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Stability of Linear Time Invariant Systems



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Stability of Linear Time Invariant Systems

Given a transfer function with two complex poles and one zero:

$$H(s) = \frac{(s/\alpha\zeta\omega_n) + 1}{(s/\omega_n)^2 + 2\zeta(s/\omega_n) + 1}$$

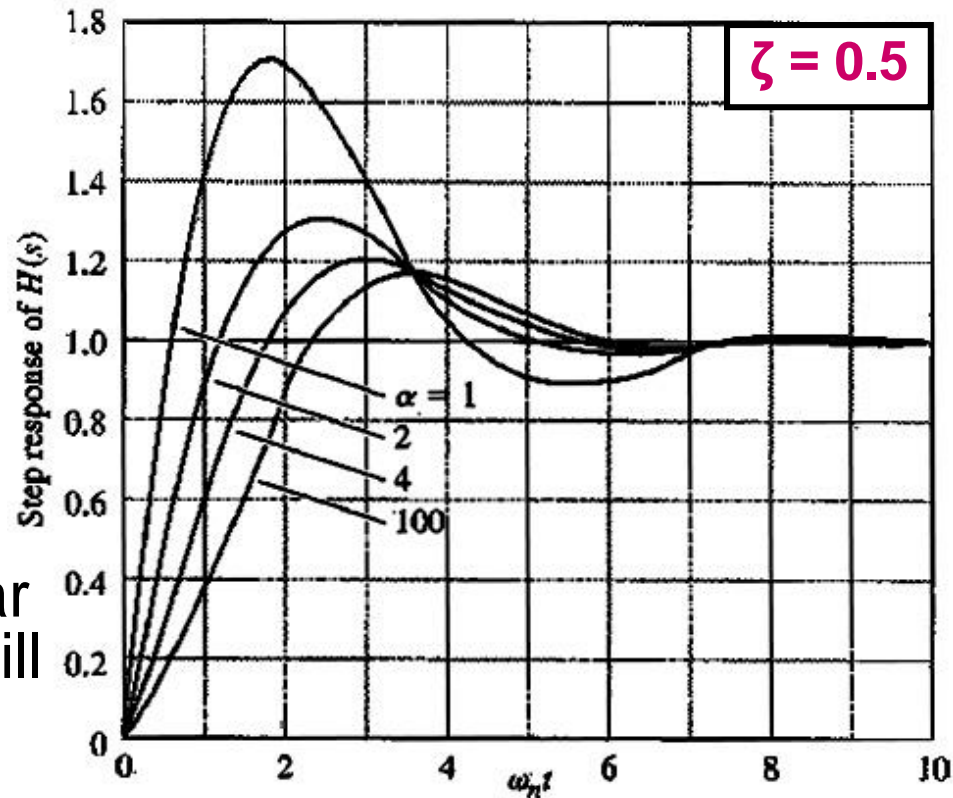
The zero is located at:

$$s = -\alpha\zeta\omega_n = -\alpha\sigma$$

The poles are located at:

$$s = -\sigma \pm j\omega_d$$

- If α is large, the zero will be far from the poles and the zero will have little effect on the response.
- If $\alpha \approx 1$, the value of the zero will be close to that of the real part of the poles and can be expected to have a substantial influence on the response.





$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)} \quad \text{Closed Loop T.F.}$$

Unity Feedback = $H(s) = 1$

Final Value Theorem

$$y(\infty) = \lim_{s \rightarrow 0} sY(s)$$

let $G(s) = \frac{1}{(s+1)(s+2)}$

$$Y(s) = \frac{G(s)}{1 + G(s)} \cdot R(s)$$

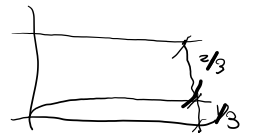
$$= \frac{1}{(s^2 + 3s + 2) + 1} = \frac{1}{s^2 + 3s + 3}$$

$$Y(s) = \frac{1}{s(s^2 + 3s + 2)}$$

$$y(\infty) = 1/3$$

Error = $(r(t) - y(t))$

$$E(\infty) = 1 - \frac{1}{3} = \frac{2}{3}$$



$\frac{1}{s} = \sum \omega_n = \infty =$ جزء الحقيقى

↑ time constant

$$T.F. = \frac{G(s)}{1 + G(s)} = \frac{\frac{B(s)}{A(s)}}{1 + \frac{B(s)}{A(s)}}$$

$$= \frac{B(s)/A(s)}{(A(s) + B(s))/A(s)}$$

$$= \frac{B(s)/A(s)}{\frac{A(s) + B(s)}{A(s)}} = \frac{B(s)}{A(s) + B(s)}$$

Effects of Zeros and Additional Poles

Given a transfer function with two complex poles and one zero:

$$H(s) = \frac{(s/\alpha\zeta\omega_n) + 1}{(s/\omega_n)^2 + 2\zeta(s/\omega_n) + 1}$$

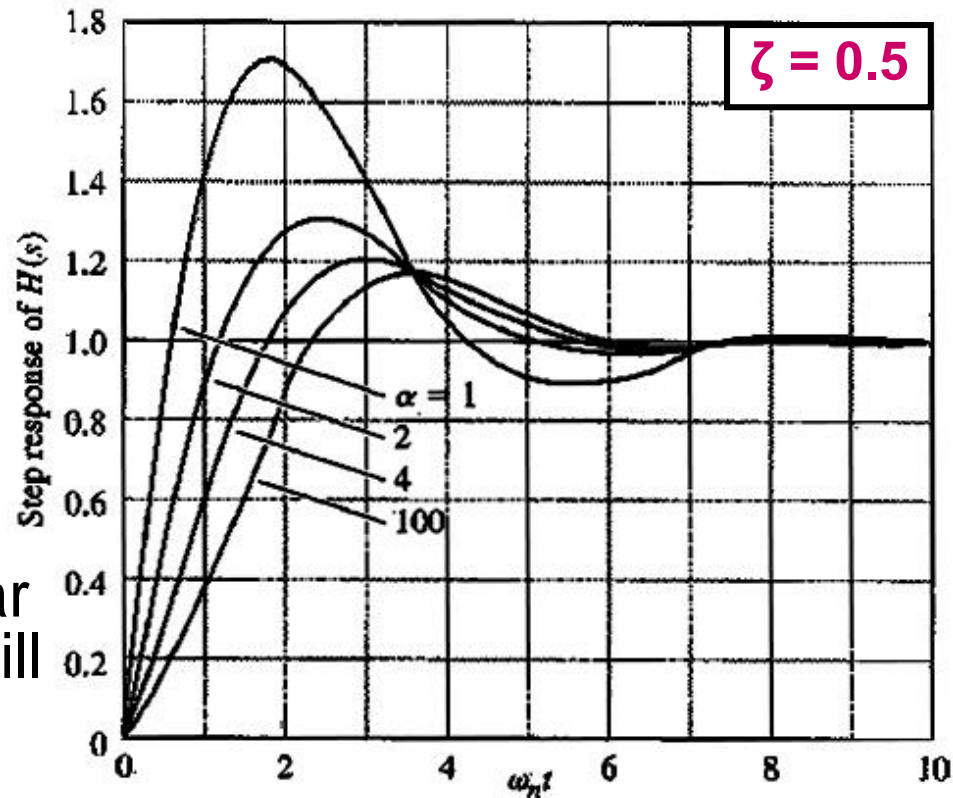
The zero is located at:

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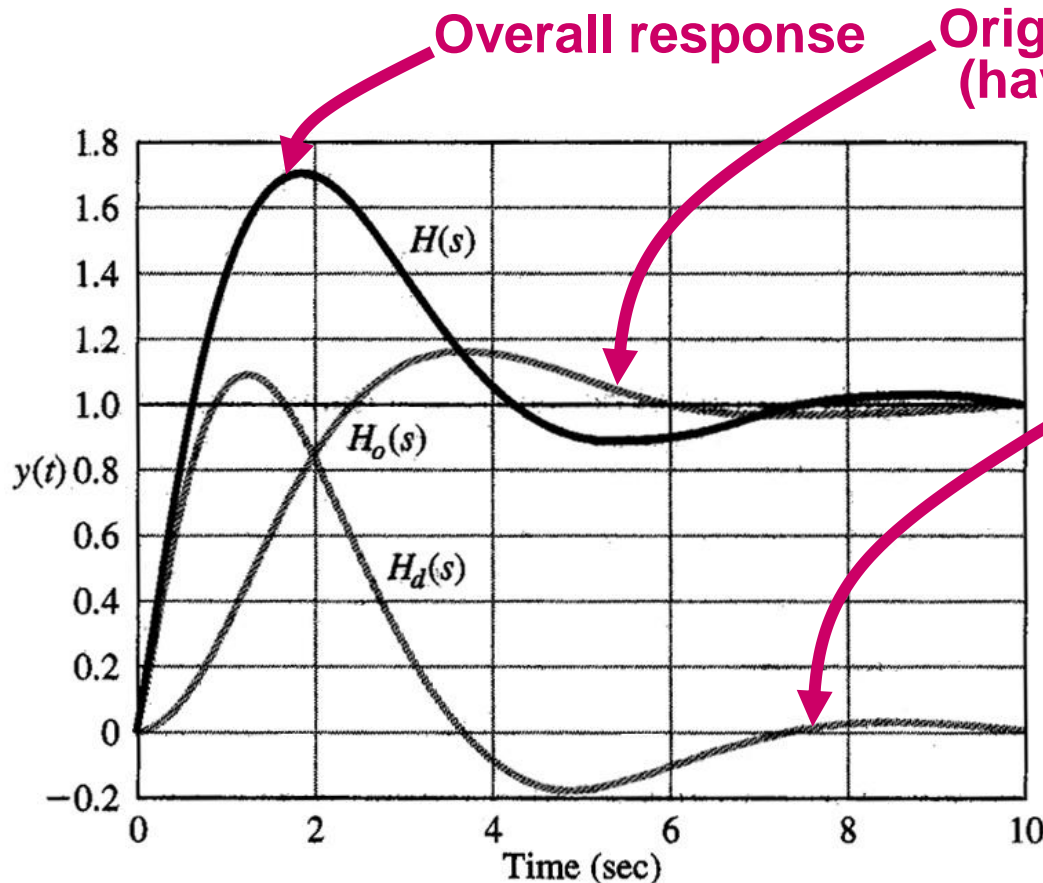
$$s = -\sigma \pm j\omega_d$$

- If α is large, the zero will be far from the poles and the zero will have little effect on the response.
- If $\alpha \approx 1$, the value of the zero will be close to that of the real part of the poles and can be expected to have a substantial influence on the response.



Effects of Zeros and Additional Poles

Step response of a second-order system with zero in the LHP



- The major effect of the zero in the LHP is to increase the overshoot.
- It has very little influence on the settling time.

$$s_{1,2} = \alpha \pm j\omega_d$$

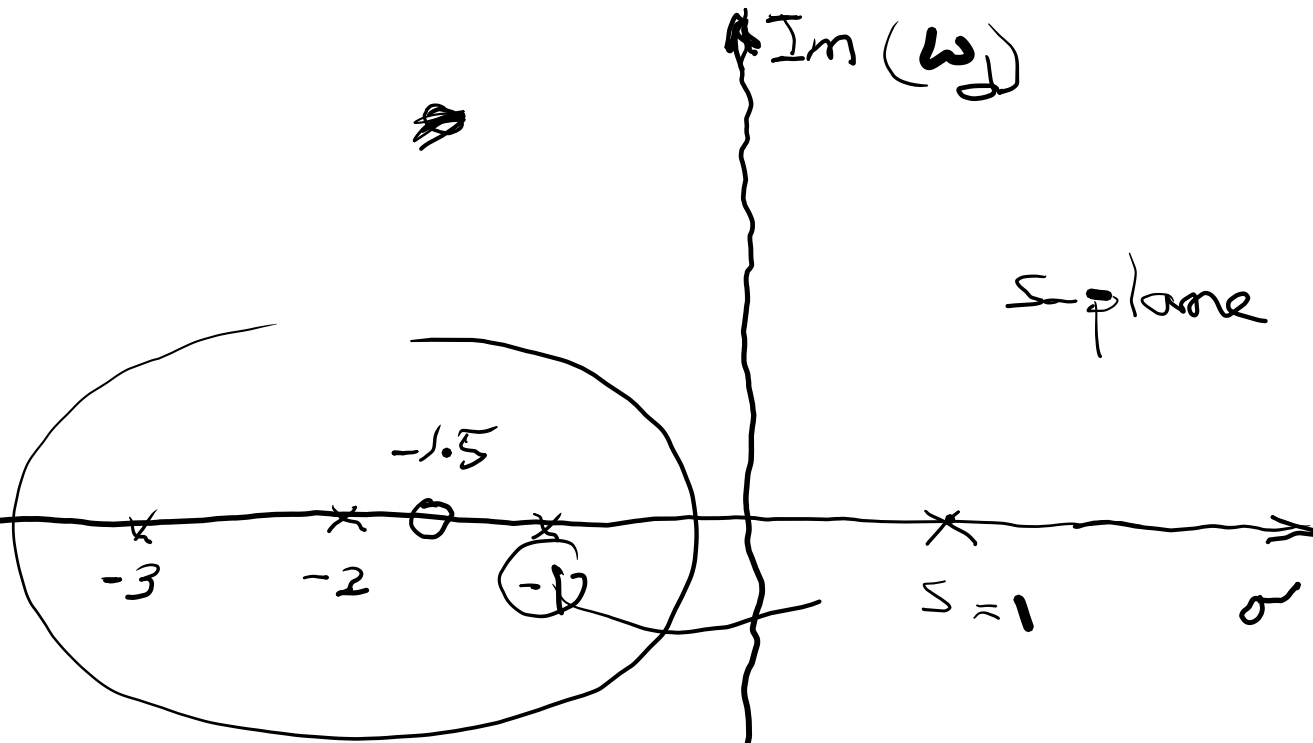
$$T.F. = \frac{(s + 1.5)}{(s + 3)(s + 2)(s + 1)(s - 1)}$$

$$s = 1$$

$$= \frac{(s^2 + 3s + 12)}{(s^3 + 6s^2 + 11s + 6)}$$

$$(s + 1)(s + 2)(s + 3)$$

$$(s + 1)(s + 2)(s + 3)$$



$$= \frac{A}{s + 3} + \frac{B}{s + 2} + \frac{C}{s - 1}$$

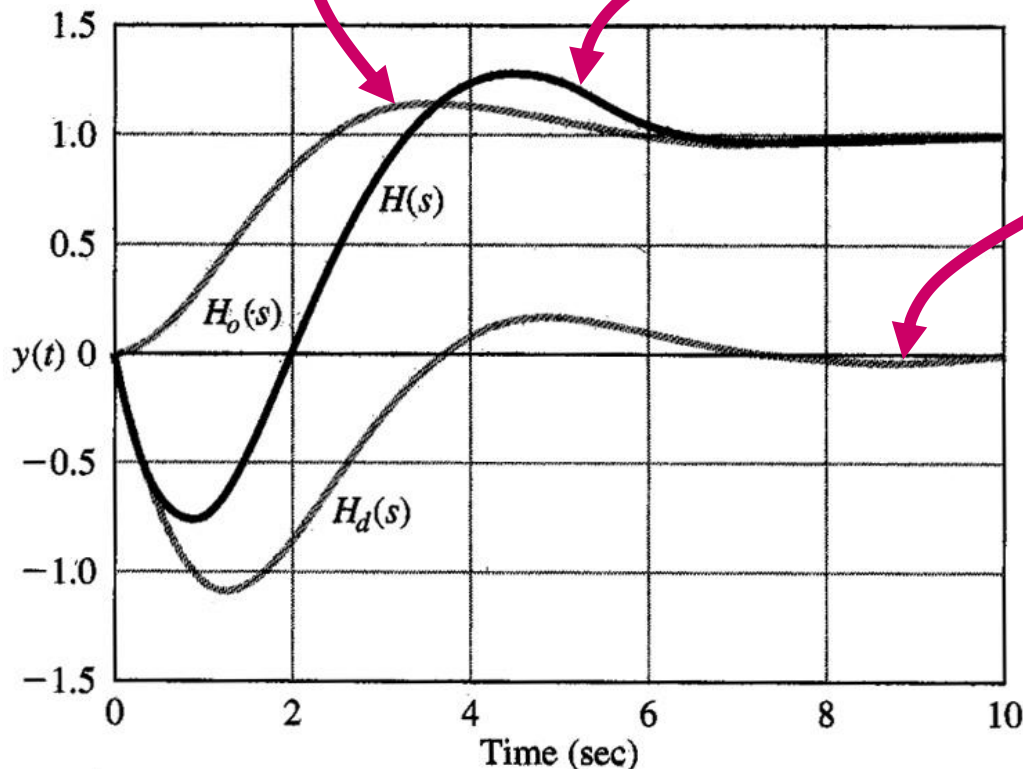
$$y(t) = Ae^{-3t} + Be^{-2t} + Ce^t$$

Effects of Zeros and Additional Poles

Step response of a second-order system with zero in the RHP

Original response
(having no zero)

Overall response



Response introduced
by the zero

- The major effect of the zero in the RHP is to depress the overshoot.
- It may cause an initial undershoot (the step response starts out in the wrong direction).

Effects of Zeros and Additional Poles

Given a transfer function with two complex poles and one real pole:

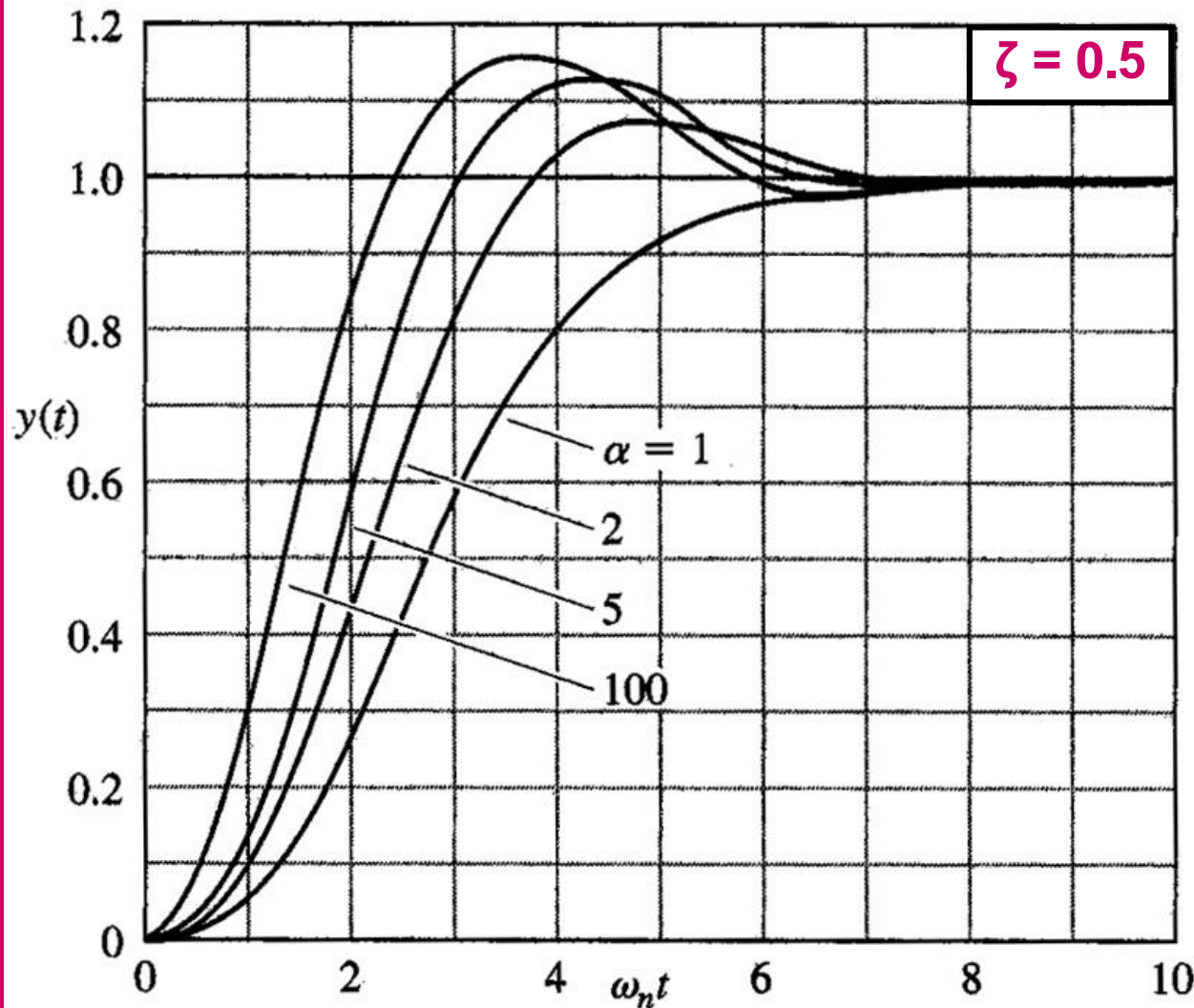
$$H(s) = \frac{1}{(s/\alpha\zeta\omega_n + 1) \left[(s/\omega_n)^2 + 2\zeta(s/\omega_n) + 1 \right]}$$

The real pole is located at :

$$s = -\alpha\zeta\omega_n = -\alpha\sigma$$

Effects of Zeros and Additional Poles

Step response of several third-order systems with $\zeta=0.5$



- The major effect of an extra pole is to increase the rise time.
- If α is large (hence far from the other poles), the extra pole have little effect on the response.

$$\begin{pmatrix} s+8 \\ +5 \\ +3 \end{pmatrix} \frac{1}{(s+3)(0.5s+1+1)}$$

x

x
-8
rd
3

-8t
Ae

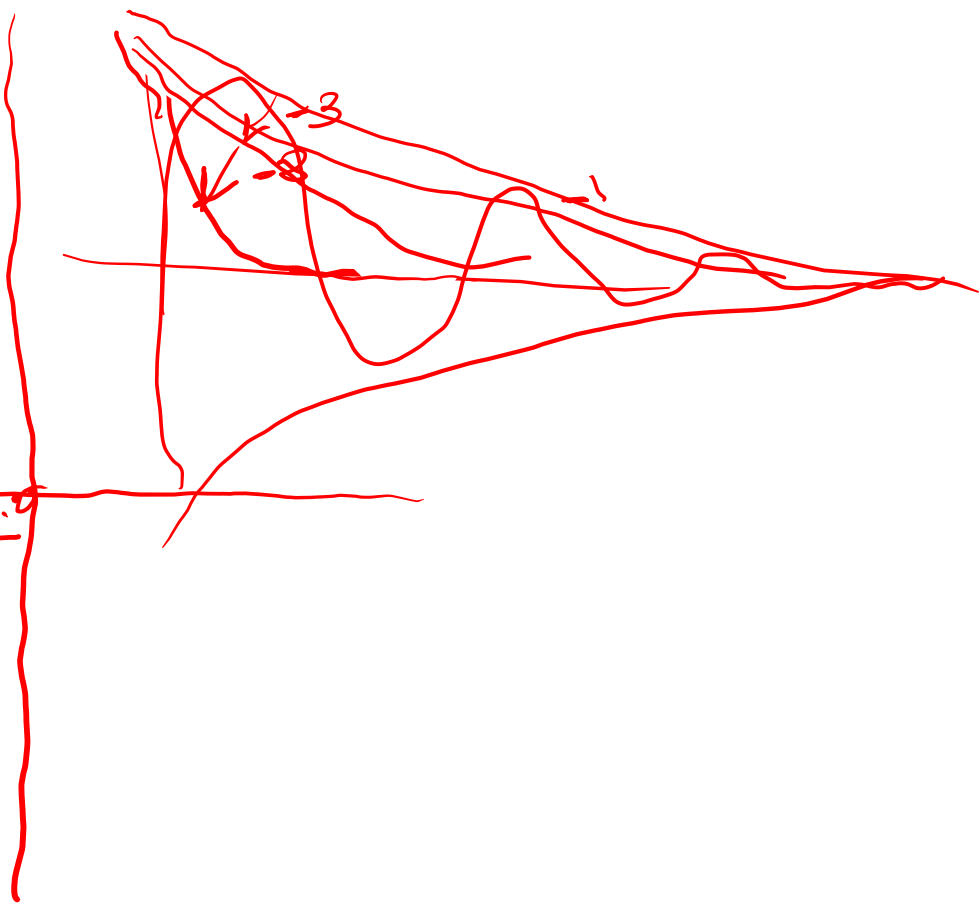
x
-5
rd
3

-5t
Ae

x
-3
rd
3

-3t
Ae

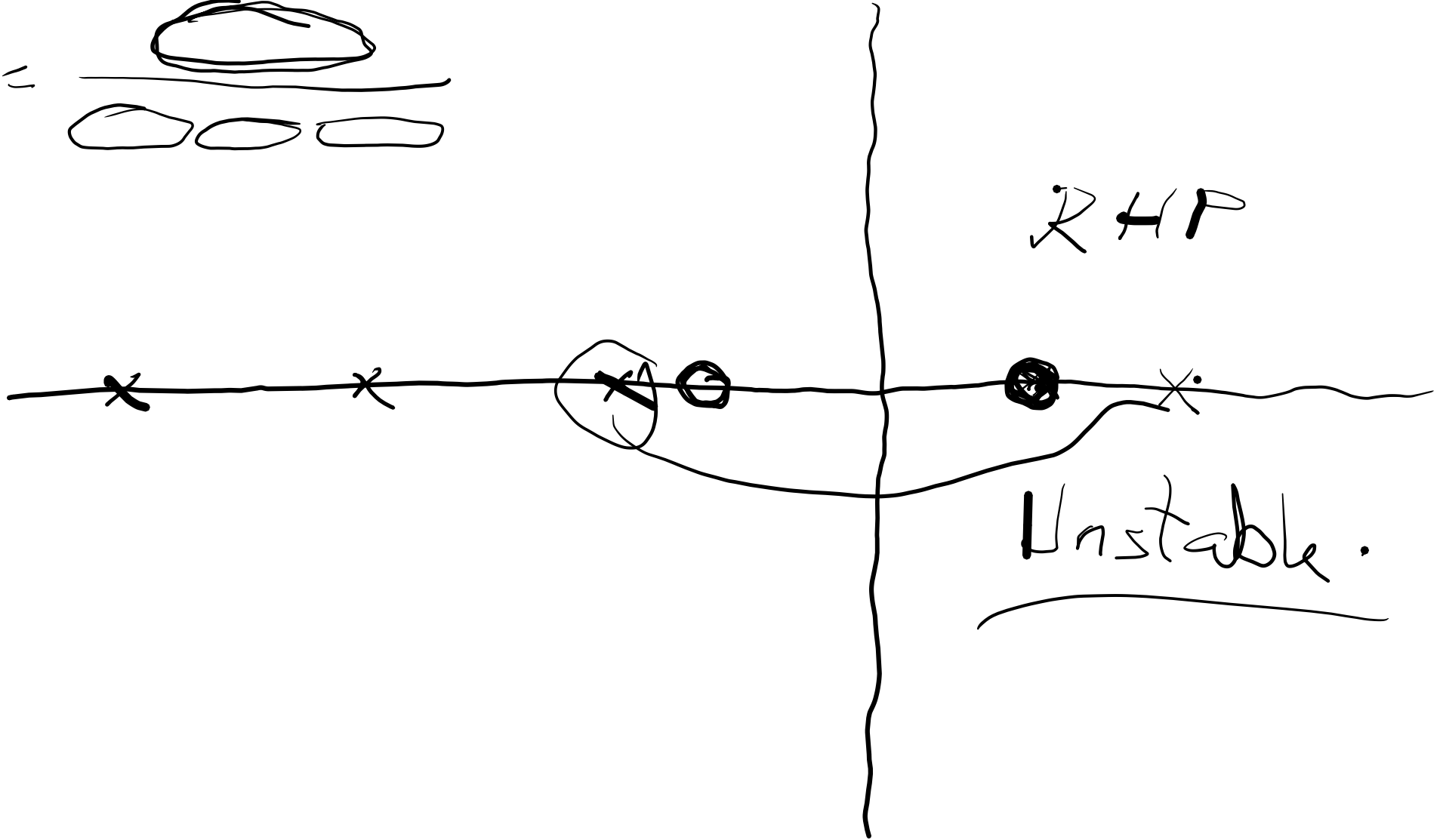
x



Summary of Effects of Pole and Zero

- The pole and zero locations determine the character of the transient response.
 - A zero in the LHP will increase the overshoot if the zero is within a factor of 4 of the real part of the complex poles (σ).
 - A zero in the RHP will depress the overshoot (and may cause the step response to start out in the wrong direction).
 - An additional pole in the LHP will decrease the rise time significantly (the response will be faster) if the extra pole is within a factor of 4 of the real part of the complex poles.
 - If the extra pole is more than a factor of 6 of the real part of the complex poles, the effect is negligible.
- An additional pole in the RHP will ...?





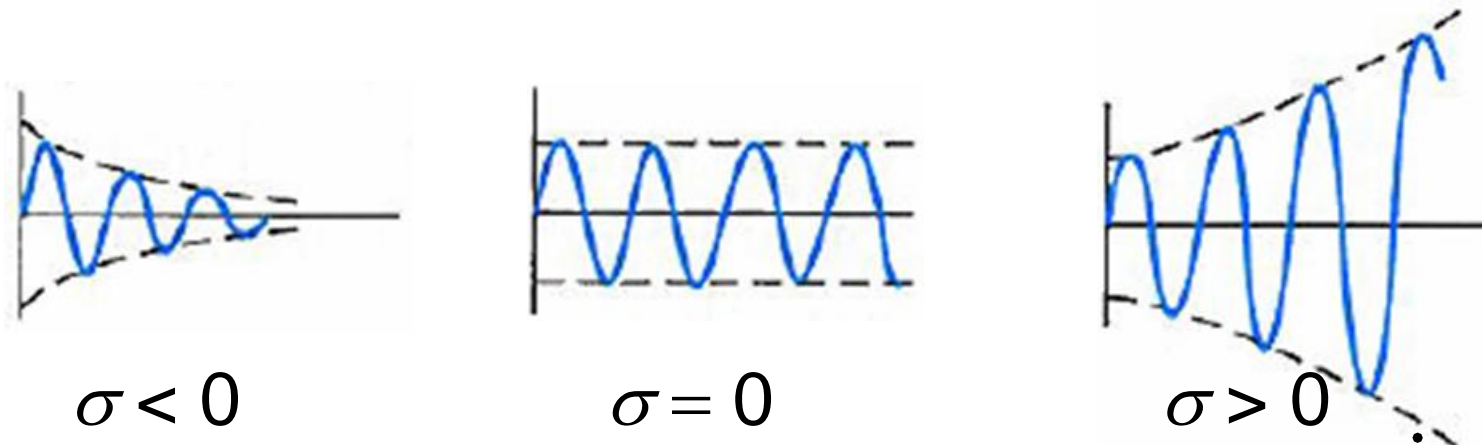
RHP

Unstable.

Stability

Consider the linear time-invariant system (LTI system).
For those systems, the following condition for stability applies:

“ A linear time-invariant system is said to be stable if **all** the roots of the transfer function denominator polynomial have negative real parts (i.e., they are **all** in the left-hand s-plane) and is unstable otherwise. ”



A system is stable if its initial conditions decay to zero and is unstable if they diverge.

Stability of Linear Time-Invariant Systems

Consider the linear time-invariant system whose transfer function denominator polynomial leads to the **characteristic equation**

$$s^n + a_1s^{n-1} + a_2s^{n-2} + \dots + a_{n-1}s + a_n = 0$$

Assume that the roots $\{p_{ij}\}$ of the characteristic equation are real or complex, but are distinct; so that the transfer function can be given as:

$$T(s) = \frac{Y(s)}{R(s)} = \frac{b_0s^m + b_1s^{m-1} + b_2s^{m-2} + \dots + b_{m-1}s + b_m}{s^n + a_1s^{n-1} + a_2s^{n-2} + \dots + a_{n-1}s + a_n}$$

$$= \frac{K \prod_{i=1}^m (s - z_i)}{\prod_{i=1}^n (s - p_i)}, \quad m \leq n$$



Stability of Linear Time-Invariant Systems

The solution of the system response, found using partial fraction expansion, may be written as:

$$y(t) = \sum_{i=1}^n K_i e^{p_i t}$$

The system is stable **if and only if** (necessary and sufficient condition) every term in the equation above goes to zero as $t \rightarrow \infty$.

$$e^{p_i t} \rightarrow 0 \text{ for all } p_i$$

This situation will happen if **all** the poles of the system are strictly in the LHP.

$$\operatorname{Re}\{p_i\} < 0$$

Stability of Linear Time-Invariant Systems

If any LHP poles are repeated, the response will change because a polynomial in t must be included in place of K_j . However, the conclusion is the same: as $t \rightarrow \infty$, $y(t) \rightarrow 0$.

$$\lim_{t \rightarrow \infty} \frac{1}{(n-1)!} t^{n-1} e^{-at} \equiv 0 \quad \text{for any } n \geq 1$$

$f(t)$	$F(s)$
$\frac{t^{n-1} e^{-at}}{(n-1)!} 1(t)$ $n \geq 1$	$\frac{1}{(s+a)^n}$

- Thus, the stability of a system can be determined by computing the location of the roots of the characteristic equation and determining whether they are all in the LHP. This is called **internal stability**.
- If a system has any poles in the RHP, it is **unstable**.
- If a system has nonrepeated $j\omega$ axis poles, then it is said to be **neutrally stable**.
- If the system has repeated poles on the $j\omega$ axis, then it is **unstable**, as it results in $te^{\pm j\omega t}$.



Routh's Stability Criterion

The roots of the characteristic equation **determine** whether the system is stable or unstable.

Consider the characteristic equation

$$a(s) = s^n + a_1s^{n-1} + a_2s^{n-2} + \dots + a_{n-1}s + a_n$$

Routh's stability criterion allows us to make certain statements about the stability of the system without actually solving for the roots of the polynomial.

Routh's stability criterion is also useful for determining the ranges of coefficients of polynomials for stability, especially when the coefficients are in symbolic (nonnumerical) form.

Routh's Stability Criterion

A *necessary condition for stability* of the system is that all of the roots of its characteristic equation have negative real parts, which in turn requires that all the coefficients $\{a_i\}$ be positive.

“ A necessary (but not sufficient) condition for stability is that *all* the coefficients of the characteristic polynomial be positive. ”



“ If a system is stable, then *all* the coefficients of the characteristic polynomial are positive. ”

Implication $p \rightarrow q$:

- If p , then q .
- p implies q .
- p only if q .
- p is the sufficient condition for q .
- q is the necessary condition for p .

Routh's Stability Criterion

Once the elementary necessary conditions have been satisfied, a more powerful test is needed.

Routh in 1874 proposed a test that requires the computation of a triangular array that is a function of the coefficients of the characteristic equation.

“ A system is stable if and only if *all* the elements in the first column of the Routh array are positive. ”



“ If a system is stable then *all* the elements in the first column of the Routh array are positive, and vice versa. ”

Bi-Implication $p \leftrightarrow q$:

- p if and only if q .
- p is the sufficient and necessary conditions for q .
- if p then q , and if q then p .

Routh's Stability Criterion

Consider the characteristic equation

$$a(s) = s^n + a_1 s^{n-1} + a_2 s^{n-2} + \dots + a_{n-1} s + a_n$$

$$1s^4 + 5s^3 + 2s^2 + 3s + 1$$

First, arrange the coefficients of the characteristic polynomial in two rows, beginning with the first and second coefficients and followed by the even-numbered and odd-numbered coefficients:

$$s^n : \quad 1 \quad a_2 \quad a_4 \quad \dots$$

$$s^{n-1} : \quad a_1 \quad a_3 \quad a_5 \quad \dots$$

s^4	1	2
s^3	5	0
s^2	a	b
s^1	c	d
s^0	e	

$$a = \frac{5 \times 2 - 3 \times 1}{5}$$

$$= \frac{7}{5}$$

$$b = \frac{5 \times 1 - 0 \times 1}{5}$$

$$= 1$$

$$c = \frac{3a - 5b}{a}$$

$$d = 0$$

$$e = \frac{c \times b - 0}{c} = b$$



Routh's Stability Criterion

Then add subsequent rows to complete the **Routh array**:

Row n	$s^n :$	1	a_2	a_4	\dots	$b_1 = \frac{a_1 a_2 - a_3}{a_1}, c_1 = \frac{b_1 a_3 - b_2 a_1}{b_1},$
Row $n-1$	$s^{n-1} :$	a_1	a_3	a_5	\dots	$b_2 = \frac{a_1 a_4 - a_5}{a_1}, c_2 = \frac{b_1 a_5 - b_3 a_1}{b_1},$
Row $n-2$	$s^{n-2} :$	b_1	b_2	b_3	\dots	$b_3 = \frac{a_1 a_6 - a_7}{a_1}, c_3 = \frac{b_1 a_7 - b_4 a_1}{b_1}, \dots$
Row $n-3$	$s^{n-3} :$	c_1	c_2	c_3	\dots	
\vdots	\vdots	\vdots	\vdots	\vdots		
Row 2	$s^2 :$	*	*			
Row 1	$s^1 :$	*				
Row 0	$s^0 :$	*				

First column of Routh's array

- If the elements of the first column are all positive, then all the roots are in the LHP.
- If the elements are not all positive, then the number of roots in the RHP equals the number of sign changes in the column.



Example 1: Routh's Test

All the coefficients of the characteristic equation

$$a(s) = s^6 + 4s^5 + 3s^4 + 2s^3 + 1s^2 + 4s + 4$$

are positive \rightarrow the system maybe stable or maybe not.

Now, we determine whether all of the roots are in the LHP

$$a(s) = s^6 + 4s^5 + 3s^4 + 2s^3 + 1s^2 + 4s + 4$$

$$s^6 : \quad 1 \quad 3 \quad 1 \quad 4$$

$$s^5 : \quad 4 \quad 2 \quad 4 \quad 0$$

$$s^4 : \quad ? \quad ? \quad ?$$

$$s^3 : \quad ? \quad ?$$

$$s^2 : \quad ? \quad ?$$

$$s^1 : \quad ?$$

$$s^0 : \quad ?$$

Example 1: Routh's Test

$$s^6 : 1 \quad 3 \quad 1 \quad 4$$

$$s^5 : 4 \quad 2 \quad 4 \quad 0$$

$$s^4 : \boxed{2.5} \quad \boxed{0} \quad \boxed{4}$$

$$s^3 : ? \quad ?$$

$$s^2 : ? \quad ?$$

$$s^1 : ?$$

$$s^0 : ?$$

$$b_1 = \frac{4 \cdot 3 - 2}{4} = 2.5$$

$$b_2 = \frac{4 \cdot 1 - 4}{4} = 0$$

$$b_3 = \frac{4 \cdot 4 - 0}{4} = 4$$

Example 1: Routh's Test

$$s^6: \quad 1 \quad 3 \quad 1 \quad 4$$

$$s^5: \quad 4 \quad 2 \quad 4 \quad 0$$

$$s^4: \quad 2.5 \quad 0 \quad 4$$

$$s^3: \quad \boxed{2} \quad \boxed{-2.4}$$

$$s^2: \quad \boxed{3} \quad \boxed{4}$$

$$s^1: \quad ?$$

$$s^0: \quad ?$$

$$c_1 = \frac{2.5 \cdot 2 - 0 \cdot 4}{2.5} = 2$$

$$c_2 = \frac{2.5 \cdot 4 - 4 \cdot 4}{2.5} = -2.4$$

$$d_1 = \frac{2 \cdot 0 - (-2.4) \cdot 2.5}{2} = 3$$

$$d_2 = \frac{2 \cdot 4 - 0 \cdot 2.5}{2} = 4$$

Example 1: Routh's Test

s^6 :	1	3	1	4
s^5 :	4	2	4	0
s^4 :	2.5	0	4	
s^3 :	2	-2.4		
s^2 :	3	4		
s^1 :	-5.067			
s^0 :	4			

$$e_1 = \frac{3 \cdot (-2.4) - 4.2}{3} = -5.067$$

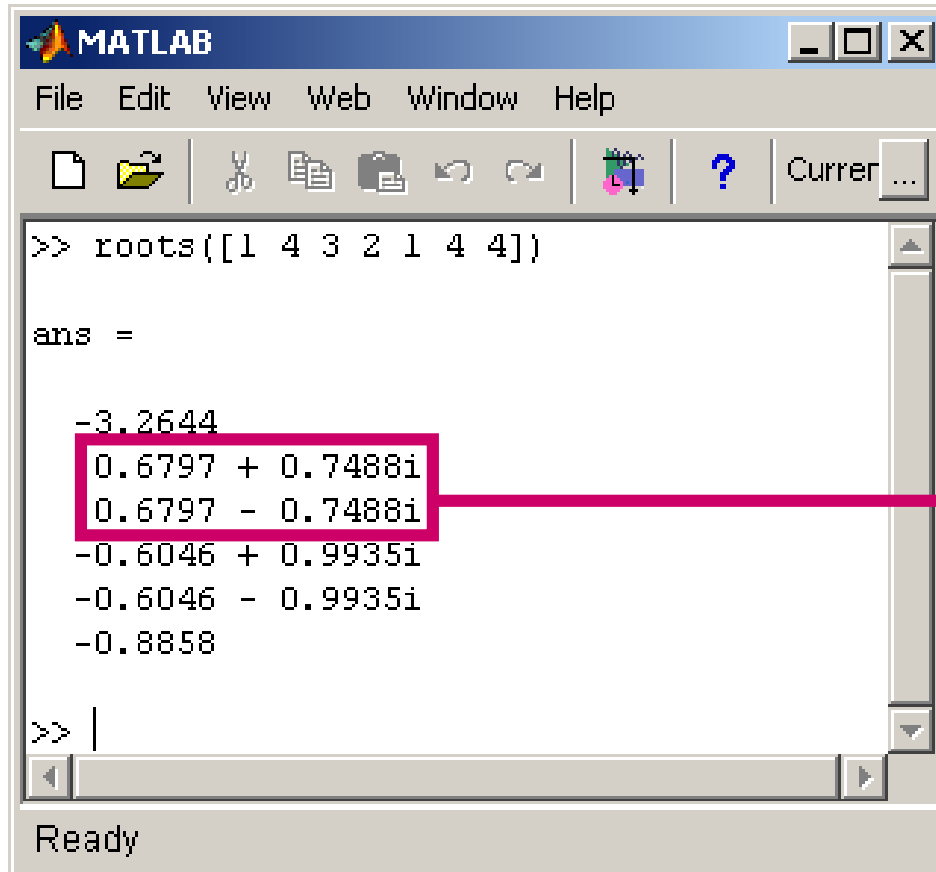
$$f_1 = \frac{-5.067 \cdot 4 - 0.3}{-5.067} = 4$$

- The elements of the first column are not all positive
 - The characteristic equation has at least one RHP root
 - The system is unstable
- There are two sign changes (+ to - and - to +)
 - There are two poles in the RHP



Example 1: Routh's Test

Finding roots of polynomials in MATLAB:



The image shows a MATLAB command window with the following content:

```
>> roots([1 4 3 2 1 4 4])  
  
ans =  
  
-3.2644  
0.6797 + 0.7488i  
0.6797 - 0.7488i  
-0.6046 + 0.9935i  
-0.6046 - 0.9935i  
-0.8858  
  
>> |
```

The two roots $0.6797 + 0.7488i$ and $0.6797 - 0.7488i$ are highlighted with a red box. A red arrow points from this box to the first bullet point of the list on the right.

- Roots in the RHP, i.e., roots with positive real parts
- There are two roots of characteristic equation in the RHP
- There are two unstable poles

Example 2: Routh's Test

Given the characteristic equation:

$$a(s) = s^6 + 4s^5 - 4s^4 + 6s^3 + 8s^2 + 1s + 1$$

Is the system described by this characteristic equation stable?

““ If a system is stable, then *all* the coefficients of the characteristic polynomial are positive. ””



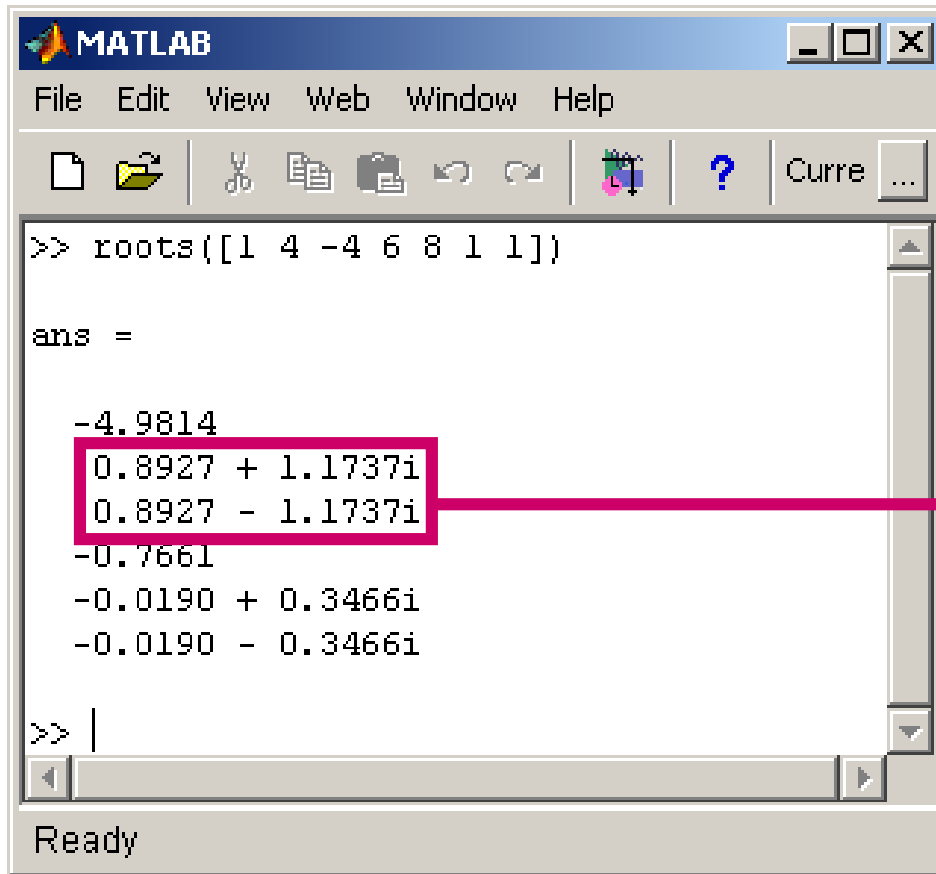
““ If **not** *all* the coefficients of the characteristic polynomial are positive, then a system is **not** stable. ””

$$\therefore p \rightarrow q \Leftrightarrow \sim q \rightarrow \sim p$$

Example 2: Routh's Test

$$a(s) = s^6 + 4s^5 - 4s^4 + 6s^3 + 8s^2 + 1s + 1$$

- There is a negative coefficient
→ The system is not stable
- Necessary condition for stability is not even fulfilled
→ No need to continue to Routh's Test



```
>> roots([1 4 -4 6 8 1 1])

ans =

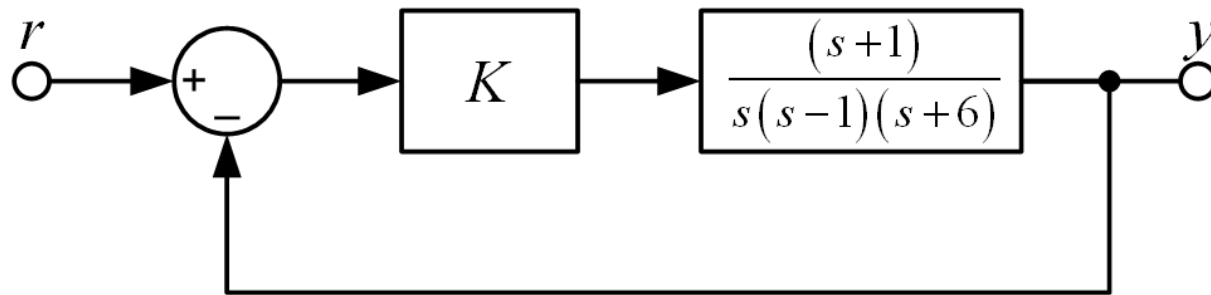
-4.9814
0.8927 + 1.1737i
0.8927 - 1.1737i
-0.7661
-0.0190 + 0.3466i
-0.0190 - 0.3466i

>> |
```

- Roots in the RHP, i.e., roots with positive real parts

Stability Versus Parameter Range

Consider the system shown below. The stability properties of the system are a function of the proportional feedback gain K . Determine the range of K over which the system is stable.



$$\frac{Y(s)}{R(s)} = \frac{K \frac{(s+1)}{s(s-1)(s+6)}}{1 + K \frac{(s+1)}{s(s-1)(s+6)}} = \frac{K(s+1)}{\underbrace{s(s-1)(s+6) + K(s+1)}} = \frac{b(s)}{a(s)}$$

- The characteristic equation
- Which is the denominator of the transfer function

Stability Versus Parameter Range

$$a(s) = s^3 + 5s^2 + (K - 6)s + K$$

$$s^3 : \quad 1 \quad (K - 6)$$

$$s^2 : \quad 5 \quad K$$

$$s^1 : \quad b_1 \quad b_2$$

$$s^0 : \quad c_1 \quad c_2$$

$$b_1 = \frac{5 \cdot (K - 6) - K}{5} = \frac{4K - 30}{5}$$

$$b_2 = 0$$

$$c_1 = \frac{b_1 \cdot K - b_2 \cdot 5}{b_1} = K$$

$$c_2 = 0$$

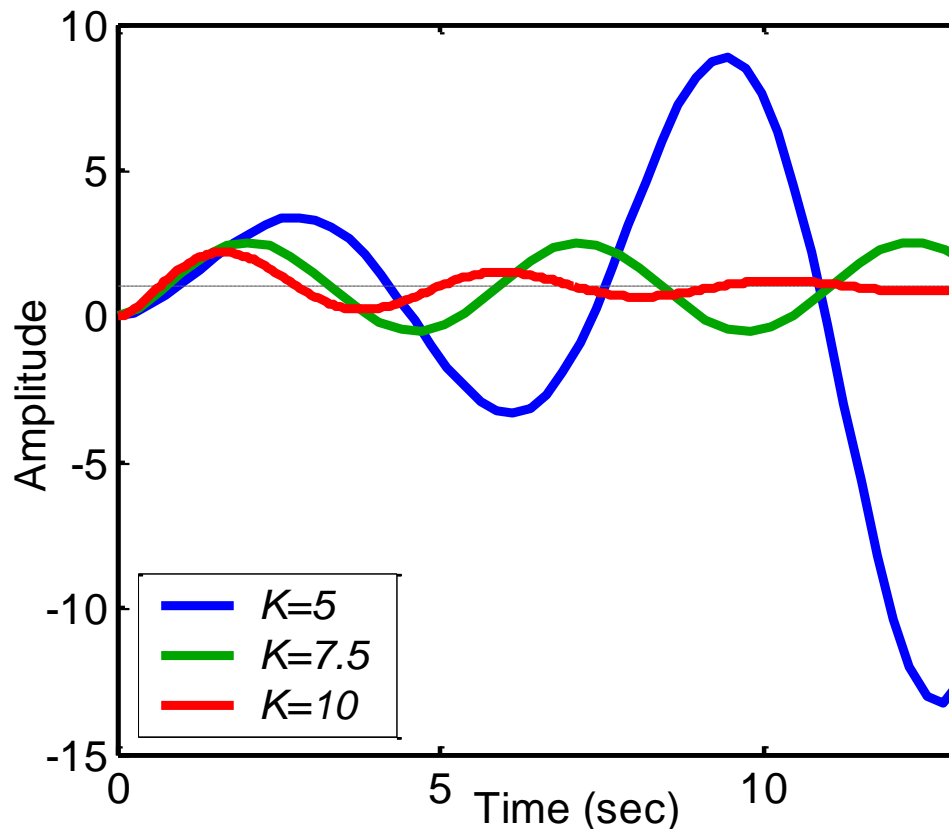
The system is stable if and only if b_1 and c_1 are positive.

$$\left. \begin{array}{l} 4K - 30 > 0 \Rightarrow K > 7.5 \\ K > 0 \end{array} \right\} \therefore \underline{\underline{K > 7.5}}$$

Stability Versus Parameter Range

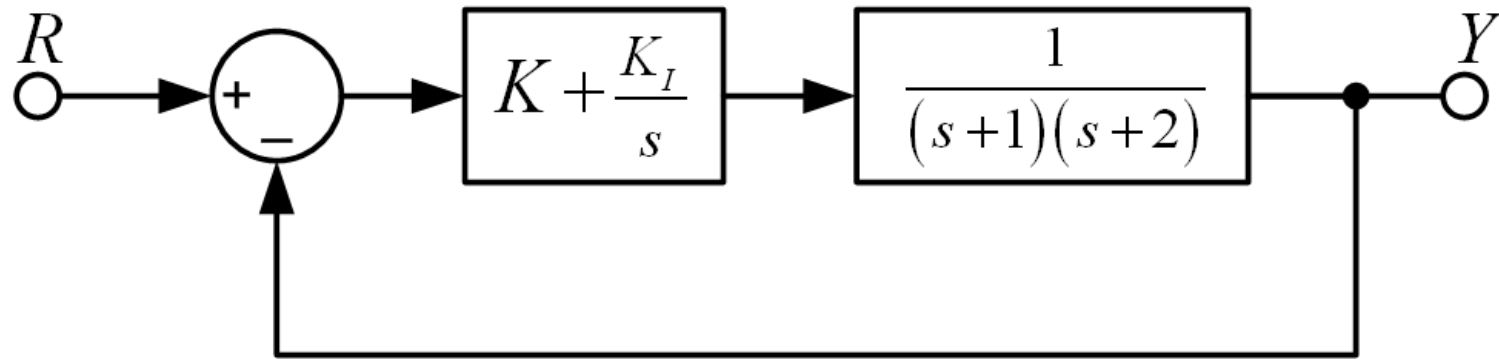
Generating the step responses of the transfer function in MATLAB, for 3 different values of K :

$$\frac{Y(s)}{R(s)} = \frac{Ks + K}{s^3 + 5s^2 + (K - 6)s + K}$$



Stability Versus Two Parameter Ranges

Find the range of the controller gains (K , K_I) so that the PI (proportional-integral) feedback system in the figure below is stable.



$$\frac{Y(s)}{R(s)} = \frac{\left(K + \frac{K_I}{s}\right) \frac{1}{(s+1)(s+2)}}{1 + \left(K + \frac{K_I}{s}\right) \frac{1}{(s+1)(s+2)}} = \frac{Ks + K_I}{\underbrace{s^3 + 3s^2 + (2 + K)s + K_I}_{\text{The characteristic equation}}}$$

Stability Versus Two Parameter Ranges

Example 3 (cont'd)

$$a(s) = s^3 + 3s^2 + (2 + K)s + K_I$$

$$s^3 : \quad 1 \quad 2 + K$$

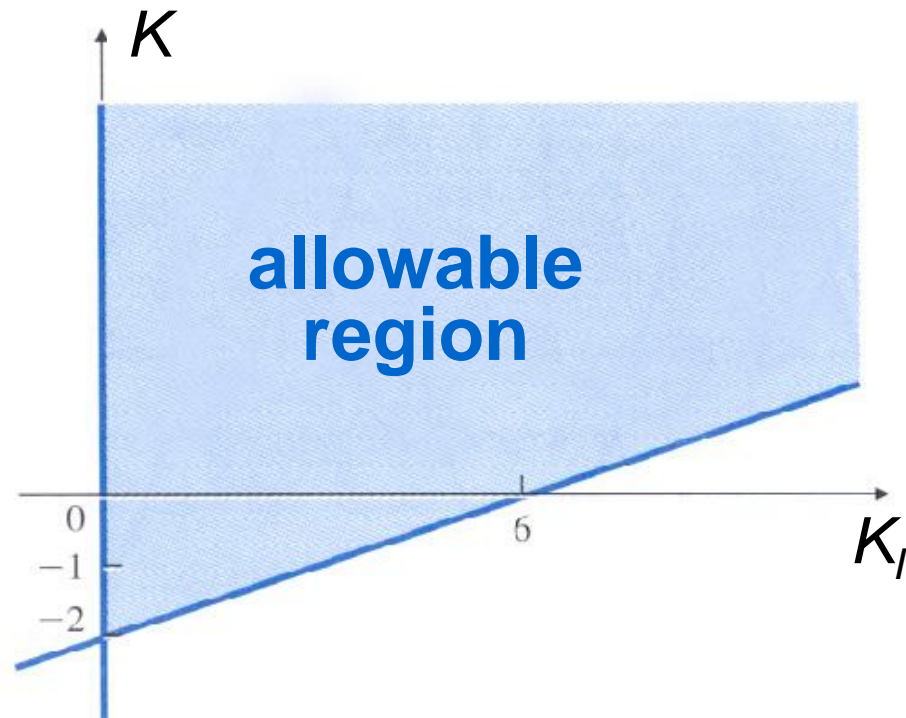
$$s^2 : \quad 3 \quad K_I$$

$$s^1 : \quad \frac{3 \cdot (2 + K) - K_I}{3} \quad 0$$

$$s^0 : \quad K_I \quad 0$$

For the stability,

$$\left. \begin{array}{l} 6 + 3K - K_I > 0 \\ K_I > 0 \end{array} \right\} \therefore \underline{\underline{K_I > 0}}$$
$$\underline{\underline{K > \frac{K_I - 6}{3}}}$$



Special Cases for Routh's Stability Criterion

- If a first-column term in any row is zero, but the remaining terms are not zero or there is no remaining term, then the zero term is replaced by a very small positive number ε and the rest of the array is evaluated as before.

Example:

$$a(s) = s^3 + 2s^2 + s + 2$$

$$s^3 : \quad 1 \quad 1$$

$$s^2 : \quad 2 \quad 2$$

$$s^1 : \quad 0 \equiv \varepsilon \quad 0$$

$$s^0 : \quad 2 \quad 0$$

Special Cases for Routh's Stability Criterion

- If the sign of the coefficient above the zero (ε) is the same as that below it, it indicates that there are a pair of imaginary roots.

Example:

$$a(s) = s^3 + 2s^2 + s + 2$$

$$s^3 : + \quad 1 \quad 1$$

$$s^2 : + \quad 2 \quad 2$$

$$s^1 : +0 \equiv \varepsilon \quad 0$$

$$s^0 : + \quad 2 \quad 0$$

The same signs above and below zero (ε)
→ A pair of imaginary roots

$$s_1 = -2$$
$$s_{2,3} = \pm j$$



Special Cases for Routh's Stability Criterion

- If the sign of the coefficient above the zero (ϵ) is opposite that below it, it indicates that there is one sign change.

Example:

$$a(s) = s^5 + 3s^4 + 2s^3 + 6s^2 + 6s + 9$$

$$s^5 : + \quad 1 \quad \boxed{1^{\text{st}}} \quad 2 \quad 6$$

$$s^4 : + \quad 3 \quad 6 \quad 9$$

$$s^3 : + \quad 0 \equiv \epsilon \quad 3$$

$$s^2 : - \quad \frac{6\epsilon - 9}{\epsilon} \quad 9$$

The opposite signs above and below ϵ
 → There is one sign change
 → The 1st root in the RHP

Another sign change between s^2 and s^1
 → The 2nd root in the RHP

$$s^1 : + \quad \frac{3 \cdot \left(\frac{6\epsilon - 9}{\epsilon} \right) - 9\epsilon}{6\epsilon - 9}$$

$$s^0 : + \quad 9$$

$$s_1 = -2.9043$$

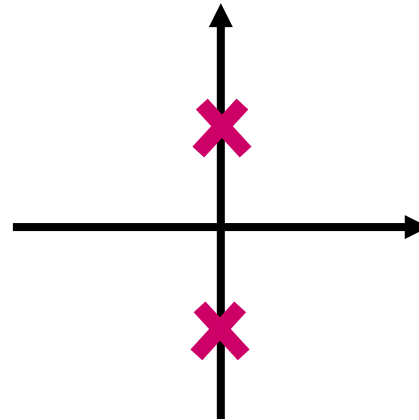
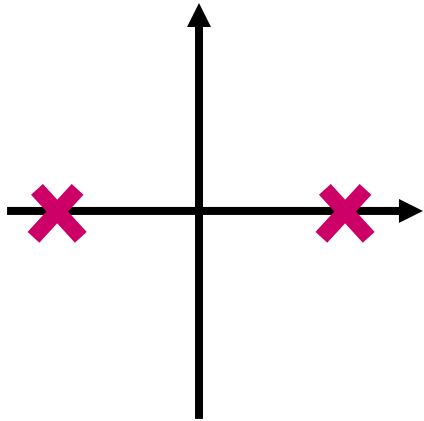
$$s_{2,3} = 0.6567 \pm j1.2881$$

$$s_{4,5} = -0.7046 \pm j0.9929$$



Special Cases for Routh's Stability Criterion

- If all the coefficients in any derived row are zero, it indicates that there are roots of equal magnitude lying radially opposite in the s-plane, that is, two real roots with equal magnitudes and opposite signs and/or two conjugate imaginary roots.



- In such a case, the evaluation of the rest of the array can be continued by forming an auxiliary polynomial from the last nonzero row, and then using the coefficients of the derivative of this auxiliary polynomial to replace the zero row.

Special Cases for Routh's Stability Criterion

Example:

$$a(s) = s^5 + 5s^4 + 11s^3 + 23s^2 + 28s + 12$$

$$s^5 : \quad \boxed{1} \quad 11 \quad 28$$

$$s^4 : \quad 5 \quad 23 \quad 12$$

$$s^3 : \quad 6.4 \quad 25.6 \quad 0$$

$$s^2 : \quad 3 \quad 12 \quad \leftarrow a_1(s) = 3s^2 + 12$$

$$s^1 : \quad \boxed{0} \quad \boxed{0} \quad \rightarrow \text{Zero row}$$

$$\text{New } s^1 : \quad \boxed{6} \quad \boxed{0} \quad \leftarrow \frac{da_1(s)}{ds} = 6s$$

$$s^0 : \quad \boxed{12}$$

\rightarrow Zero row is replaced

Auxiliary polynomial from the last nonzero row

Derivative of the auxiliary polynomial

- A zero row
 \rightarrow Radially opposite roots
- No sign change
 \rightarrow No root in the RHP

$$\begin{aligned} s_1 &= -3 \\ s_{2,3} &= \pm j2 \\ s_4 &= -1 \\ s_5 &= -1 \end{aligned}$$



Special Cases for Routh's Stability Criterion

Example:

$$a(s) = s^5 + 2s^4 + 24s^3 + 48s^2 - 25s - 50$$

Unstable

s^5 :	1	24	-25	
s^4 :	2	48	-50	$\leftarrow a_1(s) = 2s^4 + 48s^2 - 50$
s^3 :	0	0	0	\rightarrow Zero row
New s^3 :	8	96	0	$\leftarrow \frac{da_1(s)}{ds} = 8s^3 + 96s$ Derivative of the auxiliary polynomial
s^2 :	24	-50		
s^1 :	112.7	0		Zero row is replaced
s^0 :	-50			

- A zero row
 \rightarrow Radially opposite roots
- One sign change
 \rightarrow One root in the RHP

$$s_{1,2} = \pm j5$$

$$s_3 = 1$$

$$s_4 = -1$$

$$s_5 = -2$$



Homework 3

■ No.1

A system is given with a transfer function:

$$\frac{Y(s)}{R(s)} = \frac{24(s+1)}{s^4 + 6s^3 + 2s^2 + s + 3}$$

Determine whether the system is stable or not. Use Routh's stability criterion and verify with MATLAB.

■ No.2

A system has a characteristic equation:

$$s^3 + 2s^2 + (K+1)s + 6 = 0$$

Find the range of K for a stable system.

■ Deadline: 29 April 2019

■ Quiz 3: 2 May 2019, 10:30 – 10:45

