The Reasoning of Statistical Estimation

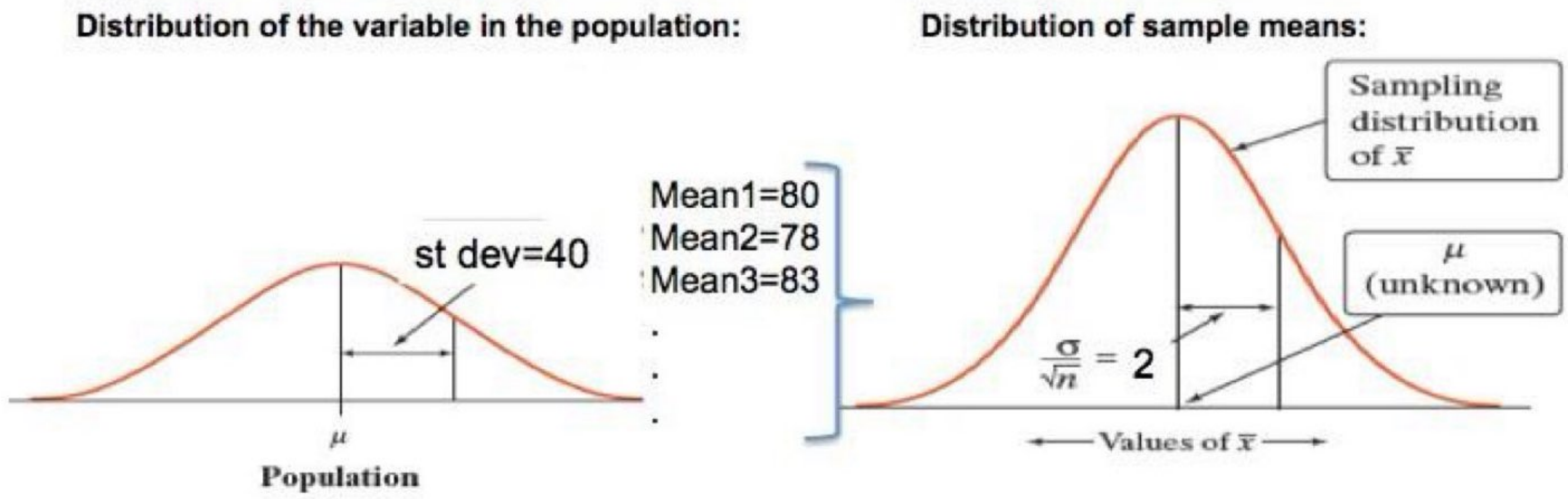
Where does the formula for computing a confidence interval come from?

- The true value of the population mean is never known – it can only be approximated or estimated.
- The best way to do this is to select a large number of random samples of the same size from the population.
- The mean from each random sample will be slightly different.
- The average of these sample means is the population mean.

How would the sample mean $\bar{x}$ vary if many SRSs were taken of the same size from the population?

Shape: Since the population is Normal, so is the sampling distribution of $\bar{x}$.

Center: The mean of the sampling distribution of $\bar{x}$ is the same as the mean of the population distribution, μ.



- For instance, the mean for the sample in the example was 80, but if another sample was selected the mean might be 78 or 83.
- If a large number of sample means were represented graphically, they would have a Normal distribution.
- The mean of this distribution is the same as the sample mean, but the standard deviation of this distribution is equal to the standard deviation of the variable in the population divided by the square root of the sample size.

- This is the reason that the standard deviation is divided by the square root of n in the formula, instead of the simple standard deviation, because this formula represents the standard deviation of the distribution of many sample means.
- When working with real data it may not be feasible to select a very large number of random samples, but if researchers were able to do so, the samples would form a Normal distribution.

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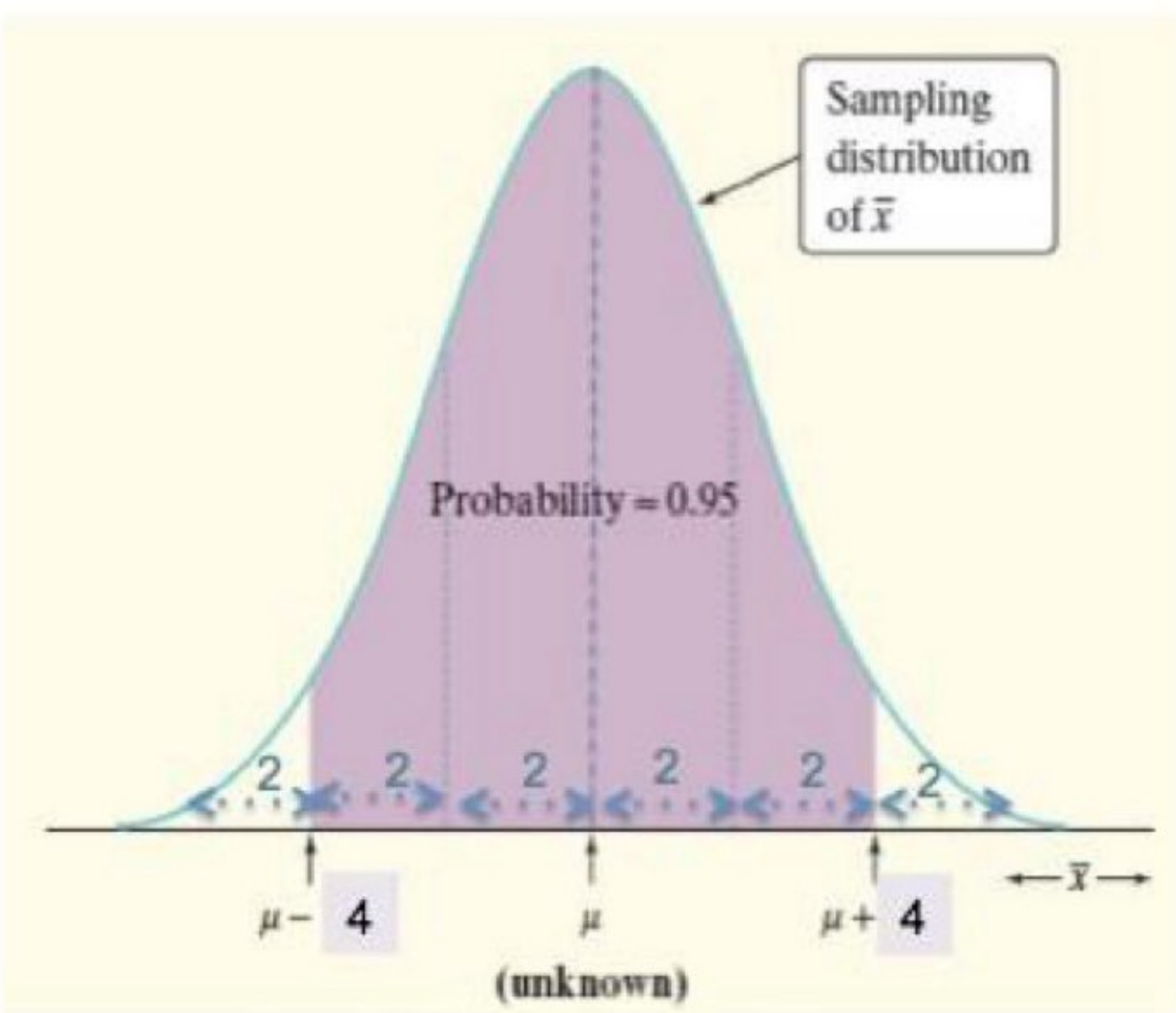
- If many random samples are collected, their means will have a Normal distribution.
- This means that the 68-95-99.7 Rule can be used to estimate the values within which the population mean would fall.
- Since 95% of values fall within two standard deviations of the mean according to the 68-95-99.7 Rule, simply add and subtract two standard deviations from the mean in order to obtain the 95% confidence interval.



✓ In repeated samples, the values of the sample mean will follow a Normal distribution with mean μ and standard deviation 2.

✓ The 68-95-99.7 Rule says that in 95% of all samples of size 400, the sample mean will be within 4 (two standard deviations) of μ.

✓ If the sample mean is within 4 points of μ, then μ is within 4 points of the sample mean.

✓ Therefore, the interval from 4 points below to 4 points above the sample mean will "capture" μ in about 95% of all samples of size 400.

If researchers estimate that μ lies somewhere in the interval 76.08 to **83.92**, they would be calculating an interval using a method that captures the true μ in about 95% of all possible samples of this size.

- Notice that with higher confidence levels the confidence interval gets large so there is less precision.

- According to the 68-95-99.7 Rule:
  - The 68% confidence interval for this example is between 78 and 82.
  - The 95% confidence interval for this example is between 76 and 84.
  - The 99.7% confidence interval for this example is between 74 and 86.
- Therefore, the larger the confidence level, the larger the interval. There is a trade-off between the two.
- If researchers want to be very certain that their interval includes the population mean, they must extend the interval, so there is less precision.
- For intervals that are not specified in the 68-95-99.7 Rule, z* can be used to obtain the upper bound and the lower bound of the interval.
- It is important to note that z* provides more precise estimates than the 68-
  95-99.7 Rule

How Confidence Intervals Behave

The z confidence interval for the mean of a Normal population illustrates several important properties that are shared by all confidence intervals in common use.

- The user chooses the confidence level and the margin of error follows.
- Researchers would prefer high confidence with a small margin of error.
  - High confidence suggests the method almost always gives correct answers.
  - A small margin of error suggests the parameter has been pinned down precisely.

How is a small margin of error obtained?

The margin of error for the z confidence interval is:

$$z^* \cdot \frac{\sigma}{\sqrt{n}}$$

The margin of error gets smaller when:

- z^* gets smaller (the same as a lower confidence level C)
- σ is smaller. It is easier to pin down μ when σ is smaller.
- n gets larger. Since n is under the square root sign, four times as many observations are needed to cut the margin of error in half.

Interpreting the Confidence Level

The confidence level is the overall capture rate if the method is used many times. The sample mean will vary from sample to sample, but when the method *estimate* ± *margin of error* is used to get an interval based on each sample, C% of these intervals capture the unknown population mean μ.

To say that there is 95% *confidence* is shorthand for "95% *of* all possible samples of a given size from this population will result in an interval that captures the unknown parameter."

Confidence Intervals: The Four-Step Process

State: What is the practical question that requires estimating a parameter?

Plan: Identify the parameter, choose a level of confidence, and select the type of confidence interval that fits the situation.

Solve: Carry out the work in two phases:

1. **Check the conditions** for the interval that has been chosen.

2. Calculate the **confidence interval**.

Conclude: Return to the practical question to describe the results in this setting.