Relativistic Effects
Lec 5

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Objectives: After completing this module, you should be able to:

- State and discuss Einstein's two postulates regarding special relativity.
- Demonstrate your understanding of time dilation and apply it to physical problems.
- Demonstrate and apply equations for relativistic length, momentum, mass, and energy.

Special Relativity

Einstein's Special Theory of Relativity, published in 1905, was based on two postulates:

I. The laws of physics are the same for all frames of reference moving at a constant velocity with respect to each other.

II. The free space velocity of light c is constant for all observers, independent of their state of motion. ($c = 3 \times 10^8$ m/s)

Rest and Motion

What do we mean when we say that an object is at rest . . . or in motion? Is anything at rest?

We sometimes say that man, computer, phone, and desk are at rest.

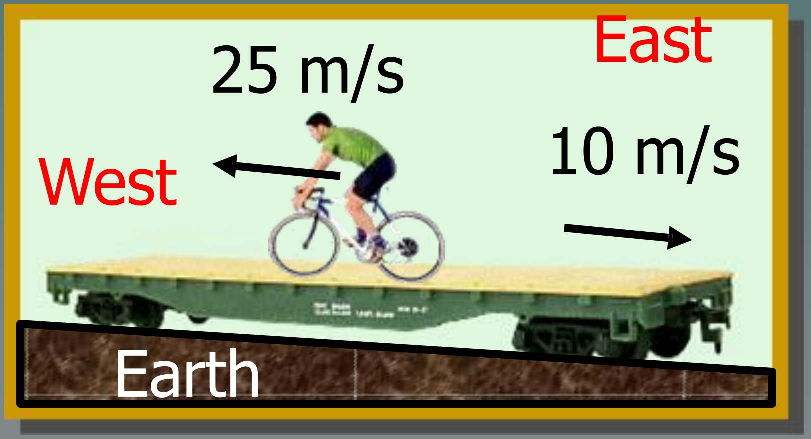
We forget that the Earth is also in motion.

What we really mean is that all are moving with the same velocity. We can only detect motion in reference to something else.

No Preferred Frame of Reference

What is the velocity of the bicyclist?

We cannot say without a frame of reference.



Assume bike moves at 25 m/s,W relative to Earth and that platform moves 10 m/s, E relative to Earth.

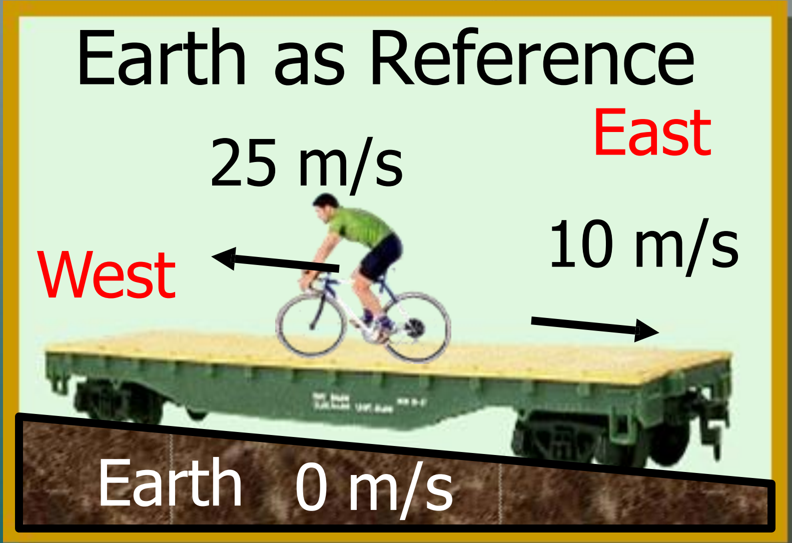
What is the velocity of the bike relative to platform?

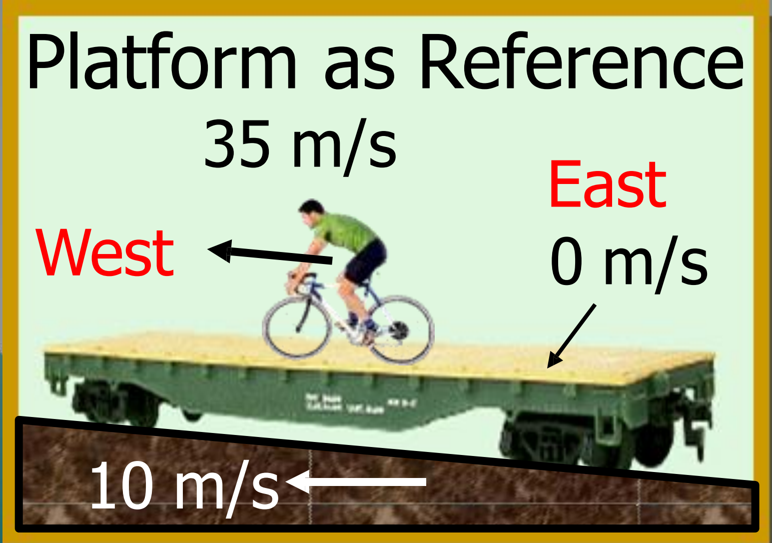
Assume that the platform is the reference, then look at relative motion of Earth and bike.

Reference for Motion (Cont.)

To find the velocity of the bike relative to platform, we must imagine that we are sitting on the platform at rest (0 m/s) relative to it.

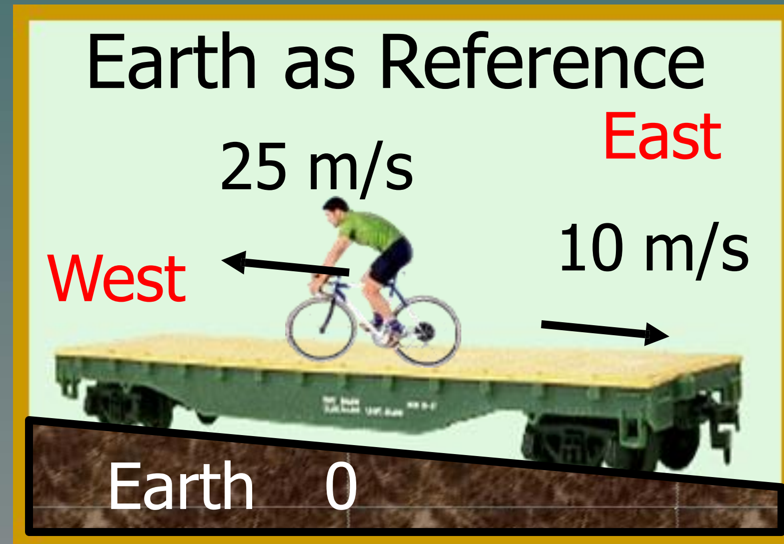
We would see the Earth moving westward at 10 m/s and the bike moving west at 35 m/s.

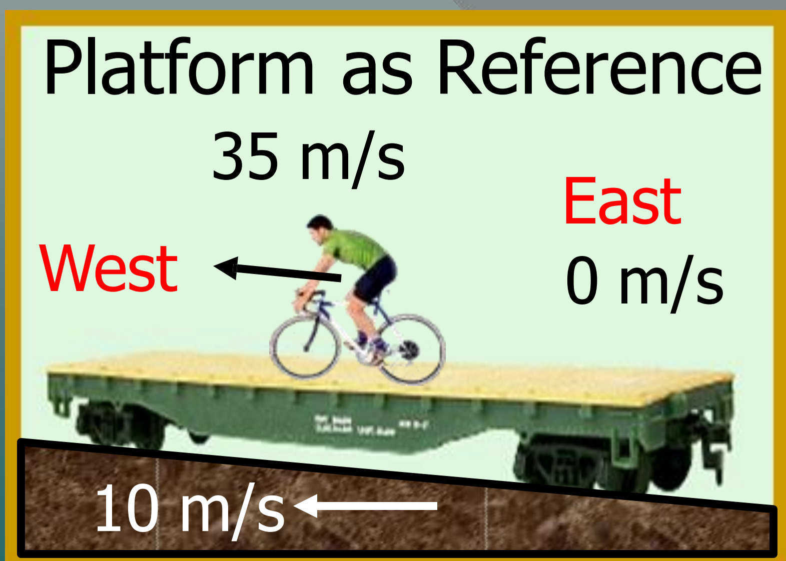


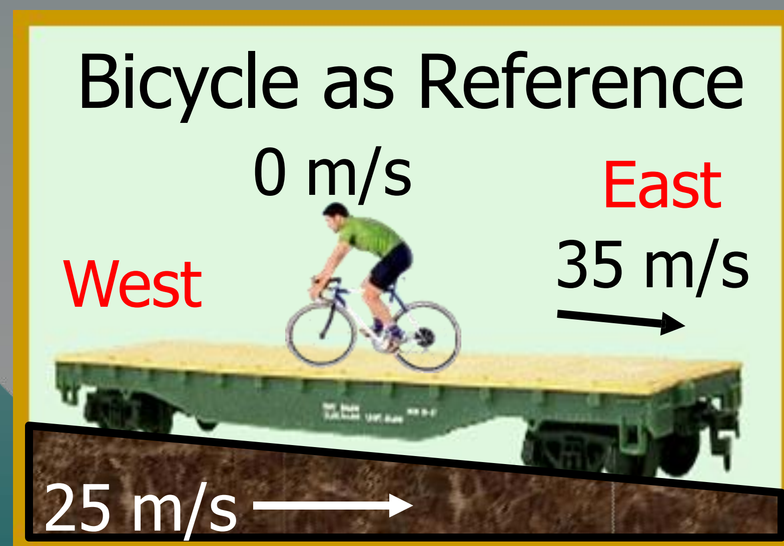


Frame of Reference

Consider the velocities for three different frames of reference.

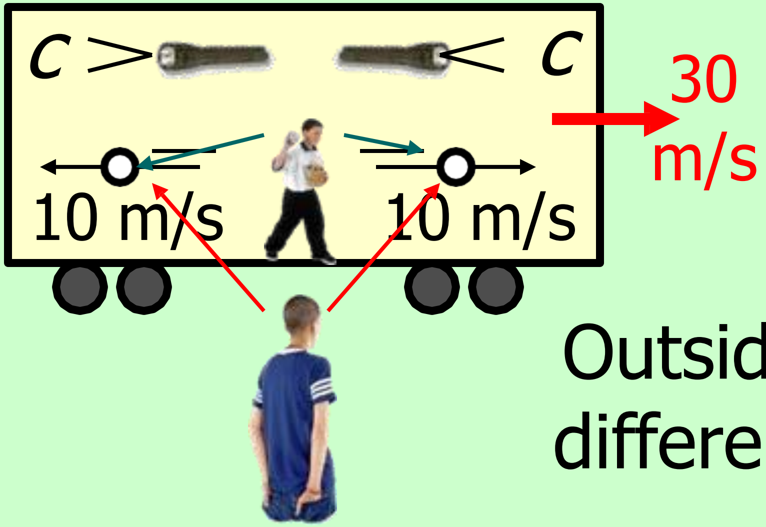






Velocity of Light (Cont.)

Platform moves 30 m/s to right relative to boy.



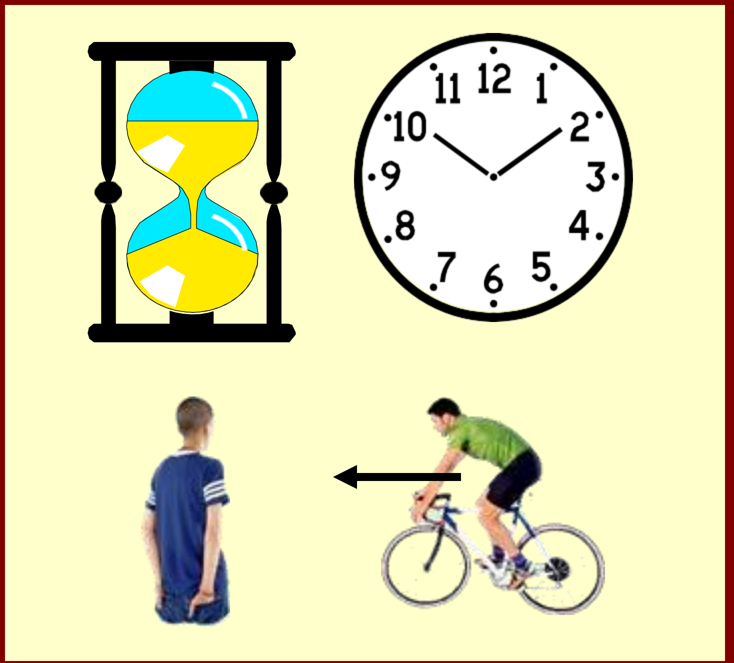
Each observer sees
$c = 3 \times 10^8$ m/s

Outside observer sees very different velocities for balls.

The velocity of light is unaffected by relative motion and is exactly equal to:

$$c = 2.99792458 \times 10^8 \text{ m/s}$$

Time Measurements

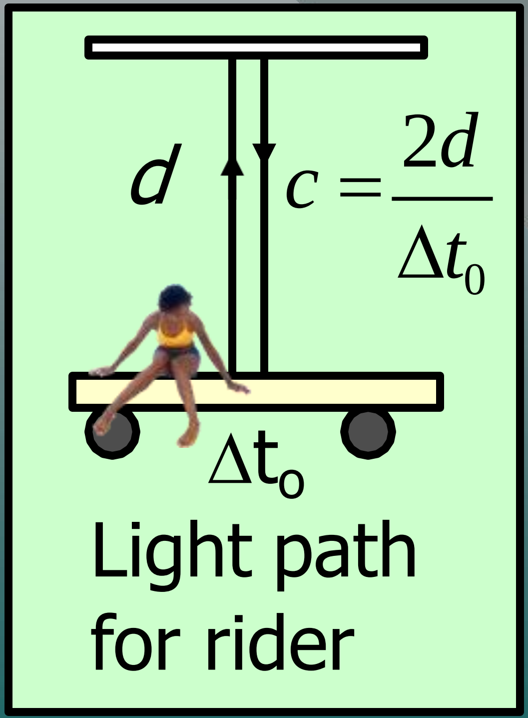


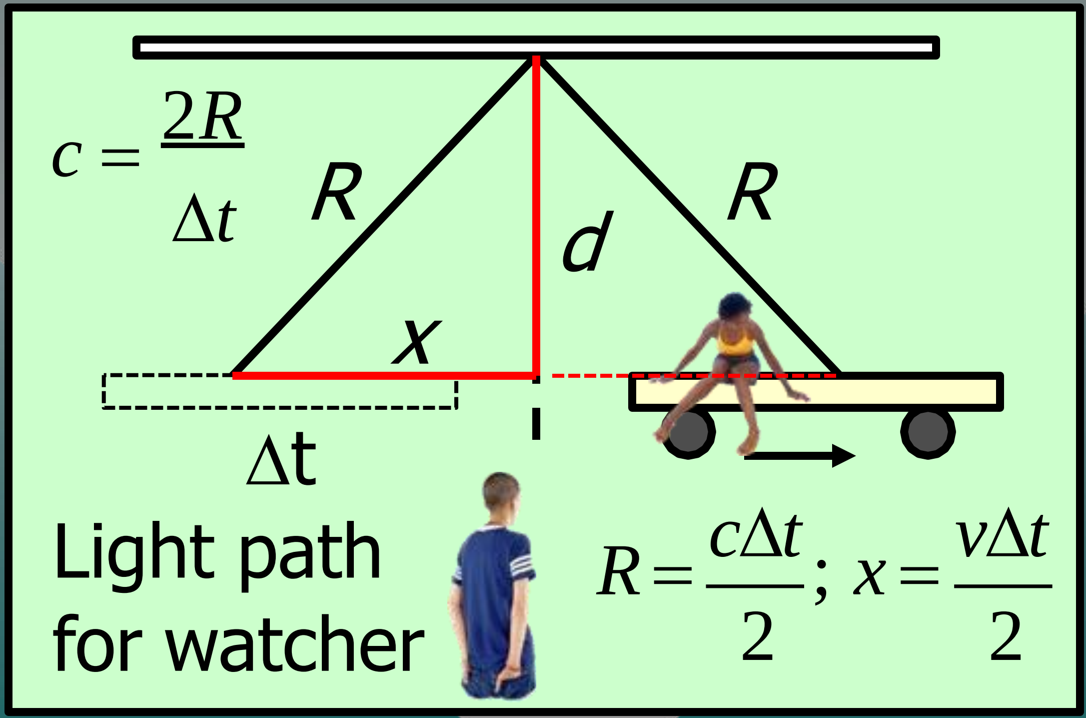
Since our measurement of time involves judgments about simultaneous events, we can see that time may also be affected by relative motion of observers.

In fact, Einstein's theory shows that observers in relative motion will judge times differently - furthermore, each is correct.

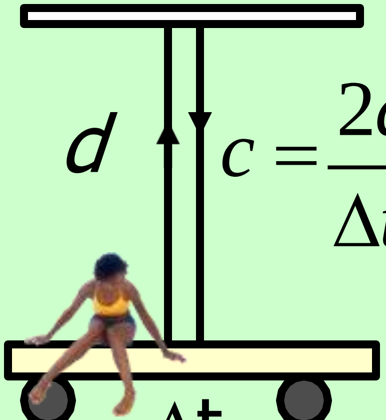
Relative Time

Consider cart moving with velocity *v* under a mirrored ceiling. A light pulse travels to ceiling and back in time Δt_o for rider and in time Δt for watcher.



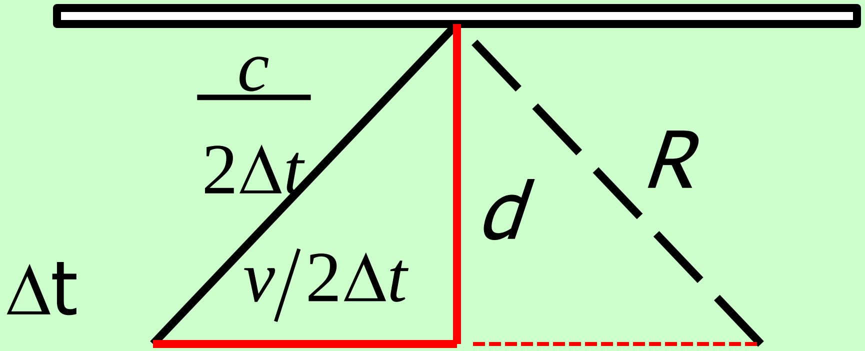


Relative Time (Cont.)



$$c = \frac{2d}{\Delta t_0}$$

Light path for rider



$$\left(\frac{c}{2\Delta t}\right)^2 + \left(\frac{v}{2\Delta t}\right)^2 = d^2$$

Substitution of:

$$d = \frac{c\Delta t_0}{2}$$

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2/c^2}}$$

Time Dilation Equation

Einstein's Time dilation Equation:

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2/c^2}}$$

Δt = Relative time (Time measured in frame moving relative to actual event).

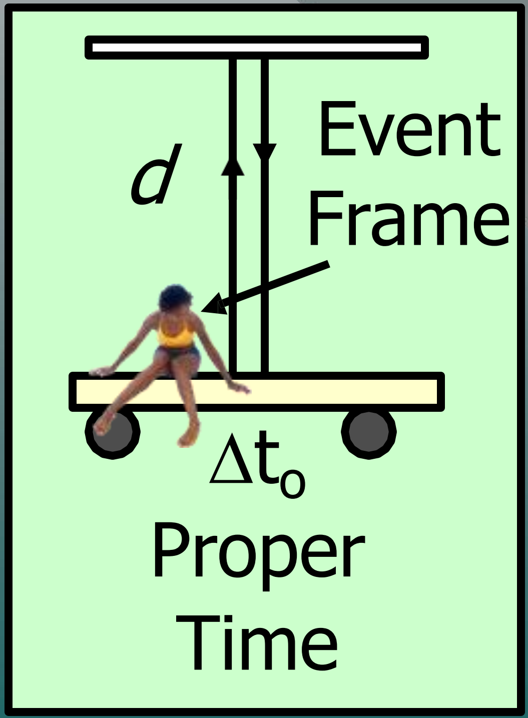
Δt_o = Proper time (Time measured in the same frame as the event itself).

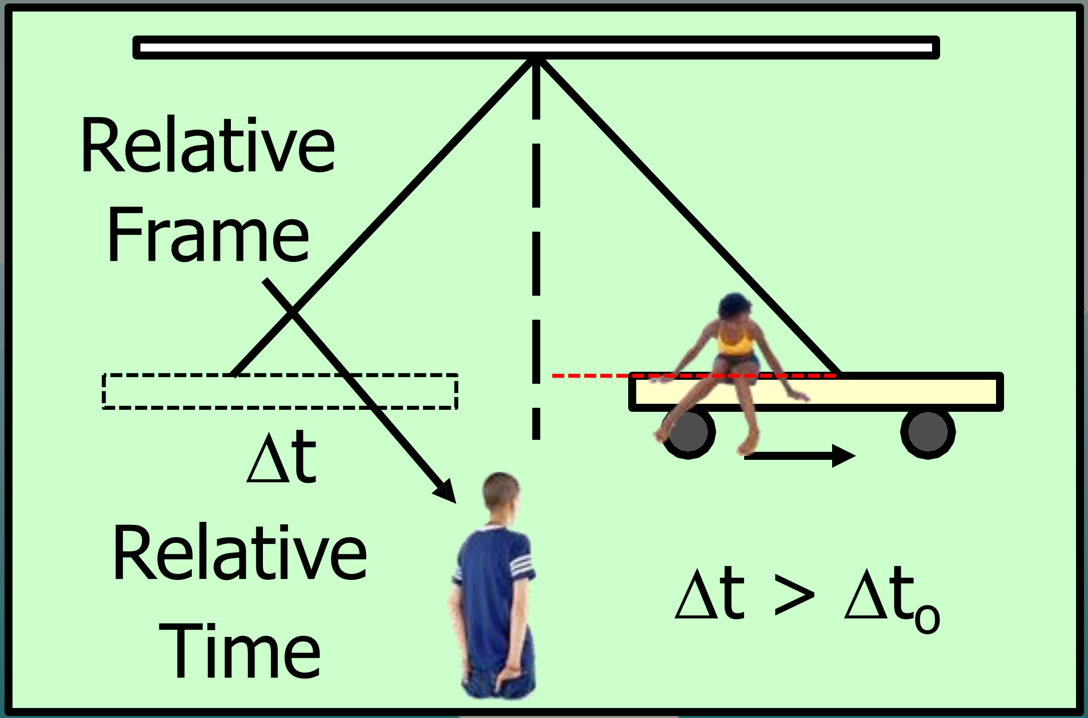
v = Relative velocity of two frames.

c = Free space velocity of light (c = 3 x 10^8 m/s).

Proper Time

The key to applying the time dilation equation is to distinguish clearly between proper time Δt_o and relative time Δt. Look at our example:



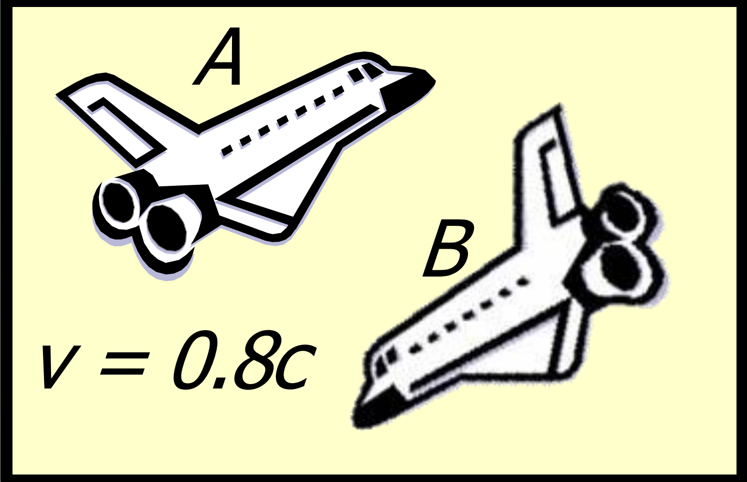


Example 1: Ship *A* passes ship *B* with a relative velocity of 0.8c (eighty percent of the velocity of light). A woman aboard Ship *B* takes 4 s to walk the length of her ship. What time is recorded by the man in Ship A?

Proper time $\Delta t_o = 4$ s

Find relative time Δt

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2/c^2}}$$



$$\Delta t = \frac{4.00\ \text{s}}{\sqrt{1 - (0.8c)^2/c^2}} = \frac{4.00\ \text{s}}{\sqrt{1 - 0.64}}$$

$\Delta t = 6.67$ s

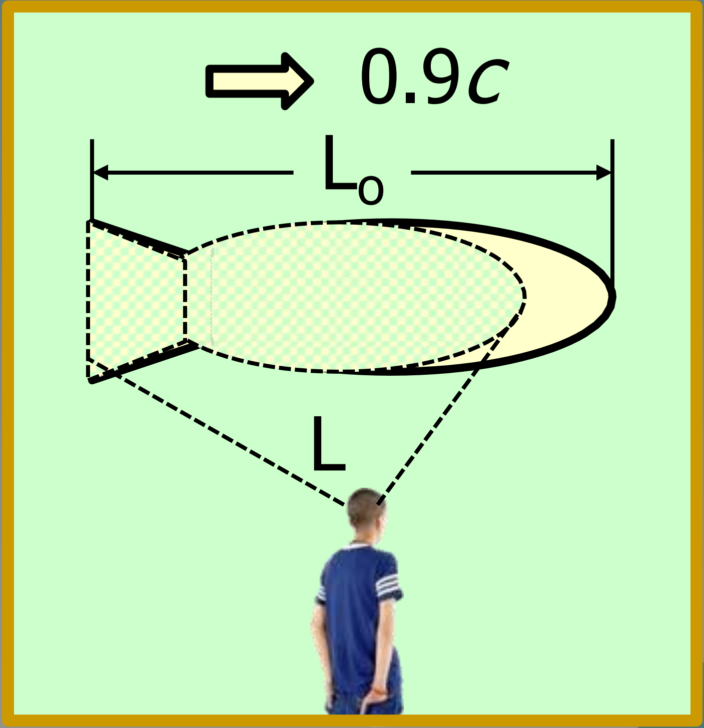
Length Contraction

Since time is affected by relative motion, length will also be different:

$$L = L_0 \sqrt{1 - v^2/c^2}$$

L_o is proper length

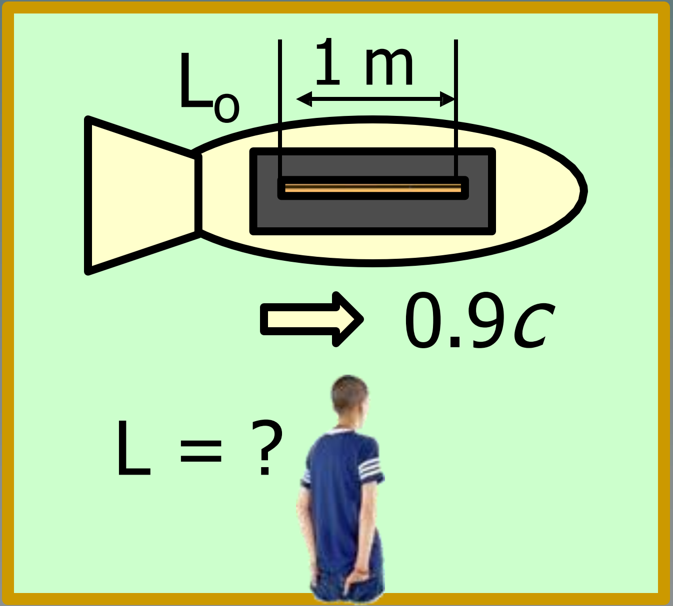
L is relative length



Moving objects are foreshortened due to relativity.

Example 2: A meter stick moves at 0.9c relative to an observer. What is the relative length as seen by the observer?

$$L = L_0\sqrt{1 - v^2/c^2}$$



$$L = (1\text{ m})\sqrt{1 - (0.9c)^2/c^2}$$

$$L = (1\text{ m})\sqrt{1 - 0.81} = 0.436\text{ m}$$

Length recorded by observer: **L = 43.6 cm**

If the ground observer held a meter stick, the same contraction would be seen from the ship.

Relativistic Momentum

The basic conservation laws for momentum and energy can not be violated due to relativity.

Newton's equation for momentum *(mv)* must be changed as follows to account for relativity*:*

Relativistic momentum:

$$p = \frac{m_0 v}{\sqrt{1 - v^2/c^2}}$$

m_o is the proper mass, often called the rest mass. Note that for large values of v, this equation reduces to Newton's equation.

Relativistic Mass

If momentum is to be conserved, the relativistic mass *m* must be consistent with the following equation:

Relativistic mass: $$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

Note that as an object is accelerated by a resultant force, its mass increases, which requires even more force. This means that:

The speed of light is an ultimate speed!

Example 3: The rest mass of an electron is 9.1 x 10^-31^ kg. What is the relativistic mass if its velocity is 0.8*c* ?

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

0.8*c*

m_o = 9.1 x 10^{-31} kg

$$m = \frac{9.1 \text{ x } 10^{-31}\text{kg}}{\sqrt{1-(0.6c)^2/c^2}} = \frac{9.1 \text{ x } 10^{-31}\text{kg}}{\sqrt{0.36}}$$

m = 15.2 x 10^{-31} kg

The mass has increased by 67% !

Total Relativistic Energy

The general formula for the relativistic total energy involves the rest mass m_o and the relativistic momentum $p = mv$.

Total Energy, *E* $$E = \sqrt{(m_0c^2) + p^2c^2}$$

For a particle with zero momentum $p = 0$:

$$E = m_oc^2$$

For an EM wave, $m_0 = 0$, and *E* simplifies to:

$$E = pc$$

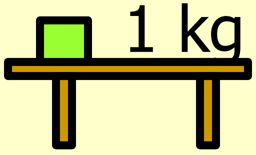
Mass and Energy (Cont.)

The conversion factor between mass m and energy E is:

$$E_o = m_o c^2$$

The zero subscript refers to proper or rest values.

A 1-kg block on a table has an energy E_o and mass m_o relative to table:



$$E_o = (1 \text{ kg})(3 \times 10^8 \text{ m/s})^2$$

$$E_o = 9 \times 10^{16} \text{ J}$$

If the 1-kg block is in relative motion, its kinetic energy adds to the total energy.

Total Energy

According to Einstein's theory, the total energy E of a particle of is given by:

Total Energy: $E = mc^2$

$)m_o c^2 + K($

Total energy includes rest energy and energy of motion. If we are interested in just the energy of motion, we must subtract $m_o c^2$.

Kinetic Energy: $K = mc^2 - m_o c^2$

Kinetic Energy: $K = (m - m_o)c^2$

Example 4: What is the kinetic energy of a proton (m_o = 1.67 x 10^{-27} kg) traveling at 0.8c?

$$m = \frac{\square_0 \; m}{\sqrt{1 - v^2/c^2}}$$

0.7c

m_o = 1.67 x 10^{-27} kg

$$m = \frac{1.67 \text{ x } 10^{-27}\text{kg}}{\sqrt{1-(0.7c)^2/c^2}} = \frac{1.67 \text{ x } 10^{-27}\text{kg}}{\sqrt{0.51}}$$; m = 2.34 x 10^{-27} kg

$$K = (m - m_o)c^2 = (2.34 \text{ x } 10^{-27} \text{ kg} - 1.67 \text{ x } 10^{-17} \text{ kg})c^2$$

Relativistic Kinetic Energy K = 6.02 x 10^{-11} J

Summary

Einstein's Special Theory of Relativity, published in 1905, was based on two postulates:

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II. The free space velocity of light c is constant for all observers, independent of their state of motion. ($c = 3 \times 10^8$ m/s(

Summary (Cont(.

Relativistic time: $$\Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2/c^2}}$$

Relativistic length: $$L = L_0 \sqrt{1 - v^2/c^2}$$

Relativistic mass: $$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

Summary(Cont.)

Relativistic momentum: $p = \frac{m_0 v}{\sqrt{1 - v^2/c^2}}$

Total energy: $E = mc^2$

Kinetic energy: $K = (m - m_o)c^2$