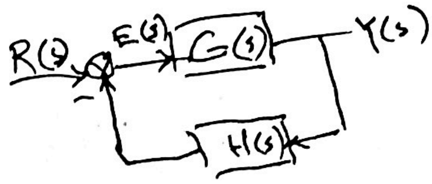
Steady state Error

$$E(s) = R(s) - Y(s)\cdot H(s) \quad \text{and} \quad Y(s) = E(s)\cdot G(s)$$



$$E(s) = R(s) - E(s)\cdot G(s)\cdot H(s)$$

$$E(s) + E(s)\,G(s)\,H(s) = R(s)$$

$$\boxed{E(s) = \frac{R(s)}{1 + G(s)H(s)}}$$

$$e_{ss} = E(\infty) = \lim_{s \to 0} \frac{s\,R(s)}{1 + G(s)H(s)}$$

① for Unity step I/P $\quad r(t) = 1\,u(t) \Longleftrightarrow R(s) = 1/s$.

a) Then e_{ss} for type 0 system with $G(s)\cdot H(s) = \dfrac{K_1 (s+Z_1)(s+Z_2)\cdots(s+Z_m)}{s^0 (s+P_1)(s+P_2)\cdots(s+P_n)}$

$$\boxed{e_{ss} = \lim_{s \to 0} \frac{s\,\frac{1}{s}}{1 + G(s)\cdot H(s)} = \frac{1}{1 + K_p}} \qquad \boxed{K_p = \lim_{s \to 0} G(s)H(s)}$$

$$K_p = \frac{K_1 \cdot Z_1 \cdot Z_2 \cdots Z_m}{P_1\,P_2 \cdots P_n} = \text{Constant}$$

~~K_p = Position Constant~~

e_{ss} = Constant

K_p = static position error Constant

b) for type 1 system with $G(s)H(s) = \dfrac{K_1 (s+Z_1)(s+Z_2)\cdots(s+Z_m)}{s^1 (s+P_1)(s+P_2)\cdots(s+P_n)}$

$$e_{ss} = \lim_{s \to 0} s\cdot E(s) = \frac{s(1/s)}{1 + \lim_{s \to 0} \frac{K_2 (s+Z_1)\cdots(s+Z_m)}{s(s+P_1)\cdots(s+P_n)}}$$

$$= \frac{1}{1 + \infty} = 0$$

مهم جداً

c) for type 2 System, also $e_{ss} = 0$

I/P	e_{ss} for Type 0	e_{ss} for Type 1	e_{ss} for Type 2
Unit step	$\frac{1}{1+K_p}$ Constant	0	0

② For Unit Ramp Input $r(t) = t$

Then $R(s) = \frac{1}{s^2}$

$$E(s) = \frac{R(s)}{1 + G(s)H(s)} = \frac{1/s^2}{1 + G(s)H(s)}$$

$$e(\infty) = \lim_{s \to 0} s.E(s)$$

$$e_{ss} = e(\infty) = \lim_{s \to 0} \frac{s(1/s^2)}{1 + G(s)H(s)}$$

$$= \lim_{s \to 0} \frac{1}{s + sG(s)H(s)} = \frac{1}{0 + K_v} = \frac{1}{K_v}$$

K_v = static velocity Error Constant $= \lim_{s \to 0} sG(s)H(s)$

a) for type 0 system $G(s)H(s) = \frac{K(s+Z_1)(s+Z_2)\cdots(s+Z_m)}{(s+P_1)(s+P_2)(s+P_3)\cdots(s+P_n)}$

$$= \frac{K.Z_1 Z_2 \cdots Z_m}{P_1 P_2 P_3 \cdots P_n} = \text{Constant}$$

$\therefore K_v = 0 \xrightarrow{e_{ss} = \frac{1}{K_v}}$ hence $e_{ss} = \frac{1}{0} = \infty$

b) for type 1 System $G(s)H(s) = \frac{K(s+Z_1)\cdots(s+Z_m)}{s(s+P_1)\cdots(s+P_n)}$

$$K_v = \lim_{s \to 0} s.G(s)H(s) = \frac{s\, K(Z_1 Z_2 \cdots Z_m)}{s(P_1 P_2 \cdots P_n)} = \text{Constant}$$

$e_{ss} = \frac{1}{K_v}$ = Constant

c) for type 2 System

$K_v = \frac{1}{0} = \infty$ hence $e_{ss} = \frac{1}{\infty} = 0$

Steady state error when i/p is Unit Ramp

$$e_{ss} = \lim_{s \to 0} s\,E(s) \quad \text{--- (1)}$$

$$E(s) = \frac{R(s)}{1 + G(s) \cdot H(s)} \quad \text{--- (2)}$$

$G(s) \cdot H(s)$ — open loop T.F.

$r(t) = t \quad t \geq 0$ unit ramp input

$$R(s) = \frac{1}{s^2}$$

$$e_{ss} = \lim_{s \to 0} \frac{s\,(1/s^2)}{1 + G(s)H(s)}$$

$$= \lim_{s \to 0} \frac{1}{s + s\,G(s)H(s)}$$

$$= \frac{1}{\lim s\,G(s)H(s)} = \frac{1}{K_v}$$

Velocity Error Constant

and it dependce on type of System

$$K_v = \lim_{s \to 0} s\,G(s)\,H(s)$$

$$e_{ss} = \lim_{s \to 0} s E(s)$$

$K_p = \lim_{s \to 0} G(s)H(s)$ static position erro Const.

$K_v = \lim_{s \to 0} s\, G(s)H(s)$ static Velocity " " .

$K_a = \lim_{s \to 0} s^2 G(s)H(s)$ static acceler. " " .

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for step input $r(t) = A \longrightarrow R(s) = \frac{A}{s}$

for Ramp input $r(t) = At \longrightarrow R(s) = \frac{A}{s^2}$

for Parabolic input $r(t) = \frac{At^2}{2} \longrightarrow R(s) = \frac{A}{s^3}$

System type:

type 0 $G(s)H(s) = \frac{K(s+z_1)\dots(s+z_m)}{s^0 (s+P_1)(s+P_2)\cdots(s+P_n)}$

type 1 $G(s)H(s) = \frac{K(s+z_1)(s+z_2)\cdots(s+z_m)}{s^1 (s+P_1)(s+P_2)\cdots(s+P_n)}$

type 2 $G(s)H(s) = \frac{K}{s^2(}$

System	e_{ss}/step i/p ($r(t) = A$)	e_{ss}/Ramp i/p ($r(t) = At$)	e_{ss}/Parabolic i/p ($r(t) = \frac{A}{2}t^2$)
type 0	$\frac{A}{1+K_p}$	∞	∞
type 1	ϕ	$\frac{A}{K_v}$ non zero	∞
type 2	ϕ	$\frac{1}{K_a}$. 0	$\frac{A}{K_a}$ non Zero