EXAMPLE 4–14

Convert each of the following Boolean expressions to SOP form:

(a) $AB + B(CD + EF)$ (b) $(A + B)(B + C + D)$ (c) $\overline{(\overline{A + B}) + C}$

Solution

(a) $AB + B(CD + EF) = AB + BCD + BEF$

(b) $(A + B)(B + C + D) = AB + AC + AD + BB + BC + BD$

(c) $\overline{(\overline{A + B}) + C} = (\overline{\overline{A + B}})\overline{C} = (A + B)\overline{C} = A\overline{C} + B\overline{C}$

EXAMPLE 4–16

Determine the binary values for which the following standard SOP expression is equal to 1:

$$ABCD + A\overline{B}\,\overline{C}D + \overline{A}\,\overline{B}\,\overline{C}\,\overline{D}$$

Solution

The term $ABCD$ is equal to 1 when $A = 1, B = 1, C = 1$, and $D = 1$.

$$ABCD = 1 \cdot 1 \cdot 1 \cdot 1 = 1$$

The term $A\overline{B}\,\overline{C}D$ is equal to 1 when $A = 1, B = 0, C = 0$, and $D = 1$.

$$A\overline{B}\,\overline{C}D = 1 \cdot \overline{0} \cdot \overline{0} \cdot 1 = 1 \cdot 1 \cdot 1 \cdot 1 = 1$$

The term $\overline{A}\,\overline{B}\,\overline{C}\,\overline{D}$ is equal to 1 when $A = 0, B = 0, C = 0$, and $D = 0$.

$$\overline{A}\,\overline{B}\,\overline{C}\,\overline{D} = \overline{0} \cdot \overline{0} \cdot \overline{0} \cdot \overline{0} = 1 \cdot 1 \cdot 1 \cdot 1 = 1$$

The SOP expression equals 1 when any or all of the three product terms is 1.

Note.

When we say "standard" (SOP) or "Standard" (POS) form we mean that the form contains ALL variables.

EXAMPLE 4–23

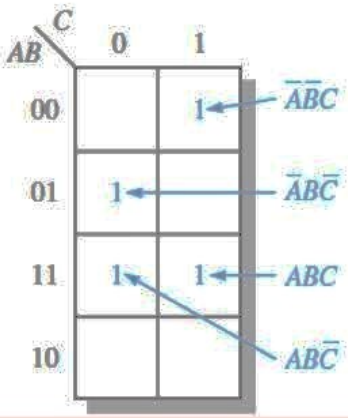
Map the following standard SOP expression on a Karnaugh map:

$$\overline{A}\overline{B}C + \overline{A}B\overline{C} + AB\overline{C} + ABC$$

Solution

Evaluate the expression as shown below. Place a 1 on the 3-variable Karnaugh map in Figure 4–29 for each standard product term in the expression.

$$\overline{A}\overline{B}C + \overline{A}B\overline{C} + AB\overline{C} + ABC$$
$$001 \quad 010 \quad 110 \quad 111$$



EXAMPLE 4–24

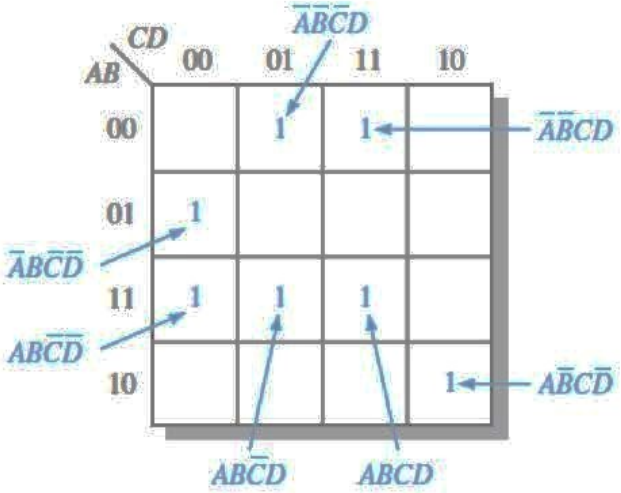
Map the following standard SOP expression on a Karnaugh map:

$$\overline{A}\overline{B}CD + \overline{A}B\overline{C}\overline{D} + AB\overline{C}D + ABCD + AB\overline{C}\overline{D} + \overline{A}\overline{B}\overline{C}D + A\overline{B}C\overline{D}$$

Solution

Evaluate the expression as shown below. Place a 1 on the 4-variable Karnaugh map in Figure 4–30 for each standard product term in the expression.

$$\overline{A}\overline{B}CD + \overline{A}B\overline{C}\overline{D} + AB\overline{C}D + ABCD + AB\overline{C}\overline{D} + \overline{A}\overline{B}\overline{C}D + A\overline{B}C\overline{D}$$
$$0011 \quad 0100 \quad 1101 \quad 1111 \quad 1100 \quad 0001 \quad 1010$$



EXAMPLE 4–27

Group the 1s in each of the Karnaugh maps in Figure 4–33.

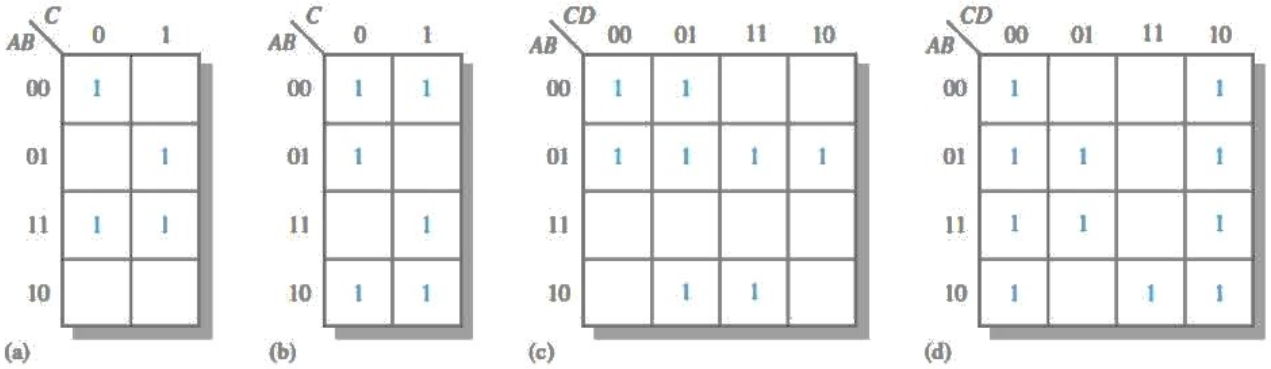


FIGURE 4–33

Solution

The groupings are shown in Figure 4–34. In some cases, there may be more than one way to group the 1s to form maximum groupings.

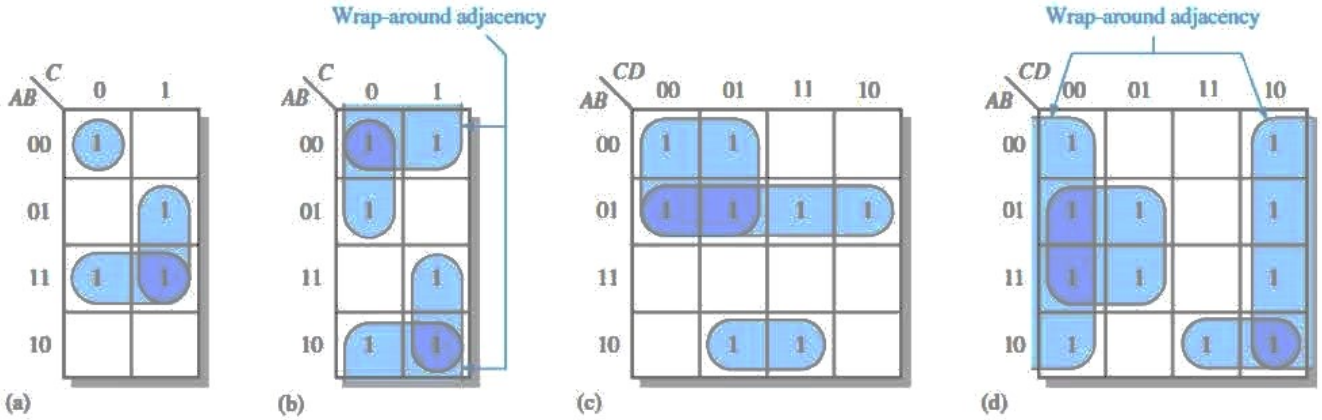


FIGURE 4–34

EXAMPLE 4–28

Determine the product terms for the Karnaugh map in Figure 4–35 and write the resulting minimum SOP expression.

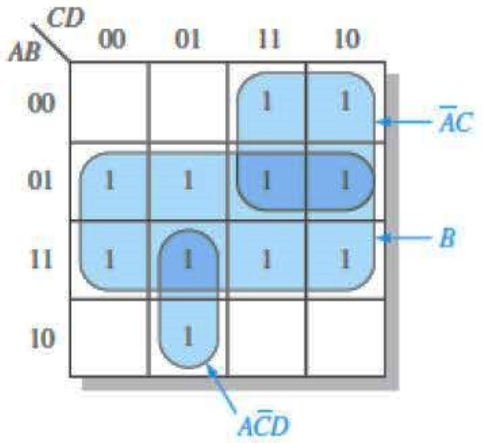


FIGURE 4–35

Solution

Eliminate variables that are in a grouping in both complemented and uncomplemented forms. In Figure 4–35, the product term for the 8-cell group is B because the cells within that group contain both A and $\overline{A}$, C and $\overline{C}$, and D and $\overline{D}$, which are eliminated. The 4-cell group contains B, $\overline{B}$, D, and $\overline{D}$, leaving the variables $\overline{A}$ and C, which form the product term $\overline{A}C$. The 2-cell group contains B and $\overline{B}$, leaving variables A, $\overline{C}$, and D which form the product term $A\overline{C}D$. Notice how overlapping is used to maximize the size of the groups. The resulting minimum SOP expression is the sum of these product terms:

$$B + \overline{A}C + A\overline{C}D$$

EXAMPLE 4–29

Determine the product terms for each of the Karnaugh maps in Figure 4–36 and write the resulting minimum SOP expression.

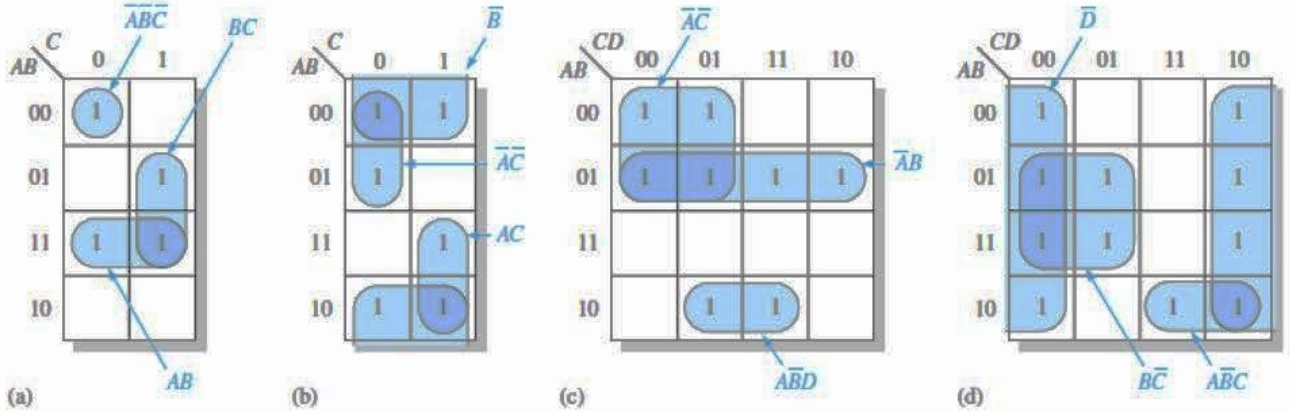


FIGURE 4–36

Solution

The resulting minimum product term for each group is shown in Figure 4–36. The minimum SOP expressions for each of the Karnaugh maps in the figure are

(a) $AB + BC + \overline{A}\overline{B}\overline{C}$

(b) $\overline{B} + \overline{A}\overline{C} + AC$

(c) $\overline{A}B + \overline{A}\overline{C} + A\overline{B}D$

(d) $\overline{D} + A\overline{B}C + B\overline{C}$

EXAMPLE 4–30

Use a Karnaugh map to minimize the following standard SOP expression:

$$A\overline{B}C + \overline{A}BC + \overline{A}\overline{B}C + \overline{A}B\overline{C} + A\overline{B}\overline{C}$$

Solution

The binary values of the expression are

$$101 + 011 + 001 + 000 + 100$$

Map the standard SOP expression and group the cells as shown in Figure 4–37.

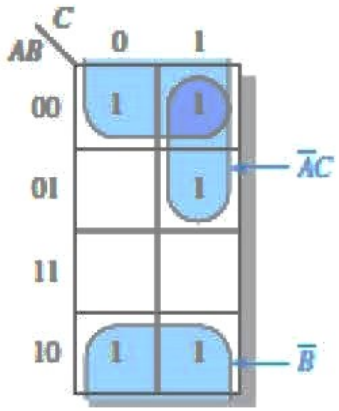


FIGURE 4–37

Notice the "wrap around" 4-cell group that includes the top row and the bottom row of 1s. The remaining 1 is absorbed in an overlapping group of two cells. The group of four 1s produces a single variable term, $\overline{B}$. This is determined by observing that within the group, $\overline{B}$ is the only variable that does not change from cell to cell. The group of two 1s produces a 2-variable term $\overline{A}C$. This is determined by observing that within the group, $\overline{A}$ and C do not change from one cell to the next. The product term for each group is shown. The resulting minimum SOP expression is

$$\overline{B} + \overline{A}C$$

Keep in mind that this minimum expression is equivalent to the original standard expression.

Working Example: Find the minimal Sum Of Product (SOP) and minimal Product of Sum (POS) forms of the function:

$$F = \Sigma\,(3,4,5,6,7,8,10,12,14)$$

Solution

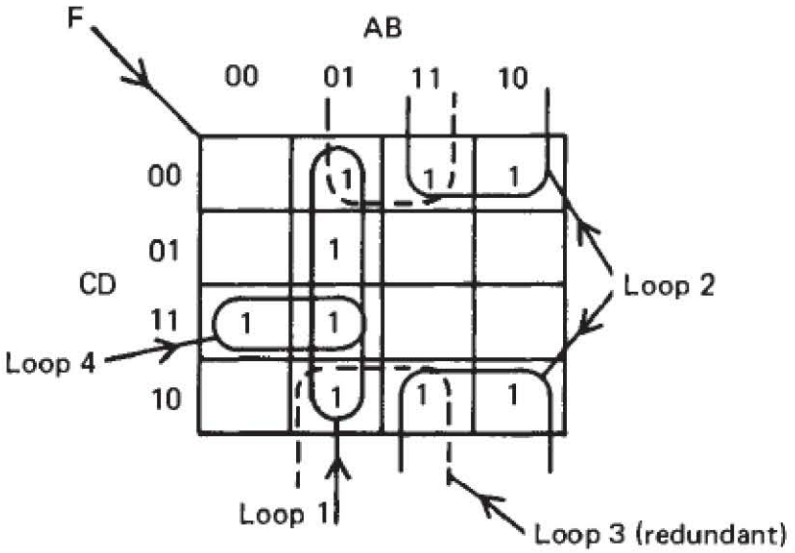
(a)minimal (SOP)

- بما ان الدالة (F) تحتوي لغاية الرقم (14) .. اذا هناك (٤) متغيرات، نفرضها (A,B,C,D).
- نرسم خارطة (كارنو) ذات الأربعة متغيرات ونضع (1) مكان الحدود الموضحة بالدالة .. (1) لان الدالة (F) هي ($\sum$). بينما نضع (0) مكان الحدود الاخرى.

F \ AB	00	01	11	10
00	0_0	1_4	1_{12}	1_8
01	0_1	1_5	0_{13}	0_9
11	1_3	1_7	0_{15}	0_{11}
10	0_2	1_6	1_{14}	1_{10}

(CD)

- بما ان المطلوب هو ايجاد (minimal SOP) اي المطلوب ان نعمل اختصار (reduction). ويتم ذلك بعمل حلقات (loops) تضم كل الخلايا المتجاورة التي فيها (1) .. لاحظ ان عدد الخلايا في الحلقة يكون (١ أو ٢ ، او ٤ ...).
- ولاجل التوضيح فقط سوف نزيل ال (0) .. و نرسم الخارطة بالخلايا التي فيها (1) فقط ونرسم كل الحلقات الممكن رسمها حول الخلايا التي تحتوي على (1).



- نلاحظ ان الحلقة (Loop 3) هي حلقة زائدة .. كون خلاياها مغطاة بحلقات اساسية اخرى .. وعليه يتم حذف هذا ال (loop 3).
وادناه شرح لكل (Loop) :-

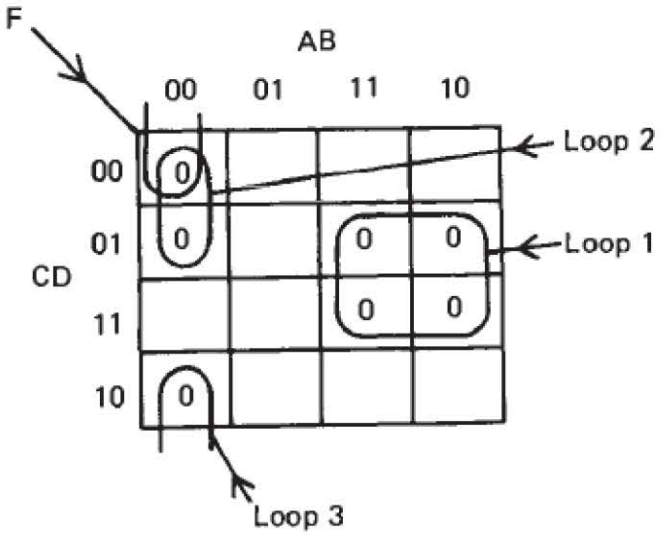
Loop 1 requires A = 0 and B = 1 but is independent of C and D.
Loop 1 is therefore $\overline{A}.B$.
Loop 2 requires A = 1 and D = 0, but is independent of B and C, giving $A\overline{D}$.
Loop 3 is redundant and can be ignored and finally loop 4 has A = 0, C = 1 and D = 1 but is independent of B, giving $\overline{A}CD$.

- اذن الـ (minimal (SOP)) في هذه الحالة هو (جمع الحدود المضروبة) :

$$F = \overline{A}B + A\overline{D} + \overline{A}CD$$

(b)minimal (POS)

- في هذه الحالة نعمل ال (Loops) حول الخلايا التي تضم ال (0) وبنفس الطريقة اعلاه :-



- في حالة الـ (POS) لاحظ اننا :- ناخذ ضرب الحدود المجموعة ، وان

$1 \longrightarrow \overline{A}$ and $0 \longrightarrow A$

- وعليه فان الحدود (terms) تكتب كالاتي:-

Loop 1 represents $(\overline{A} + \overline{D})$
loop 2 represents $(A + B + C)$
loop 3 represents $(A + B + D)$

- اذن ال (minimal (POS)) هو

$$F = (\overline{A} + \overline{D}).(A + B + C).(A + B + D)$$