min. How much salt does the tank contain when it is full; assume that the volume of the tank is 120 gal?



Sol:-

$$\frac{dw}{dt} = Q_1C_1 - Q_2\frac{w(t)}{(Q_1-Q_2)t+V_0}$$

$$\frac{dw}{dt} = 4*2 - 3\frac{w}{t+90}$$

$$\frac{dw}{dt} + \frac{3w}{t+90} = 8$$

$$I.f = e^{\int\frac{3}{t+90}dt} = e^{3\int\frac{1}{t+90}dt} = e^{3\ln(t+90)} = (t+90)^3$$

$$w(t) = \frac{1}{(t+90)^3}\int(t+90)^3 * 8\,dt + \frac{c}{(t+90)^3}$$

$$w(t) = 2(t+90) + \frac{c}{(t+90)^3}$$

B.C.1/ at $t=0$ $w=90$ Ib

$$90 = 2*90 + \frac{c}{90^3} \Rightarrow c = -90^4$$

$$\therefore w(t) = 2(t+90) - \frac{90^4}{(t+90)^3}$$

* The time required to fill the tank:

$$t = V/Q \rightarrow t = \frac{120\,\cancel{gal}}{4\,\cancel{gal}/min} \rightarrow t = 30\ min$$

$$w(30) = 2(90+30) - \frac{(90)^4}{(90+30)^3}$$

$$w(30) = 202.03\ Ib$$

② Heat transfer

EX./ A copper ball is heated to a temperature of $100\,^{\circ}C$ then at time $t=0$ it is placed in water which is maintained at temp. of $30\,^{\circ}C$. At the end of 3 min, the temp. of the ball is reduced to $70\,^{\circ}C$. Find the time at which the temp. of the ball is reduced to $31\,^{\circ}C$.

sol/

Newton's law of cooling: $\dfrac{dT}{dt} = K\,(T - T_{env.})$

$$\frac{dT}{dt} - KT = -30K \qquad \text{(linear)}$$

$$I.f = e^{\int -k\,dt} \longrightarrow I.f = e^{-kt}$$

$$T(t) = \frac{1}{e^{-kt}} \int e^{-kt} * -30K\,dt + \frac{c}{e^{-kt}}$$

$$T(t) = \frac{1}{\cancel{e^{-kt}}} * 30\,\cancel{e^{-kt}} + Ce^{+kt}$$

$$\boxed{T(t) = 30 + Ce^{kt}}$$

B.C.1 at $t=0$; $T = 100\,^{\circ}C$

$$100 = 30 + Ce^{0} \Longrightarrow C = 70$$

$$\therefore T(t) = 30 + 70\,e^{-kt}$$

B.C.2 at $t=3$ min $T=70°C$

$$70 = 30 + 70e^{3k}$$

$$k = -0.1865$$

$$\Rightarrow T(t) = 30 + 70e^{-0.1865t}$$

$$31 = 30 + 70e^{-0.1865t}$$

$$\Rightarrow t = 22.78 \text{ min}$$

EX.2/ Water at 190°F is left in a room of 70°F. At time = 0 the water is cooling at 15°F per minute.

ⓐ Find the function that models the cooling of water.

ⓑ How Long it will take for the temp to reach 143°F?

Sol:

$$\frac{dT}{dt} = k\,(T_{water} - T_{room})$$

$\frac{dT}{dt} = -15$; at $t = 0$ $T_w = 190$ and $T_r = 70$°F

$$-15 = k\,(190 - 70) \longrightarrow k = -0.125$$

$$\therefore \frac{dT}{dt} = -0.125\,(T_w - T_r)$$

$$dT = -0.125\,(T_w - 70)\,dt$$

$$\int \frac{1}{T_w - 70}\,dT = \int -0.125\,dt$$

$$\ln(T - 70) = -0.125t + c$$

$$T - 70 = e^{-0.125t + c} \longrightarrow T - 70 = e^{-0.125t} * e^{c}$$

$$\therefore T(t) = a\,e^{-0.125t} + 70 \qquad (e^{c} = a)$$

B.C.1/ At $t=0$, $T=190°F$

$190 = a e^{-0.125t} + 70 \longrightarrow a = 120$

$\Longrightarrow T(t) = 120 e^{-0.125t} + 70$

b) $T = 143\,°F$; $t = ??$

$143 = 120\, e^{-0.125t} + 70$

$143 - 70 = 120\, e^{-0.125t}$

$0.608333 = e^{-0.125t}$

$-0.49703 = -0.125t$

$\therefore t = 3.976$ min