Electromagnetic waves

Lecture 9

Directional Magnetic Field

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Introduction

Magnetic phenomena were first observed at least 2500 years ago in fragments of magnetized iron ore found near the ancient city of Magnesia (now Manisa, in western Turkey). These fragments were examples of what are now called permanent magnets; you probably have several permanent magnets on your refrigerator. Permanent magnets were found to exert forces on each other as well as on pieces of iron that were not magnetized. It was discovered that when an iron rod is brought in contact with a natural magnet, the rod also becomes magnetized. When such a rod is floated on water or suspended by a string from its center, it tends to line itself up in a north-south direction. The needle of an ordinary compass is just such a piece of magnetized iron. Before the relationship of magnetic interactions to moving charges was understood, the interactions of permanent magnets and compass needles were described in terms of magnetic poles. If a bar-shaped permanent magnet, or bar magnet, is free to rotate, one end points north. This end is called a north pole or N pole; the other end is a south pole or S pole. Opposite poles attract each other, and like poles repel each other Fig.9.1 Opposite poles attract each other, and like poles repel each other. An object that contains iron but is not itself magnetized (that is, it shows no tendency to point north or south) is attracted by either pole of a permanent magnet.

By analogy to electric interactions, we describe the interactions by saying that a bar magnet sets up a magnetic field in the space around it and a second body responds to that field. A compass needle tends to align with the magnetic field at the needle's position. The earth itself is a magnet. Its north geographic pole is close to a magnetic south pole, which is why the north pole of a compass needle points north. The earth's magnetic axis is not quite parallel to its geographic axis (the axis of rotation), so a compass reading deviates somewhat from geographic north. This deviation, which varies with location, is called magnetic declination or magnetic variation. Also،the magnetic field is not horizontal at most points on the earth's surface; its angle up or down is called magnetic inclination. At the magnetic poles the magnetic field is vertical see Fig. 9.2.

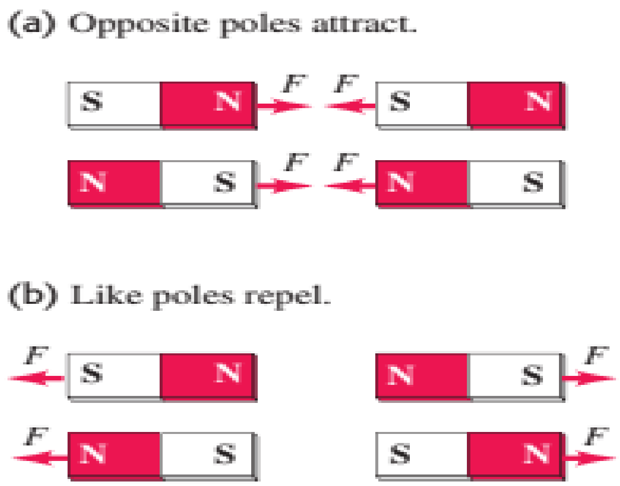

Fig 9.1

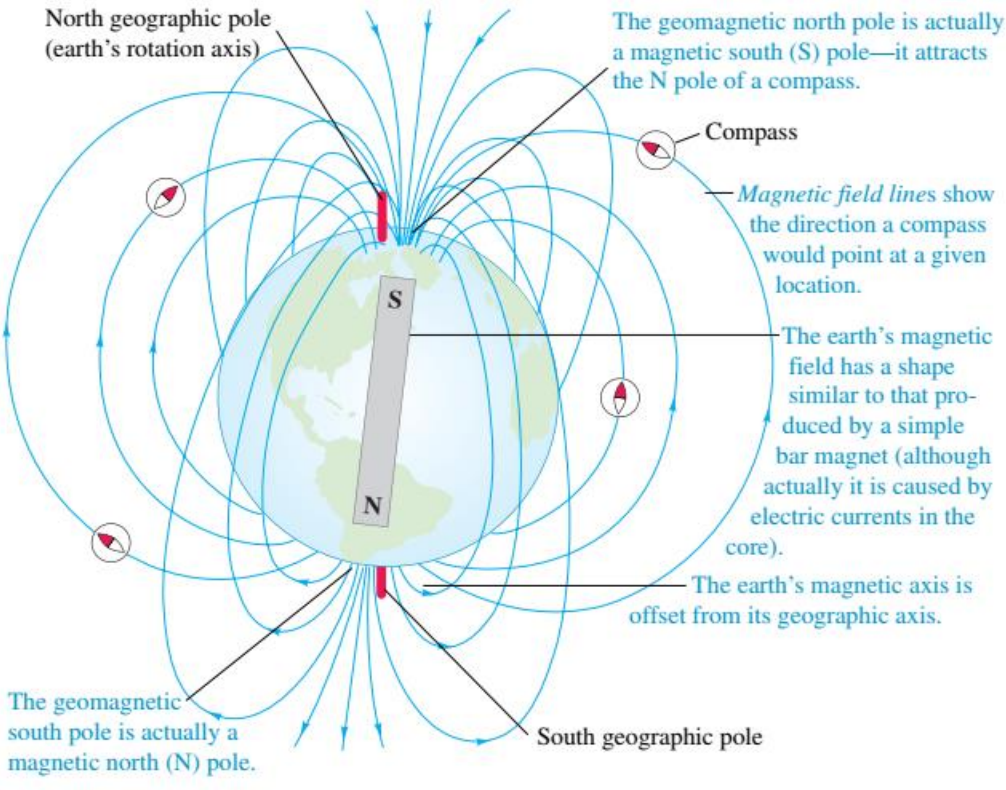

Fig 9.2

9.2 Magnetic Poles Versus Electric Charge

The concept of magnetic poles may appear similar to that of electric charge, and north and south poles may seem analogous to positive and negative charge. But the analogy can be misleading. While isolated positive and negative charges exist, there is no experimental evidence that a single isolated magnetic pole exists; poles always appear in pairs. If a bar magnet is broken in two, each broken end becomes a pole Fig. 9.3. The existence of an isolated magnetic pole, or magnetic monopole, would have sweeping implications for theoretical physics. Extensive searches for magnetic monopoles have been carried out, but so far without success.

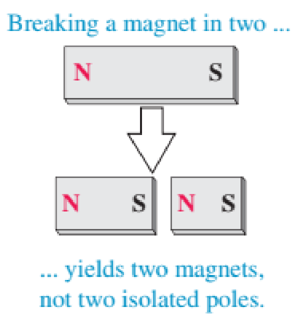

Fig 9.3

9.3 Magnetic Forces on Moving Charges

Like electric field, magnetic field is a vector field - that is, a vector quantity associated with each point in space. We will use the symbol $\vec{B}$ for magnetic field. At any position the direction of $\vec{B}$ is defined as the direction in which the north pole of a compass needle tends to point. The arrows suggest the direction of the earth's magnetic field; for any magnet, $\vec{B}$ points out of its north pole and into its south pole.

A particle with charge q moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$ experiences a force $\vec{F}$ that is perpendicular to both $\vec{v}$ and $\vec{B}$. The SI unit of magnetic field is the tesla (1 T=1 N/A. m).

Another unit of B, the gauss ,($1G = 10^{-4}T$) , is also in common use. The magnetic field of the earth is of the order of 10−4 T or 1G. However we can write the force as

$$\vec{F} = q\vec{v} \times \vec{B}$$

When a charged particle moves through a region of space where both electric and magnetic fields are present, both fields exert forces on the particle. The total force $\vec{F}$ is the vector sum of the electric and magnetic forces and it is called Lorentz force:

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

* Right Hand Rule for Magnetic Field Due to a Straight Wire

To find the direction of the magnetic field use the right hand rule.

• Point thumb in direction of current

• The fingers will curl in the direction of the magnetic field

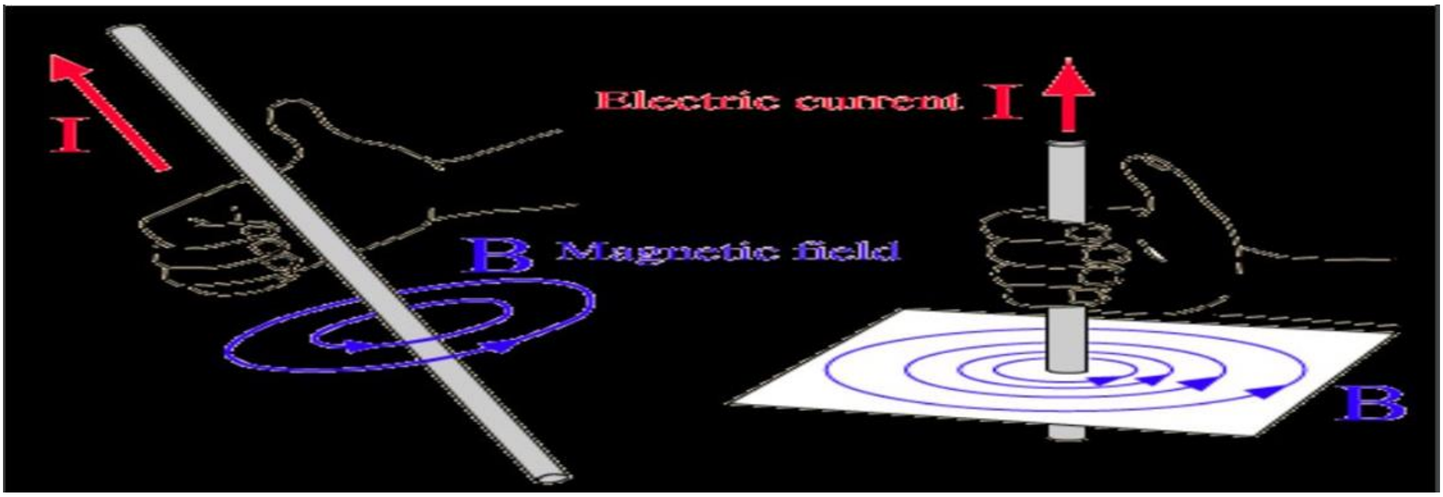

Fig 9.4

Example 1.

A beam of protons $q = 1.6\times10^{-19}$C moves at 3.0×10^{5} m/s through a uniform 2.0-T magnetic field directed along the positive z-axis. Find the force on a proton if the velocity of each proton lie in the xz-plane and is directed at 30◦ to the z-axis .

Solution:

$$F = qvBsin\theta$$
$$= (1.6 \times 10^{-19}\text{C})(3.0 \times 10^{5}\,\tfrac{\text{m}}{\text{s}})(2.0\text{ T})(\sin 30)$$
$$= 4.8 \times 10^{-14}N$$

9.4 Magnetic Flux

We define the magnetic flux through a surface just as we defined electric flux in connection with Gauss's law. We define the magnetic flux through elements of area dA see Fig.9.5 as

$$d\emptyset_B = B_{\perp}dA = Bcos\emptyset dA = \vec{B}.d\vec{A}$$

The total magnetic flux through the surface is the sum of the contributions from the individual area elements:

$$\emptyset_B = \int B_{\perp}dA = \int Bcos\emptyset dA = \int \vec{B}.d\vec{A}$$

The SI unit of magnetic flux is equal to the ، unit of magnetic field (1 T) times the unit of area ($1m^2$) . This unit is called the weber (1Wb), in honor of the German physicist Wilhelm Weber (1804-1891):

$$1\text{Wb} = 1\,T.m^2$$

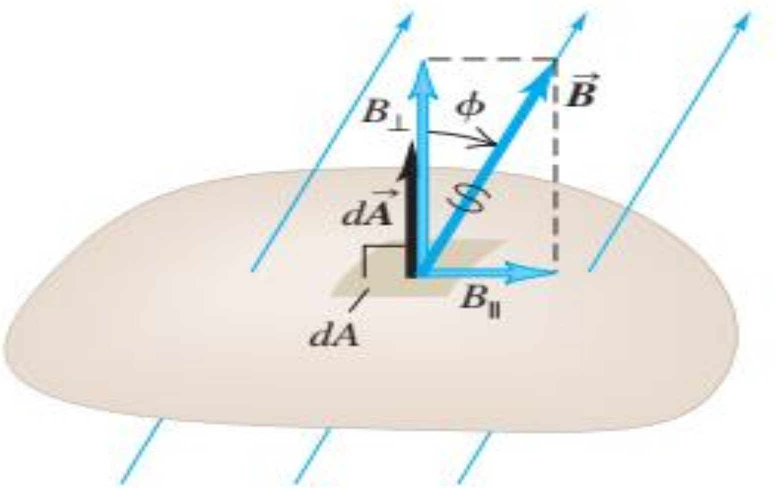

Fig 4.5

Also, 1 T = 1 N/A·m, so

$$1\text{Wb} = 1\,T.m^2 = 1\,\text{N.m/A}$$

Note: The total magnetic flux through a closed surface is always zero.

$$\oint \vec{B}.d\vec{A} = 0$$

and we can use divergence theorem to prove that

$$\nabla.\vec{B} = 0$$

The magnitude of magnetic field is equal to flux per unit area across an area at right angles φ = 90 to the magnetic field. For this reason, magnetic field is sometimes called magnetic flux density.

Example 2.

A flat surface with area 3.0 cm^2 in a uniform magnetic field $\vec{B}$ with an angle of 60°. The magnetic flux through this surface is +0.90mWb. Find the magnitude of the magnetic field.

Solution:

$$B = \frac{\emptyset_B}{A\cos\emptyset} = \frac{0.90 \times 10^{-3} Wb}{(3.0 \times 10^{-4}\, m^2)(\cos 60^{\circ})} = 6.0\, T$$

4.5 Motion of Charged Particles in a Magnetic Field

Motion in a magnetic field: The magnetic force is always perpendicular to $\vec{v}$; a particle moving under the action of a magnetic field alone moves with constant speed. In a uniform field, a particle with initial velocity perpendicular to the field moves in a circle with radius *R* as shown in Fig.9.6 that depends on the magnetic field strength *B* and the particle mass *m*, speed *v*, and charge *q*.

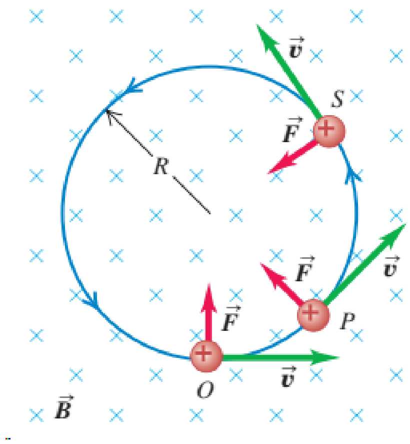

Fig 9.6

so from Newton's second law,

$$F = |q|vB = m\frac{v^2}{R}$$

The radius of rotation is given by

$$R = \frac{mv}{|q|B}$$

We can derive an expression for the total force on all the moving charges in a length *l* of conductor with cross-sectional area A as shown in Fig.

We can derive an expression for the total force on all the moving charges in a length *l* of conductor with cross-sectional area *A*. The number of charges per unit volume is *n*; a segment of conductor with length *l* has volume *Al* and contains a number of charges equal to *nAl* . The total force $\vec{F}$. on all the moving charges in this segment has magnitude

$$F = (nAl)(qv_dB) = (nqv_dA)(lB)$$

the current density is $J = nqv_d$, thus the above equation can be written as

$$F = IlB$$

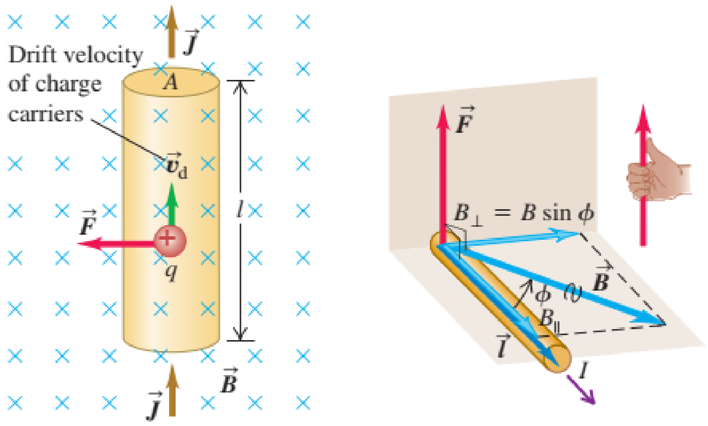

Fig 9.7

If the $\vec{B}$ field is not perpendicular to the wire but makes an angle φ with it,

$$F = IlB_\perp = IlBsin\varphi$$

so the magnetic force on a straight wire segment

$$\vec{F} = I\vec{l} \times \vec{B}$$

and the magnetic force on an infinitesimal wire section

$$\overrightarrow{dF} = Id\vec{l} \times \vec{B}$$

Example 3.

A straight horizontal copper rod carries a current of 50.0 A in a magnetic field at an angle of 45° with the magnitude 1.20 T. (a) Find the magnitude of the force on a 1.00-m section of rod. (b) Find the maximum value of the force on this section

Solution:

(a)

$$F = IlBsin\varphi = (50.0\text{ A})(1.00\ m)(1.20\text{ T})(sin45^{\circ}) = 42.4\ N$$

(b)

$$F = IlBsin\varphi = (50.0\text{ A})(1.00\ m)(1.20\text{ T}) = 60\ N$$

4.6 Magnetic Torque

Figure 9.8 shows a rectangular loop of wire with side lengths *a* and *b*. A line perpendicular to the plane of the loop (i.e., a normal to the plane) makes an angle φ with the direction of the magnetic

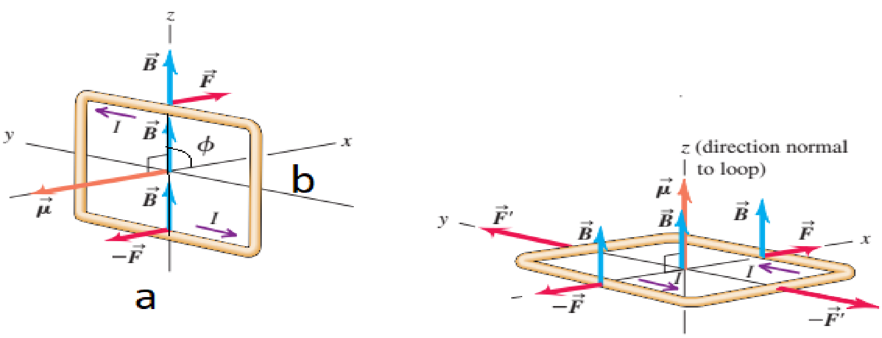

Fig 9.8

field $\vec{B}$, and the loop carries a current I . The wires leading the current into and out of the loop and the source of emf are omitted to keep the diagram simple.

The force $\vec{F}$ on the right side of the loop (length a) is to the right, in the +x-direction as shown. On this side, $\vec{B}$ is perpendicular to the current direction, and the force on this side has magnitude

$$F = IaB$$

the magnitude of the net torque is

$$\tau = 2F\left(\frac{b}{2}\right)sin\emptyset = (IBa)(b\ sin\emptyset)$$

The area of the loop is equal to so we can rewrite previous equation $\tau = \dots$

$$\tau = IBAsin\emptyset$$

The product $I\,A$ is called the **magnetic dipole moment or magnetic moment** of the loop, for which we use the symbol μ (the Greek letter mu):

$$\mu = IA$$

thus, we can write the torque

$$\tau = \mu Bsin\emptyset$$

or

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

4.7 The Biot-Savart Law

Biot–Savart's law states that the differential magnetic field dB produced at a point P, by the differential current element Idl is proportional to the product Idl and the sine of the angle a between the element and the line joining P to the element and is inversely proportional to the square of the distance r between P and the element.

$$dB = \frac{\mu_\circ}{4\pi}\frac{Idl\ sin\emptyset}{r^2}$$

The constant $\mu_\circ$ is called the permeability of free space:

$$\mu_\circ = 4\pi \times 10^{-7}\frac{N}{A^2}$$

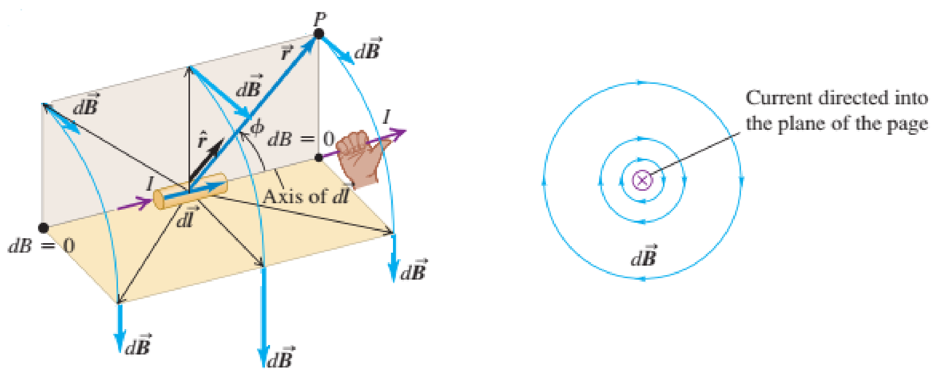

Fig 9.9

In a vector form we can write

$$d\vec{B} = \frac{\mu_\circ}{4\pi} \frac{Id\vec{l} \times \hat{r}}{r^2}$$

the Bio-Savart law is given by

$$\vec{B} = \frac{\mu_\circ}{4\pi} \int \frac{Id\vec{l} \times \hat{r}}{r^2}$$

9.7.1 Magnetic Field of a Straight Current-Carrying Conductor

For long conductor, *B* must have the same magnitude at all points on a circle centered on the conductor and lying in a plane perpendicular to it, and the direction of *B* must be everywhere tangent to such a circle. Thus, at all points on a circle of radius r around the conductor, the magnitude B is

$$B = \frac{\mu_\circ I}{2\pi r}$$

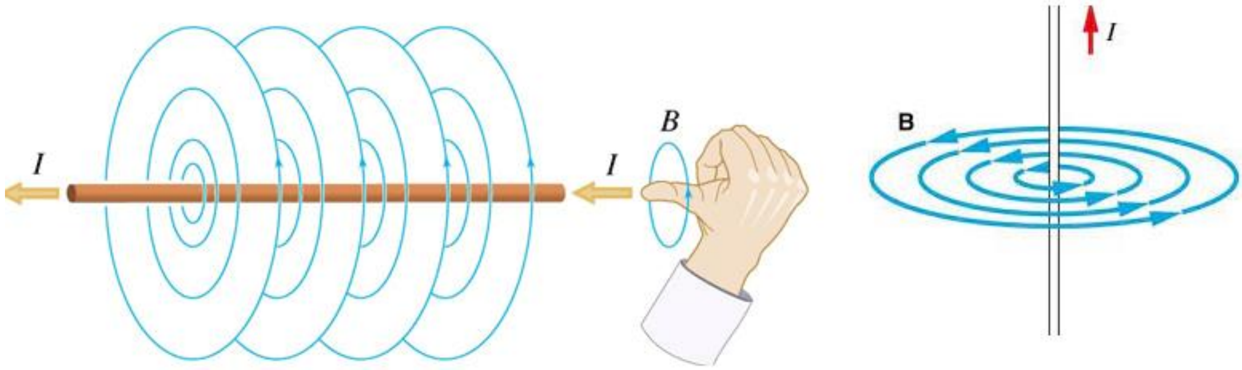

Fig 9.10

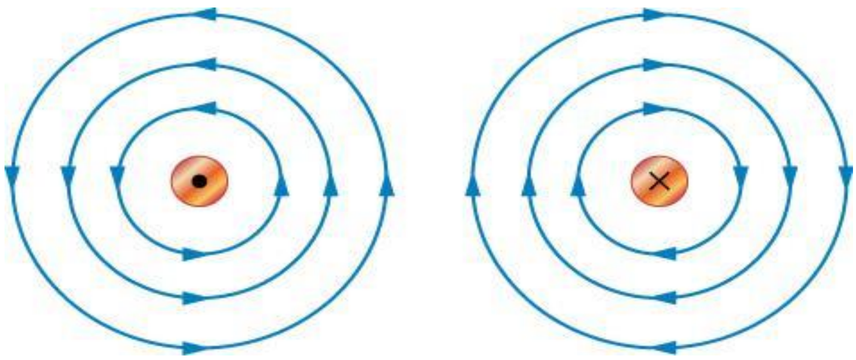

Fig 9.11

9.7.2 Magnetic Field of a Circular Current Loop

The direction of themagnetic field on the axis of a current-carrying loop is given by a right-hand rule. If you curl the fingers of your right hand around the loop in the direction of the current, your

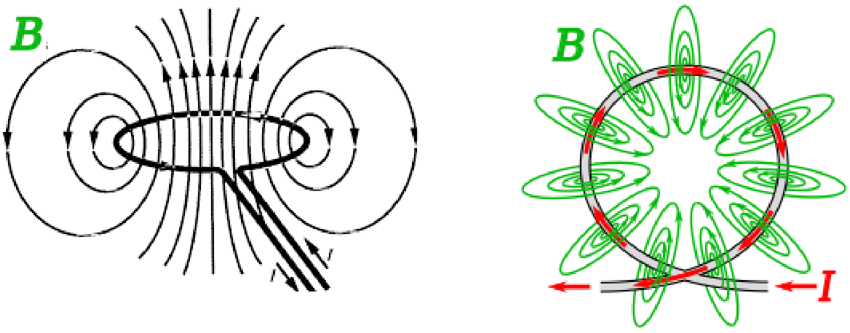

Fig 9.12

right thumb points in the direction of the field the magnetic field at center of the loop of radius *a* is given by

$$B = \frac{\mu_\circ I}{2a}$$

the direction of the magnetic field is perpendicular to the loop surface. Now suppose that instead of the single loop, we have a coil consisting of N loops, all with the same radius. Then the total field is *N* times the field of a single loop:

$$B = \frac{\mu_\circ IN}{2a}$$

9.8 Ampere's Law

Ampere's law states that the line integral of $\vec{B}$ around any closed path equals $\mu_\circ$ times the net current through the area enclosed by the path. The positive sense of current is determined by a right-hand rule.(see the figure below)

$$\oint \vec{B} d\vec{l} = \mu_\circ I_{encl}$$

We can use Stockes theorem to write Ampere Law in the following form:

$$\nabla \times \vec{B} = \mu_\circ \vec{J}$$

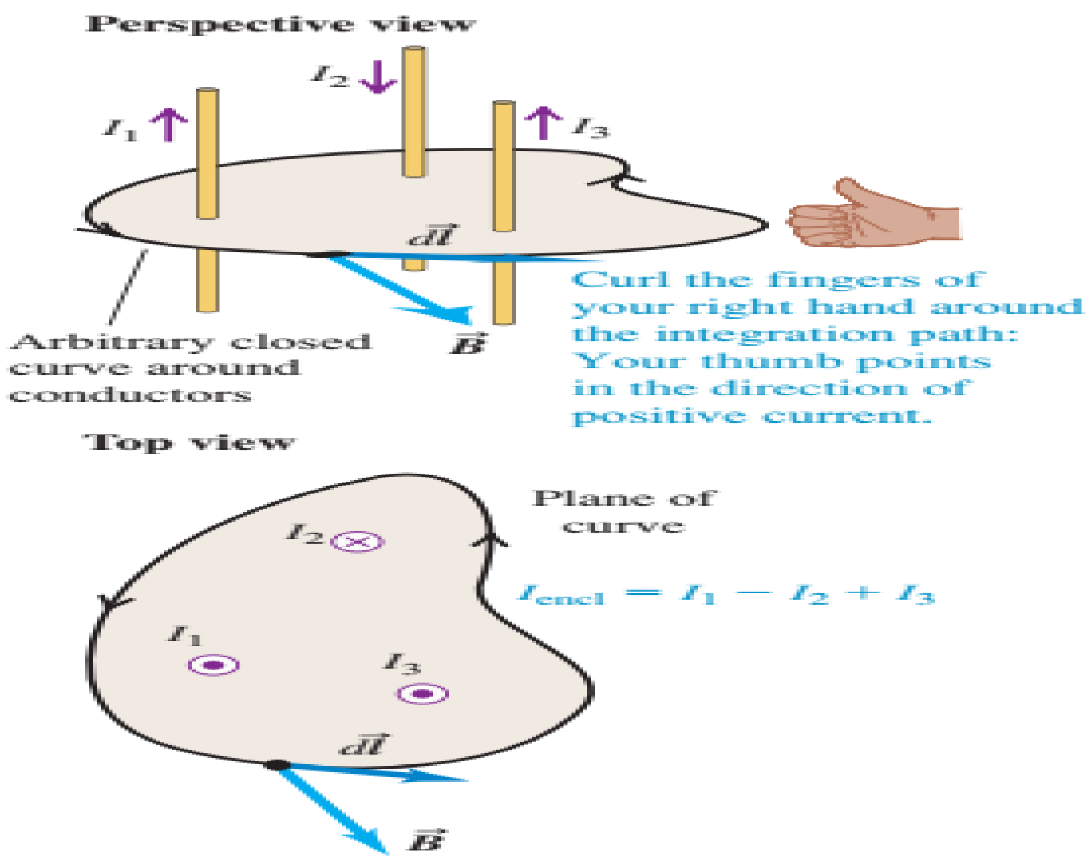

Fig 9.13

Example 4 .

For a long, straight, current-carrying conductor, use Ampere's law to find $\vec{B}$.

Solution:

$$\oint \vec{B}.d\vec{l} = \mu_\circ I$$

$$\oint \vec{B}.d\vec{l} = B\oint dl = 2\pi r B$$

using the two equation we get

$$B = \frac{\mu_\circ I}{2\pi r}$$

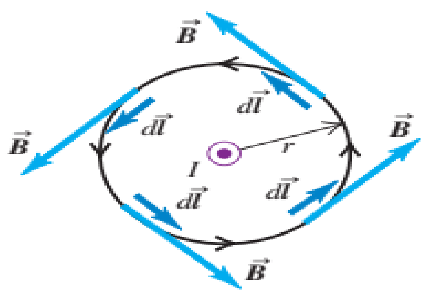

Fig 9.14

9.9 Field of a Solenoid

An intuitive argument can also be used to show that the field outside the solenoid is actually zero. Magnetic field lines only exist as loops, they cannot diverge from or converge to a point like electric field lines can (see Gauss's law for magnetism). The magnetic field lines follow the longitudinal path of the solenoid inside, so they must go in the opposite direction outside of the solenoid so that the lines can form a loop. However, the volume outside the solenoid is much greater than the volume inside, so the density of magnetic field lines outside is greatly reduced. Now recall that the field outside is constant. In order for the total number of field lines to be conserved, the field outside must go to zero as the solenoid gets longer.

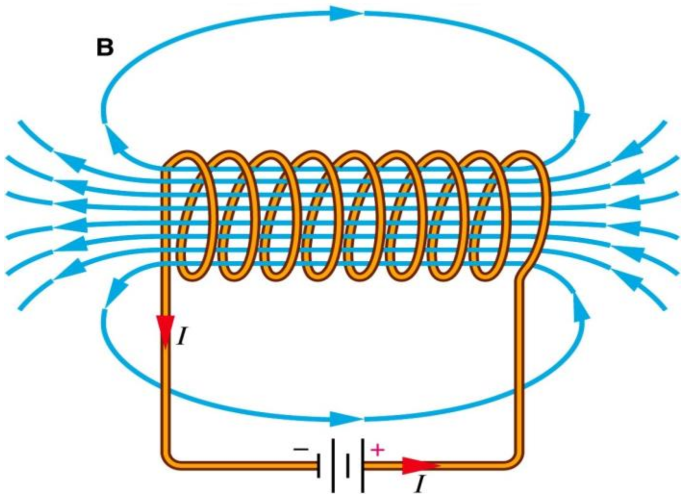

Fig 9.15

A solenoid consists of a helical winding of wire on a cylinder, usually circular in cross section. IDENTIFY and SET UP: We assume that $\vec{B}$ is uniform inside the solenoid and zero outside. Figure below shows the situation and our chosen integration path, rectangle *abcd*. Side *ab*, with length *L*, is parallel to the axis of the solenoid. Sides *bc* and *da* are taken to be very long so that side *cd* is far from the solenoid; then the field at side *cd* is negligibly small. We assume that is uniform

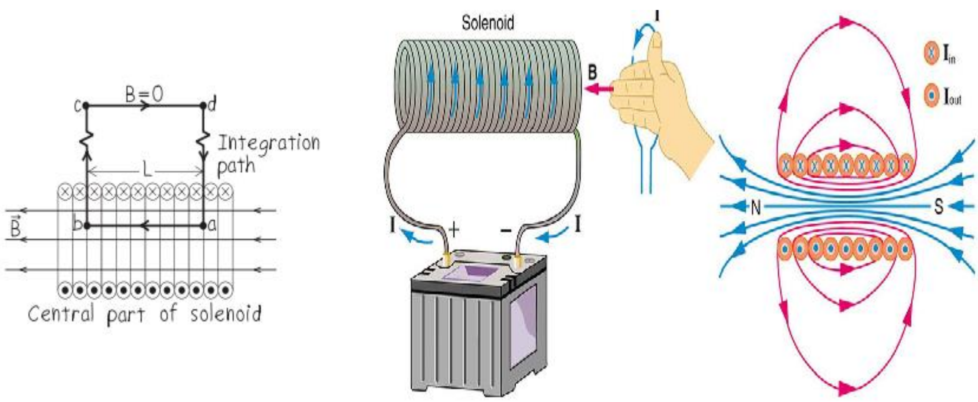

Fig 9.16

inside the solenoid and zero outside

$$\int_a^b \vec{B}.d\vec{l} = BL$$

Along sides *bc* and *da*, $\vec{B}$ is perpendicular to the path and so $B_{\parallel} = 0$; alongside *cd*, $\vec{B}$ = **0** and so $B_{\parallel} = 0$. Around the entire closed path, then, we have $\int \vec{B}.d\vec{l} = BL$

In a length *L* there are *N* = *nL* turns, each of which passes once through abcd carrying current *I* . Hence the total current enclosed by the rectangle is

$$I_{encl} = NI = nLI$$

We can write

$$BL = \mu_\circ nLI$$

Thus

$$B = \mu_\circ nI$$

Side ab need not lie on the axis of the solenoid, so this result demonstrates that the field is uniform over the entire cross section at the center of the solenoid's length.

Homework :

Find the force on a moving particle due to the magnetic field of a wire