جامـــــعـة المـــــسـتـقـبـل
AL MUSTAQBAL UNIVERSITY

Al-Mustaqbal University

College of Science

Intelligent Medical System Department

University of Information Technology and Communications

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Lecture 8- Aliasing Process

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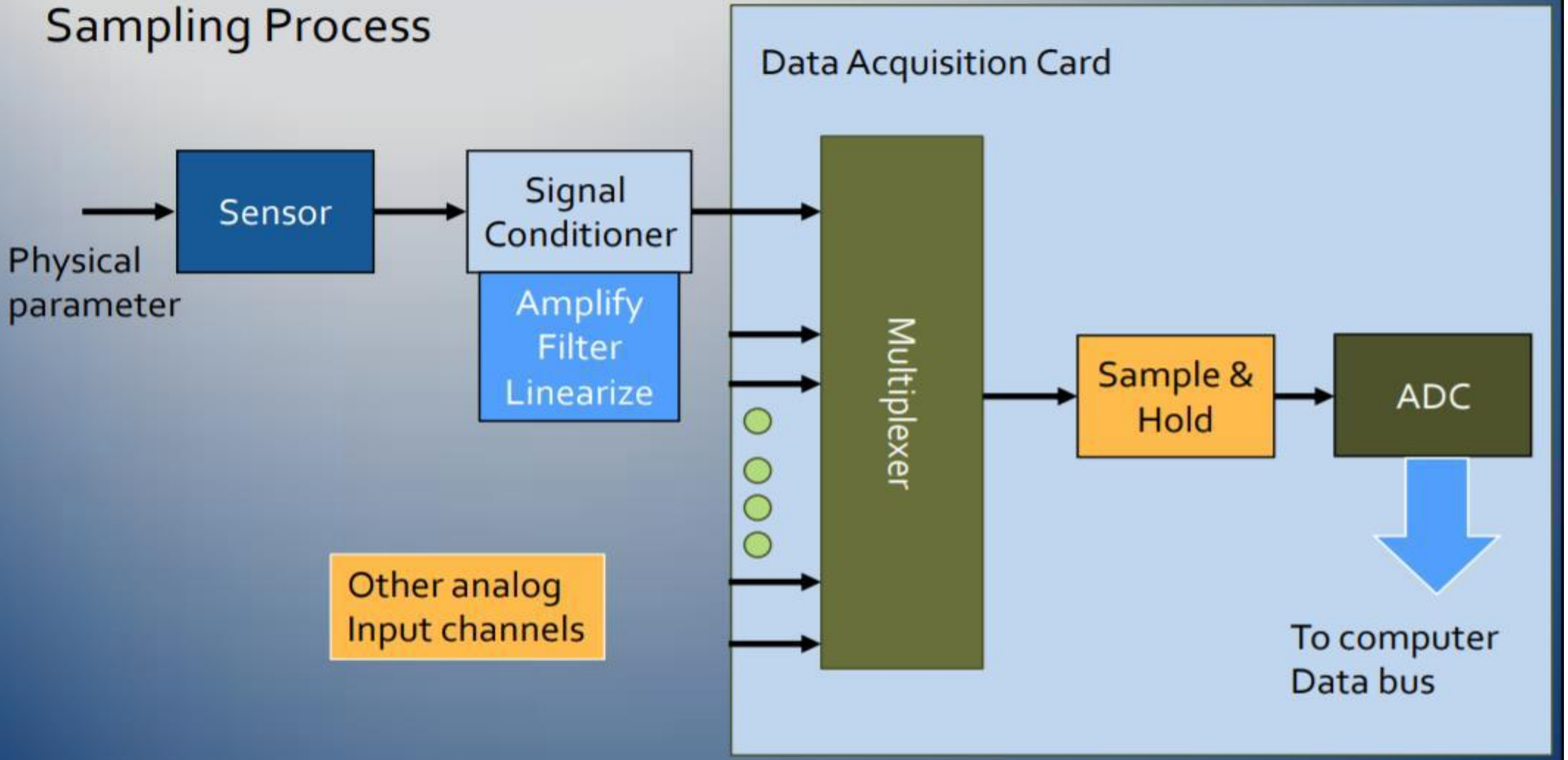
Sampled Signals

Sampling Process



Frequency Spectrum

Sampling is modulation. Shifts all signal frequency components and generates harmonics

f_c=1000 Hz f_{l1}=50 Hz f_{l2}=25 Hz

$$v(t) = 1\cdot\sin(2\pi\cdot1000\cdot t)\left[1\cdot\sin(2\pi\cdot50\cdot t) + 1\cdot\sin(2\pi\cdot25\cdot t)\right]$$

Carrier Information

Modulation produces sums and differences of carrier and information frequencies

$f_{h1} = f_c \pm f_{l1}$ for the 1st information frequency
$f_{h2} = f_c \pm f_{l1}$ for the 2nd information frequency
$f_{hi} = f_c \pm f_{li}$ for the i-th information frequency

Lesson 6_et438b.pptx 9

Nyquist Frequency and Minimum Sampling Rate

To accurately reproduce the analog input data with samples the sampling rate, f_s, must be twice as high as the highest frequency expected in the input signal. (Two samples per period) This is known as the Nyquist frequency.

$$f_{s(min)} = 2f_h$$

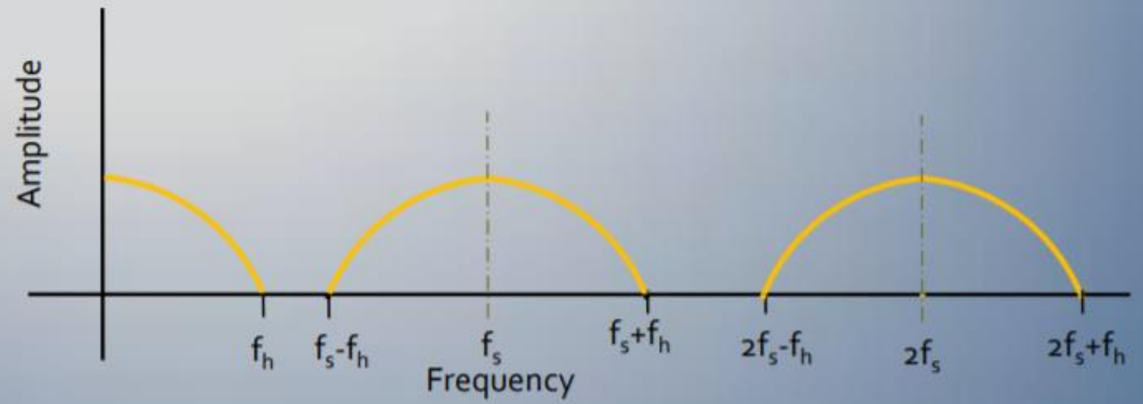
Where f_h = the highest discernible f component in input signal

$f_{s(min)}$ = minimum sampling f

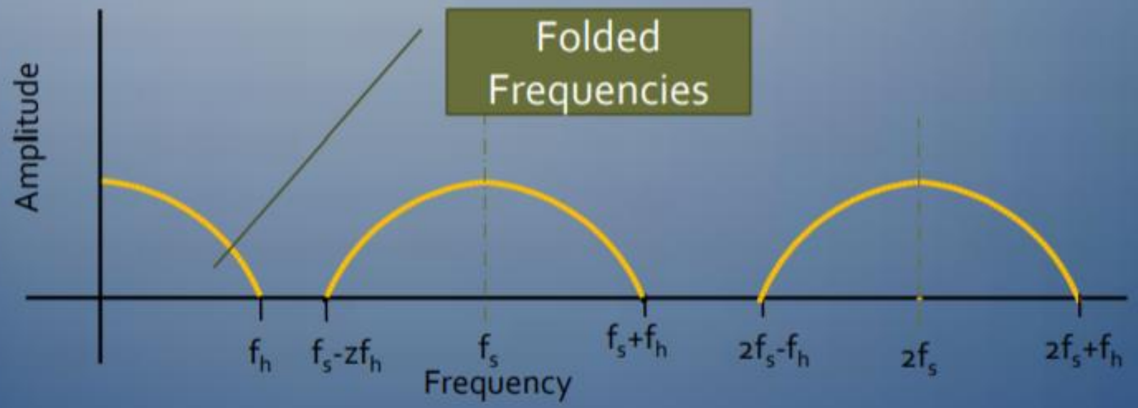
Nyquist rate is the <u>minimum</u> frequency and requires an ideal pulse to reconstruct the original signal into an analog value

Sampled Signal Frequency Spectrum

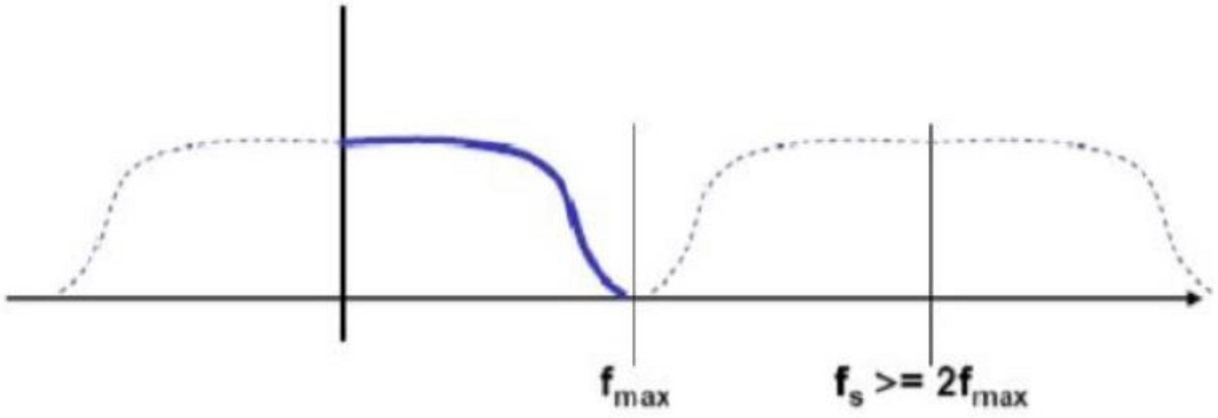
Sampling with $f_s > 2f_h$



Sampling at less than $2f_h$ causes aliasing and folding of sampled signals.



Nyquist Sampling Theorem

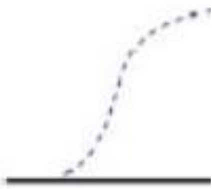


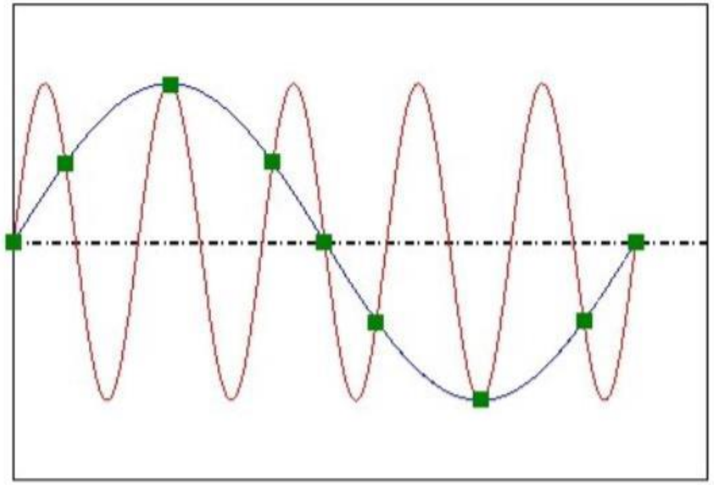
A signal that has energy to f_{max} must be sampled at a rate ($2 \times f_{max}$) or greater

Sampling creates an "alias" copy of a signal

Nyquist Sampling Theorem

Nyquist Frequency and Aliasing (1)



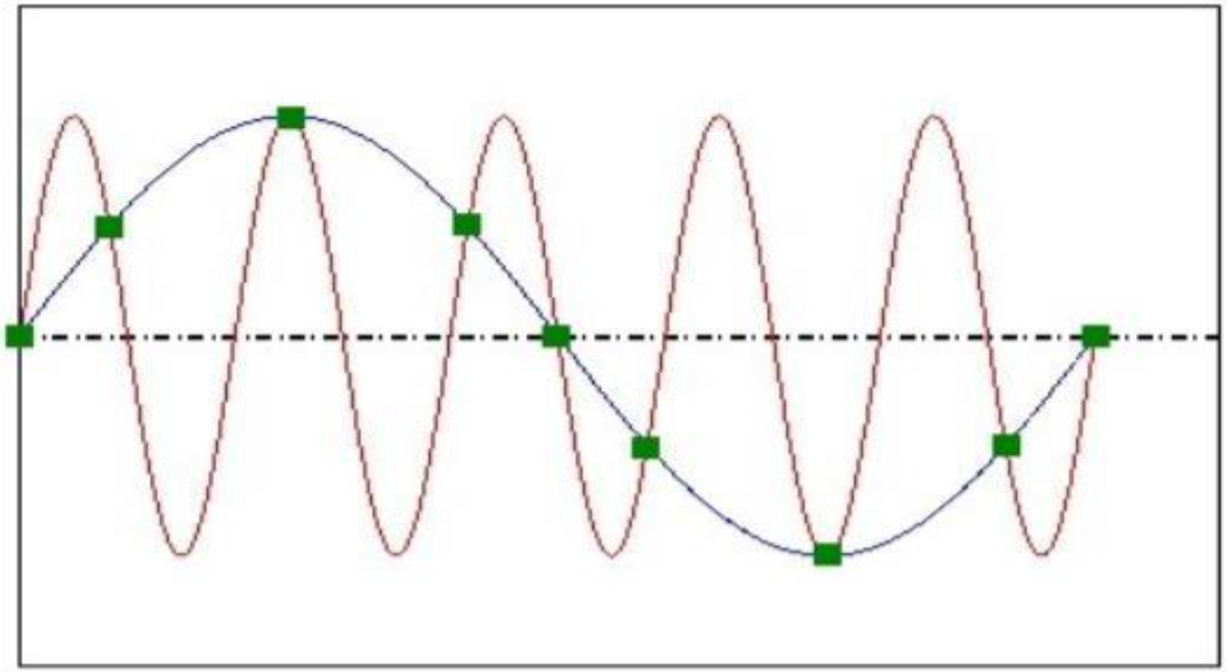


A signal that or greater

High frequency signal to be sampled by a low sampling rate may cause to “fold” the sampled data into a false lower frequency signal. This phenomena is known as *aliasing*.

If the sampling rate is less than twice the highest frequency, the alias overlaps the original, creating distortion

Nyquist Frequency and Aliasing (1)



High frequency signal to be sampled by a low sampling rate may cause to “fold” the sampled data into a false lower frequency signal. This phenomena is known as *aliasing*.

Aliasing Formulas

$f_a =$ Alias frequency

$f_N =$ Folding frequency (Nyquist frequency)

$$f_N = f_s / 2$$

$$f_a = (f_a / f_N) f_N$$

Given that sampling frequency equals 250Hz and the ratio of folding frequency to alias frequency equals 0.5, find the alias frequency.

Solution:

$$f_s = 250$$

$$f_a / f_N = 0.5$$

$$f_N = 250 / 2 = 125$$

$$f_a = (f_a / f_N) f_N = 0.5 \times 125 = 62.5 Hz$$

Nyquist Frequency and Aliasing

Example: Given the following signal, determine the minimum sampling rate (Nyquist frequency)

$$s(t) = 1.5 \cdot \sin(175\pi t) + 3 \cdot \sin(250\pi t) + 0.5\cos(800\pi t) + 1.75 \cdot \sin(900\pi t)$$

Convert the radian frequency to frequency in Hz by dividing values by 2π

$$f_1 = \frac{175\pi}{2\pi} = 87.5\text{ Hz} \quad f_2 = \frac{250\pi}{2\pi} = 125\text{ Hz} \quad f_3 = \frac{800\pi}{2\pi} = 400\text{ Hz} \quad f_4 = \frac{900\pi}{2\pi} = 450\text{ Hz}$$

Find the highest frequency component: 450 Hz

$$f_{s(min)} = 2f_h$$

$$f_{s(min)} = 2(450\text{ Hz}) = 900\text{Hz}$$

Aliased Frequencies

Sampling analog signal below $2f_h$ produces false frequencies. Aliased frequencies determined by:

$$f_{alias} = \left| f_I - n \cdot f_s \right|$$

$$0 \le f_{alias} \le f_{nyquist}$$

$$f_{nyquist} = \frac{f_s}{2}$$

Where: f_I = sampled information signal with $f_I > f_{nyquist}$
f_s = sampling frequency (Hz)
n = sampling harmonic number
f_{alias} = aliased frequency
$f_{nyquist}$ = one-half sampling frequency

Samples/Period and Aliasing

Correct signal representation requires at least two samples/period

$$N_s = \frac{f_s}{f_I} = \frac{T_I}{T_s}$$

$$f_s > f_I \text{ and } T_I > T_s$$

Where N_s = number input signal samples per period of sampling frequency
f_s = sampling frequency (Hz)
f_I = highest information signal frequency (Hz)
T_s = sampling period, $1/f_s$, (seconds)
T_I = period information signal's highest frequency ($1/f_I$)

Sampling/Aliasing Examples

Example 1: A f_s=1000 Hz sampling frequency samples an information signal of f_I=100 Hz . Determine samples/period, the resulting recovered signal ,and aliased frequencies if present

Determine the number of samples/ period

$$N_s = \frac{1000\ \text{Hz}}{100\ \text{Hz}} = \frac{0.01\ \text{S}}{0.001\ \text{S}} = 10\ \text{samples/period}$$

Above Nyquist rate of 2

$$f_{nyquist} = \frac{f_s}{2} = \frac{1000\ \text{Hz}}{2} = 500\ \text{Hz}$$

Signals below 500 Hz reproduced without aliasing

View the frequency spectrum using FFT of samples

Sampling/Aliasing Examples

Example 2: A f_s=60 Hz sampling frequency samples an information signal of f_I=100 Hz . Determine samples/period, the resulting recovered signal ,and aliased frequencies if present

Determine the number of samples/ period

$$N_s = \frac{60\ \text{Hz}}{100\ \text{Hz}} = \frac{0.01\ \text{S}}{0.001666\ \text{S}} = 0.6\ \text{samples/period}$$

Below Nyquist rate of 2

Aliased signals will occur due to low sampling rate

$$f_{nyquist} = \frac{f_s}{2} = \frac{60\ \text{Hz}}{2} = 30\ \text{Hz}$$

Signals below 30 Hz reproduced without aliasing

Now compute the aliased frequency for 1st sampling harmonic

Sampling/Aliasing Examples

Alias frequencies for 1st harmonic of sampling f (n=1)

$$f_{alias} = |f_I - n \cdot f_s|$$
$$0 \le f_{alias} \le f_{nyquist}$$
$$f_{nyquist} = \frac{f_s}{2}$$

$$f_{alias} = |100\ \text{Hz} - 1 \cdot 60\ \text{Hz}| = 40\ \text{Hz}$$
$$f_{nyquist} = \frac{60\ \text{Hz}}{2} = 30\ \text{Hz}$$
$$0 \le f_{alias} \le 30\ \text{Hz}$$

The f_{alias} is outside range 0-30 Hz, (40 Hz > 30 Hz) No recovered signal

Find alias frequencies of 2nd sampling harmonic f (n=2)

$$f_{alias} = |100\ \text{Hz} - 2 \cdot 60\ \text{Hz}| = 20\ \text{Hz}$$

The f_{alias} in range 0-30 Hz, 20 Hz recovered signal