Higher Order D.E:

1. Homogenous Higher O.D.E

Ex.1/ Solve $y''' + 3y'' + 3y' + y = 0$

sol/

$m^3 + 3m^2 + 3m + 1 = 0$

$m_1 = -1$

$\Rightarrow m + 1 = 0$

$$
\begin{array}{r|l}
 & m^2 + 2m + 1 \\
\hline
m+1 & m^3 + 3m^2 + 3m + 1 \\
 - & m^3 + m^2 \\
\hline
 & 2m^2 + 3m + 1 \\
 & 2m^2 + 2m \\
\hline
 & m + 1 \\
 & m + 1 \\
\hline
 & 0
\end{array}
$$

$(m+1)(m^2 + 2m + 1) = 0$

$(m+1)(m+1)(m+1) = 0$

$\therefore m_1 = m_2 = m_3 = -1$

$\Rightarrow y_c = C_1 e^{-X} + C_2 X e^{-X} + C_3 X^2 e^{-X}$

② Non-Homogenous higher D.E

Ex/ $y''' + 5y'' + 9y' + 5y = 3e^{2x}$

sol/

$m^3 + 5m^2 + 9m + 5 = 0$

$m_1 = -1$

$m + 1 = 0$

$(m+1)(m^2 + 4m + 5) = 0$

$$
\begin{array}{r|l}
 & m^2 + 4m + 5 \\
\hline
m+1 & m^3 + 5m^2 + 9m + 5 \\
 - & m^3 + m^2 \\
\hline
 & 4m^2 + 9m + 5 \\
 - & 4m^2 + 4m \\
\hline
 & 5m + 5 \\
 - & 5m + 5 \\
\hline
 & \text{Zero}
\end{array}
$$

$m_{2,3} = \dfrac{-4 \mp \sqrt{16 - 4 * 1 * 5}}{2 \times 1} = -2 \mp i$

$y_c = C_1 e^{-x} + e^{-2x}\left[C_2 \cos x + C_3 \sin x\right]$

$y_p = Ae^{2x}$, $y_p' = 2Ae^{2x}$, $y_p'' = 4Ae^{2x}$

$y_p''' = 8Ae^{2x}$

$8Ae^{2x} + 20Ae^{2x} + 18Ae^{2x} + 5Ae^{2x} = 3e^{2x}$

$8A + 20A + 18A + 5A = 3$

$\therefore A = 1/17$

$y_p = \frac{1}{17} e^{2x}$

$\therefore y(x) = C_1 e^{-x} + e^{-2x}\left[C_2 \cos x + C_3 \sin x\right] + \frac{1}{17} e^{2x}$