



Logic Gate



College of
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Level 1 , Semester 1
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Arithmetic Operations & Boolean Algebra -2

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Boolean Addition

In Boolean algebra, a **variable** is a symbol used to represent an action, a condition, or data. A single variable can only have a value of 1 or 0.

The **complement** represents the inverse of a variable and is indicated with an overbar. Thus, the complement of A is \bar{A} .

A **literal** is a variable or its complement.

Addition is equivalent to the OR operation. The sum term is 1 if one or more of the literals are 1. The sum term is zero only if each literal is 0.

Example Determine the values of A , B , and C that make the sum term of the expression $\bar{A} + B + \bar{C} = 0$?

Solution Each literal must = 0; therefore $A = 1$, $B = 0$ and $C = 1$.

Boolean Multiplication

In Boolean algebra, multiplication is equivalent to the AND operation. The product of literals forms a product term. The product term will be 1 only if all of the literals are 1.

Example What are the values of the A , B and C if the product term of $A \cdot \overline{B} \cdot \overline{C} = 1$?

Solution Each literal must = 1; therefore $A = 1$, $B = 0$ and $C = 0$.

Commutative Laws

The **commutative laws** are applied to addition and multiplication. For addition, the commutative law states

In terms of the result, the order in which variables are ORed makes no difference.

$$A + B = B + A$$

For multiplication, the commutative law states

In terms of the result, the order in which variables are ANDed makes no difference.

$$AB = BA$$

Associative Laws

The **associative laws** are also applied to addition and multiplication. For addition, the associative law states

When ORing more than two variables, the result is the same regardless of the grouping of the variables.

$$A + (B + C) = (A + B) + C$$

For multiplication, the associative law states

When ANDing more than two variables, the result is the same regardless of the grouping of the variables.

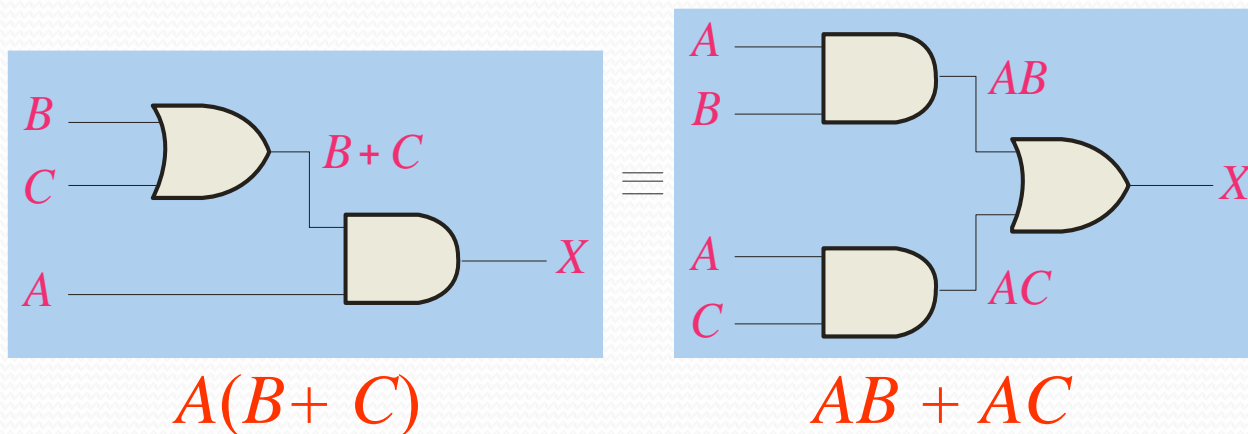
$$A(BC) = (AB)C$$

Distributive Law

The **distributive law** is the factoring law. A common variable can be factored from an expression just as in ordinary algebra. That is

$$AB + AC = A(B + C)$$

The distributive law can be illustrated with equivalent circuits:



Rules of Boolean Algebra

$$1. A + 0 = A$$

$$2. A + 1 = 1$$

$$3. A \cdot 0 = 0$$

$$4. A \cdot 1 = A$$

$$5. A + A = A$$

$$6. A + \bar{A} = 1$$

$$7. A \cdot A = A$$

$$8. A \cdot \bar{A} = 0$$

$$9. \bar{\bar{A}} = A$$

$$10. A + AB = A$$

$$11. A + \bar{A}B = A + B$$

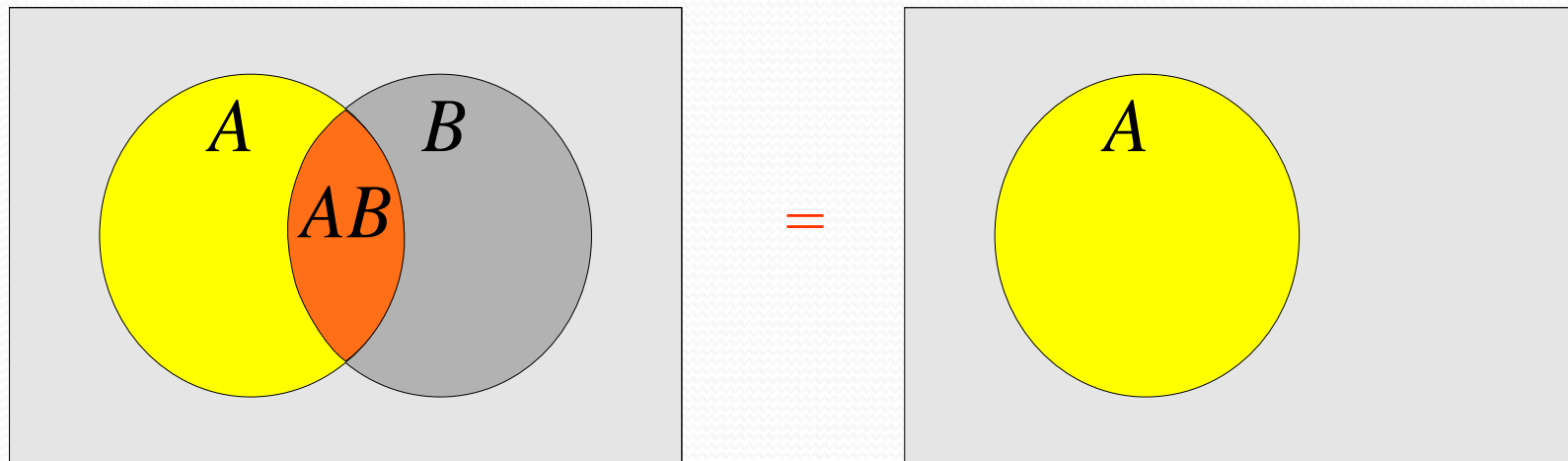
$$12. (A + B)(A + C) = A + BC$$

Rules of Boolean Algebra

Rules of Boolean algebra can be illustrated with *Venn* diagrams. The variable A is shown as an area.

The rule $A + AB = A$ can be illustrated easily with a diagram. Add an overlapping area to represent the variable B .

The overlap region between A and B represents AB .



The diagram visually shows that $A + AB = A$. Other rules can be illustrated with the diagrams as well.

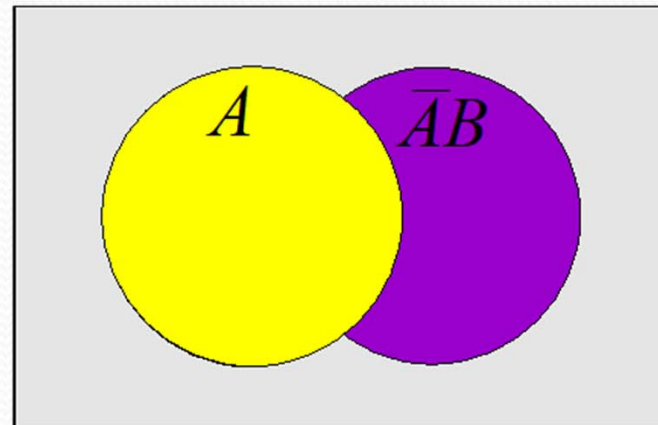
Rules of Boolean Algebra

Example

Illustrate the rule $A + \bar{A}B = A + B$ with a Venn diagram.

Solution

This time, \bar{A} is represented by the blue area and B again by the red circle. The intersection represents $\bar{A}B$. Notice that $A + \bar{A}B = A + B$



Rules of Boolean Algebra

Rule 12, which states that $(A + B)(A + C) = A + BC$, can be proven by applying earlier rules as follows:

$$\begin{aligned}(A + B)(A + C) &= AA + AC + AB + BC \\ &= A + AC + AB + BC \\ &= A(1 + C + B) + BC \\ &= A \cdot 1 + BC \\ &= A + BC\end{aligned}$$

This rule is a little more complicated, but it can also be shown with a Venn diagram, as given on the following slide...

Three areas represent the variables A , B , and C .

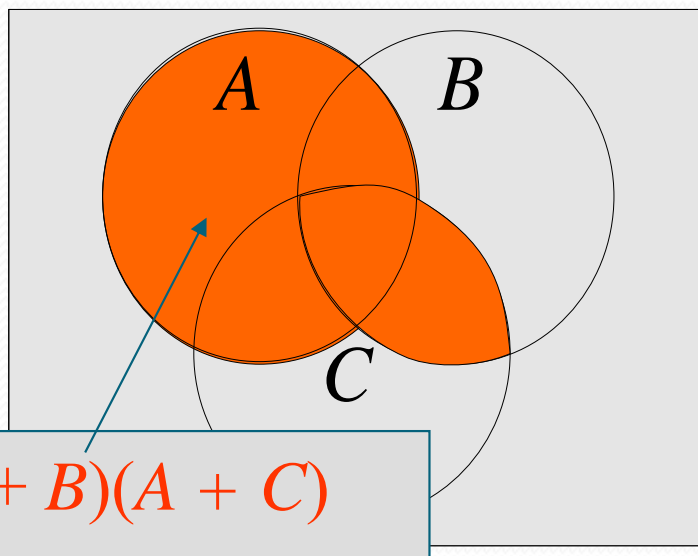
The area representing $A + B$ is shown in yellow.

The area representing $A + C$ is shown in red.

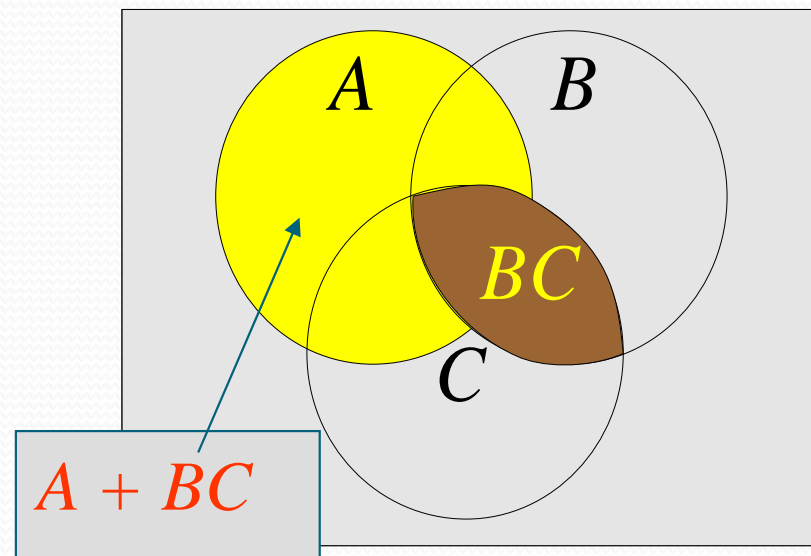
The overlap of red and yellow is shown in orange.

The overlapping area between B and C represents BC .

ORing with A gives the same area as before.



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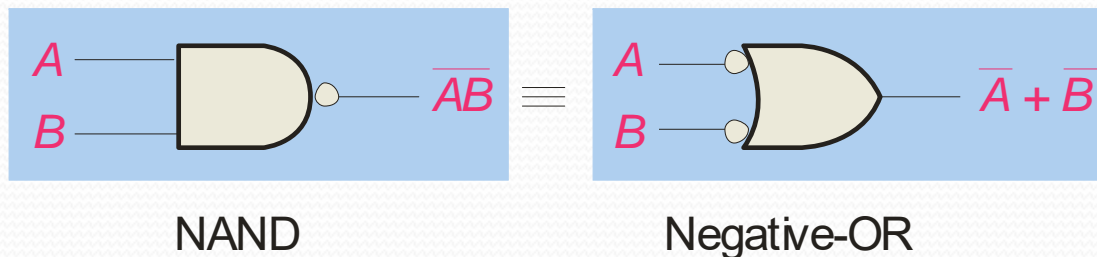
DeMorgan's Theorem

DeMorgan's 1st Theorem

The complement of a product of variables is equal to the sum of the complemented variables.

$$\overline{AB} = \overline{A} + \overline{B}$$

Applying DeMorgan's first theorem to gates:



Inputs		Output	
A	B	\overline{AB}	$\overline{A} + \overline{B}$
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0

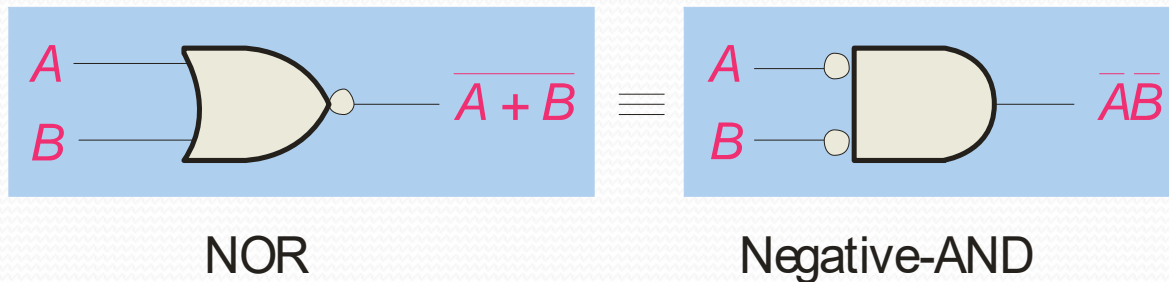
DeMorgan's Theorem

DeMorgan's 2nd Theorem

The complement of a sum of variables is equal to the product of the complemented variables.

$$\overline{A + B} = \overline{A} \cdot \overline{B}$$

Applying DeMorgan's second theorem to gates:



Inputs		Output	
<i>A</i>	<i>B</i>	$\overline{A + B}$	\overline{AB}
0	0	1	1
0	1	0	0
1	0	0	0
1	1	0	0

DeMorgan's Theorem

Example

Apply DeMorgan's theorem to remove the overbar covering both terms from the

expression $X = \overline{\overline{C} + D}$.

Solution

To apply DeMorgan's theorem to the expression, you can break the overbar covering both terms and change the sign between the terms.

This results in

$X = \overline{\overline{C}} \cdot \overline{D}$. Deleting the double bar gives $X = C \cdot \overline{D}$.

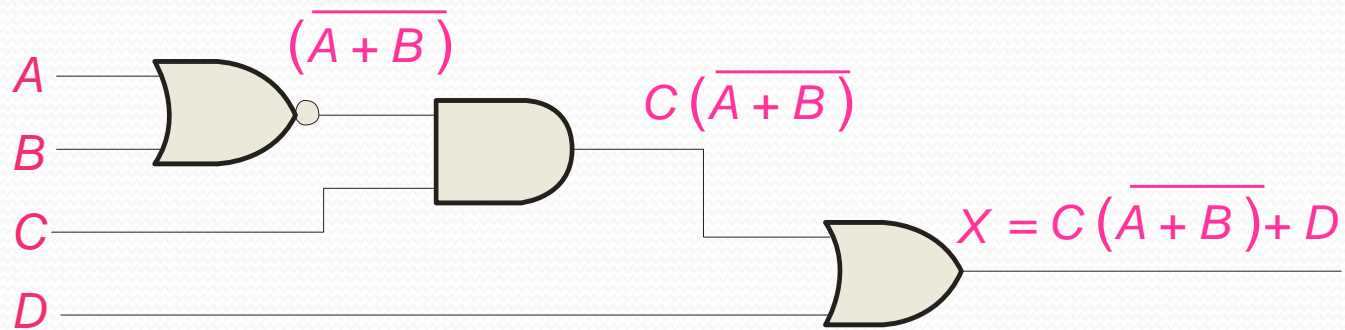
Boolean Analysis of Logic Circuits

Combinational logic circuits can be analyzed by writing the expression for each gate and combining the expressions according to the rules for Boolean algebra.

Example Solution

Apply Boolean algebra to derive the expression for X.

Write the expression for each gate:



Applying DeMorgan's theorem and the distribution law:

$$X = C (\overline{A} \overline{B}) + D = \overline{A} \overline{B} \overline{C} + D$$

SOP and POS forms

Boolean expressions can be written in the **sum-of-products** form (**SOP**) or in the **product-of-sums** form (**POS**). These forms can simplify the implementation of combinational logic, particularly with PLDs. In both forms, an overbar cannot extend over more than one variable.

An expression is in SOP form when two or more product terms are summed as in the following examples:

$$\bar{A}\bar{B}\bar{C} + AB$$

$$AB\bar{C} + \bar{C}\bar{D}$$

$$CD + \bar{E}$$

An expression is in POS form when two or more sum terms are multiplied as in the following examples:

$$(A + B)(\bar{A} + C)$$

$$(A + B + \bar{C})(B + D)$$

$$(\bar{A} + B)C$$

SOP Standard form

In **SOP standard form**, every variable in the domain must appear in each term. This form is useful for constructing truth tables or for implementing logic in PLDs.

You can expand a nonstandard term to standard form by multiplying the term by a term consisting of the sum of the missing variable and its complement.

Example Solution

Convert $X = \bar{A} \bar{B} + A B C$ to standard form.

The first term does not include the variable C . Therefore, multiply it by the $(C + \bar{C})$, which = 1:

$$\begin{aligned} X &= \bar{A} \bar{B} (C + \bar{C}) + A B C \\ &= \bar{A} \bar{B} C + \bar{A} \bar{B} \bar{C} + A B C \end{aligned}$$

POS Standard form

In **POS standard form**, every variable in the domain must appear in each sum term of the expression.

You can expand a nonstandard POS expression to standard form by adding the product of the missing variable and its complement and applying rule 12, which states that $(A + B)(A + C) = A + BC$.

Example Convert $X = (\bar{A} + \bar{B})(A + B + C)$ to standard form.

Solution The first sum term does not include the variable C . Therefore, add $C\bar{C}$ and expand the result by rule 12.

$$\begin{aligned} X &= (\bar{A} + \bar{B} + C\bar{C})(A + B + C) \\ &= (\bar{A} + \bar{B} + C)(\bar{A} + \bar{B} + \bar{C})(A + B + C) \end{aligned}$$

Binary representation of SOP and POS forms

SOP standard form $\overline{A}\overline{B}\overline{C}\overline{D} = 1 \cdot \overline{0} \cdot 1 \cdot \overline{0} = 1 \cdot 1 \cdot 1 \cdot 1 = 1$

POS standard form $A + \overline{B} + C + \overline{D} = 0 + \overline{1} + 0 + \overline{1} = 0 + 0 + 0 + 0 = 0$

Converting standard SOP to POS

SOP standard form

$$\overline{A}\overline{B}\overline{C} + \overline{A}B\overline{C} + \overline{A}BC + A\overline{B}\overline{C} + ABC$$

$$000 + 010 + 011 + 101 + 111$$

The equivalent POS standard form contains the other three remaining terms 001, 100 *and* 110

$$(A + B + \overline{C})(\overline{A} + B + C)(\overline{A} + \overline{B} + C)$$

Converting SOP to truth table

1. First list all possible combinations of binary values of the variables in the expression.
2. Convert the SOP to standard form if it is not already.
3. Place a 1 in the output column for each binary value that makes the standard SOP expression a 1 and place a 0 for all the remaining binary values

Example: Develop a Truth Table for the standard SOP Expression :-

$$ABC + \overline{A}\overline{B}C + A\overline{B}\overline{C}$$

Solution : Three variables, then 8 possible combinations.

For each product term in the Expression, place (1) in o/p, and place (0) in for the other terms in o/p

I/P			O/P		PRODUCT TERM
A	B	C	X		
0	0	0	0		
0	0	1	1	$\overline{A}\overline{B}C$	
0	1	0	0		
0	1	1	0		
1	0	0	1	$A\overline{B}\overline{C}$	
1	0	1	0		
1	1	0	0		
1	1	1	1	ABC	

Karnaugh maps

- Array of cells, each cell represents one possible product term
- It is a tool for simplifying combinational logic with 3 or 4 variables.
- For 3 variables, 8 cells are required (2^3).
- Each cell is adjacent to cells that are immediately next to it on any of its four sides.
- A cell is not adjacent to the cells that diagonally touch any of its corners.
- “wrap-around” adjacency means the top row is adjacent to the bottom row and left column to right column.

Grouping the 1s

The goal is to maximize the size of the groups and to minimize the number of the groups

- A group must contain either 1, 2, 4, 8, or 16 cells.
- Each cell in a group must be adjacent to one or more cells in that same group.
- Include the largest possible # of 1s in a group in accordance with rule 1
- Each 1 on the map must be included in at least one group.

Karnaugh maps

K-maps can simplify combinational logic by grouping cells and eliminating variables that change.

Example Group the 1's on the map and read the minimum logic.

Solution

1. Group the 1's into two overlapping groups as indicated.
2. Read each group by eliminating any variable that changes across a boundary.
3. The vertical group is read $\bar{A}\bar{C}$.
4. The horizontal group is read $\bar{A}B$.

$$X = \bar{A}\bar{C} + \bar{A}B$$

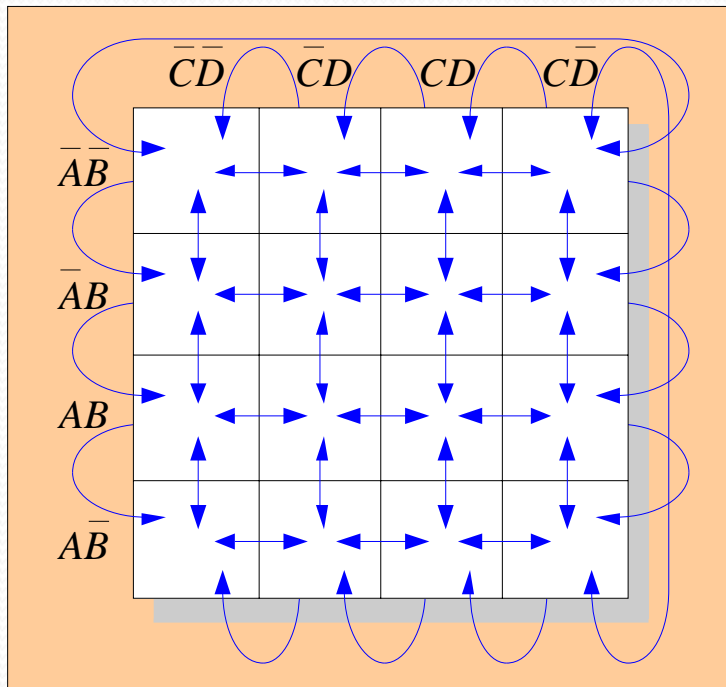
B changes across this boundary

C changes across this boundary

	<i>C</i>	
<i>AB</i>	0	1
00	1	
01	1	1
11		
10		

Karnaugh maps

A 4-variable map has an adjacent cell on each of its four boundaries as shown.



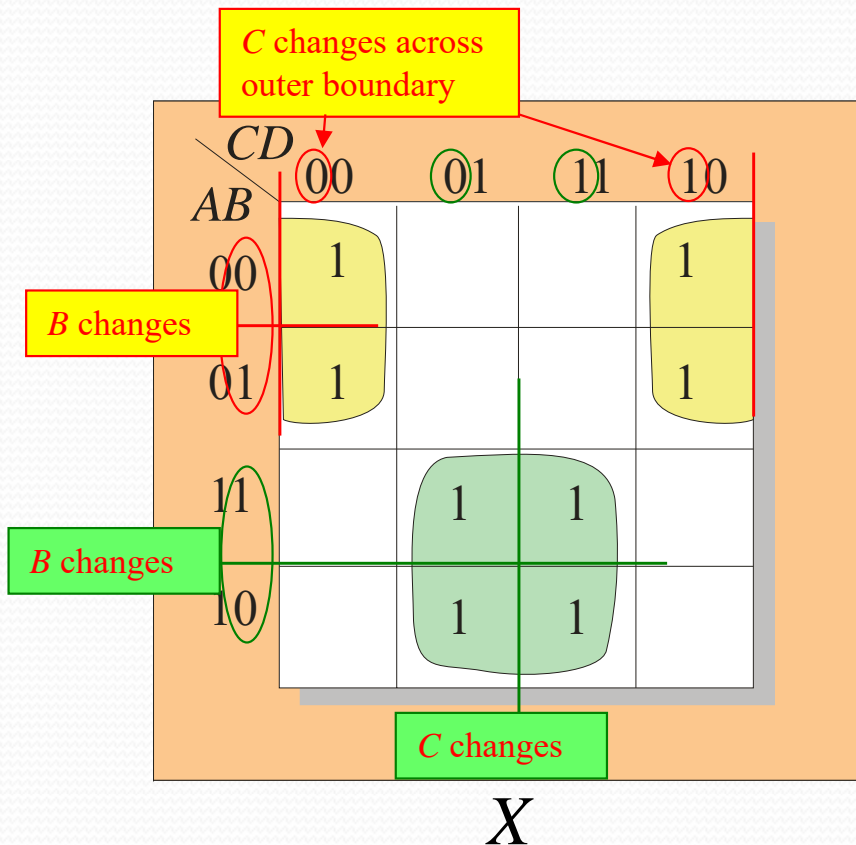
Each cell is different only by one variable from an adjacent cell.

Grouping follows the rules given in the text.

The following slide shows an example of reading a four variable map using binary numbers for the variables...

Karnaugh maps

Example Group the 1's on the map and read the minimum logic.



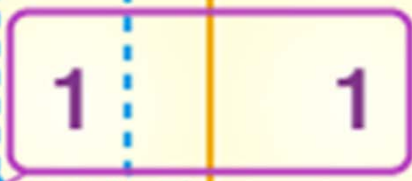
Solution

1. Group the 1's into two separate groups as indicated.
2. Read each group by eliminating any variable that changes across a boundary.
3. The upper (yellow) group is read as $\bar{A}\bar{D}$.
4. The lower (green) group is read as AD .

$$X = \bar{A}\bar{D} + AD$$

A \ BC	00	01	11	10
0	0	1	1	0
1	0	0	1	1

The table cells are numbered in green: 0 (row 0, col 00), 1 (row 0, col 01), 2 (row 0, col 10), 3 (row 0, col 11), 4 (row 1, col 00), 5 (row 1, col 01), 6 (row 1, col 10), 7 (row 1, col 11).



No need of this group as

PQ \ RS	00	01	11	10
00	1 0	0 1	0 3	1 2
01	0 4	1 5	1 7	0 6
11	0 12	1 13	1 15	0 14
10	1 8	0 9	0 11	1 10

$f_1 = \overline{P} \overline{Q} \overline{R} S + \overline{P} \overline{Q} R \overline{S} + \overline{P} Q \overline{R} S + \overline{P} Q R \overline{S}$
 $f_2 = \overline{P} \overline{Q} \overline{R} S + \overline{P} \overline{Q} R \overline{S} + P \overline{Q} \overline{R} S + P \overline{Q} R \overline{S}$
 $f_3 = \overline{P} \overline{Q} \overline{R} S + \overline{P} \overline{Q} R \overline{S} + \overline{P} Q \overline{R} S + P \overline{Q} \overline{R} S + P \overline{Q} R \overline{S} + P Q \overline{R} S + P Q R \overline{S}$

$f_4 = \overline{P} \overline{Q} \overline{R} S + \overline{P} \overline{Q} R \overline{S} + \overline{P} Q \overline{R} S + P \overline{Q} \overline{R} S + P \overline{Q} R \overline{S} + P Q \overline{R} S + P Q R \overline{S}$
 $f_5 = \overline{P} \overline{Q} \overline{R} S + \overline{P} \overline{Q} R \overline{S} + \overline{P} Q \overline{R} S + P \overline{Q} \overline{R} S + P \overline{Q} R \overline{S} + P Q \overline{R} S + P Q R \overline{S}$