Numerical analysis

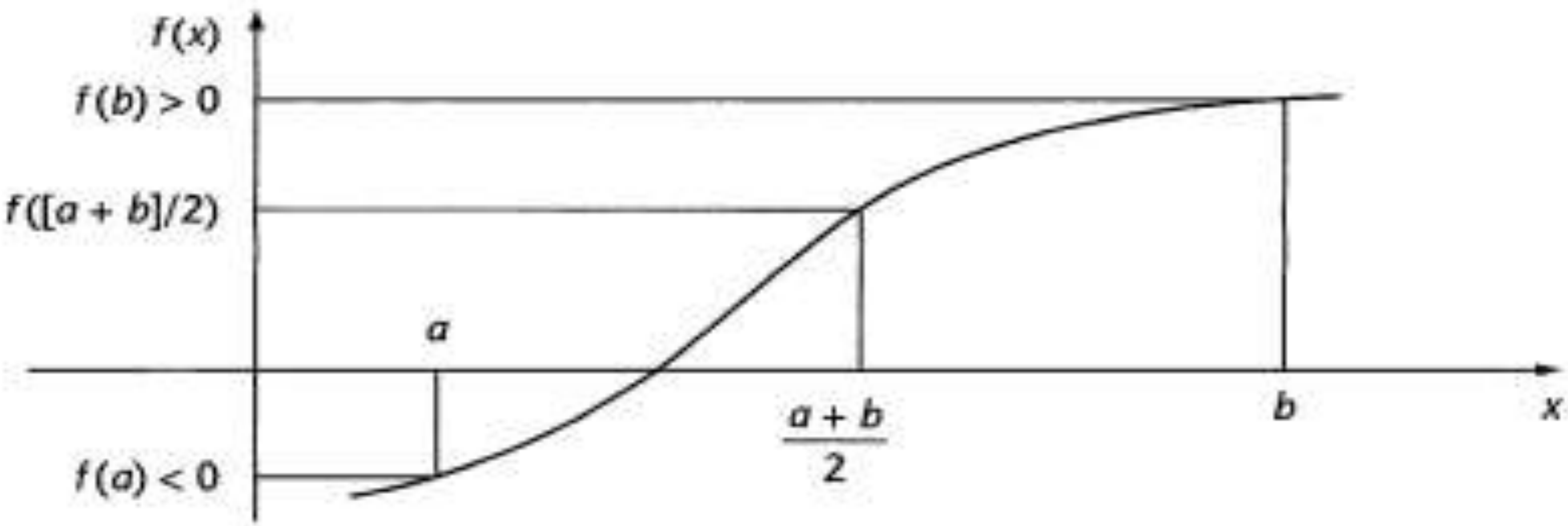
1. Bisection method.

The bisection method of finding a solution to the equation $f(x) = 0$ consists of

Finding a value of x, say $x = a$, such that $f(a) < 0$

Finding a value of x, say $x = b$, such that $f(b) > 0$

The solution to the equation $f(x) = 0$ must then lie between a and b. Furthermore, it must lie either in the first half of the interval between a and b or in the second half.



Find the value of $f([a+b]/2)$ – that is halfway between a and b.

If $f([a+b]/2) > 0$ then the solution lies in the first half and if $f([a+b]/2) < 0$ then it lies in the second half. This procedure is repeated, narrowing down the width of the interval by a half each time. An example should clarify all this.

Example 1:

Find the positive value of x that satisfies the equation $x^2 - 2 = 0$.

Firstly we note that if $x = 1$ then $x^2 - 2 < 0$, and that if $x = 2$ then $x^2 - 2 > 0$, so the solution that we seek must lie between 1 and 2.

We look for the ....

The mid-point between 1 and 2 which is 1·5

Now, when $x = 1{\cdot}5$, $x^2 - 2 = 0{\cdot}25 > 0$

so the solution must lie between ....

1 and 1·5

The mid-point between 1 and 1·5 is 1·25. When $x = 1{\cdot}25$, $x^2 - 2 = -0{\cdot}4375 < 0$

so the solution must lie between ....

1·25 and 1·5

The mid-point between 1·25 and 1·5 is 1·375. We now evaluate $x^2 - 2$ at this point and determine in which half interval the solution lies. This process is repeated and the following table displays the results. In each block of six numbers the first column lists the end points of the interval and the mid-point. The second column contains the respective values $f(x) = x^2 - 2$. Construct the table as follows.

a	1·0000	−1·0000	1·0000	−1·0000	1·5000	0·2500	1·5000	0·2500
b	2·0000	2·0000	1·5000	0·2500	1·2500	−0·4375	1·3750	−0·1094
$(a+b)/2$	1·5000	0·2500	1·2500	−0·4375	1·3750	−0·1094	1·4375	0·0664
a	1·3750	−0·1094	1·4375	0·0664	1·4063	−0·0225	1·4219	0·0217
b	1·4375	0·0664	1·4063	−0·0225	1·4219	0·0217	1·4141	−0·0004
$(a+b)/2$	1·4063	−0·0225	1·4219	0·0217	1·4141	−0·0004	1·4180	0·0106
a	1·4141	−0·0004	1·4141	−0·0004	1·4141	−0·0004	1·4141	−0·0004
b	1·4180	0·0106	1·4160	0·0051	1·4150	0·0023	1·4146	0·0010
$(a+b)/2$	1·4160	0·0051	1·4150	0·0023	1·4146	0·0010	1·4143	0·0003
a	1·4141	−0·0004	1.4143	0·0003	1·4142	−0·0001		
b	1·4143	0·0003	1·4142	−0·0001	1·4142	0·0001		
$(a+b)/2$	1·4142	−0·0001	1·4142	0·0001	1·4142	0·0000		

The final result to four decimal places is $x = 1{\cdot}4142$ which is the correct answer to that level of accuracy – but it has taken a lot of activity to produce it. A much faster way of solving this equation is to use an iteration formula that was first devised by Newton.

Let us take another explanation

1. Find the points a and b such that a $< b$
2. Find f(a)
3. Find f((b)
4. If f(a) . f(b)< 0
5. Then find x_0 = (a + b)/2
6. Find f(x_0)
7. If f(x_0) = 0 then x_0 = the exact root
8. If f(a). f(x_0) < 0 $then \quad b = x_0$
9. If f(b). f(x_0) < 0 $then \quad a = x_0$
10. Repeat the above steps until f(x_i)=0

Example: find the root of the following equation

f(x) = x^3 - x -1

If x = 0 , 1 , 2

F(x) = -1 , -1 ,5

Find the root of the equation f(x) = x^3 - x -1

The solution x^3 - x -1=0

X=0 , 1 , 2

f(x) = -1 , -1 , 5

first iteration

x_1 = (1+ 2) /2 = 1.5

F(1.5) = 0.875> 0 $then\ \ in\ the\ first\ \ i.e\ between\ 1\ and\ 1.5$

Second iteration

x_2 = (1+ 1.5)/2 = 1.25

f(1.25) = -0.29688 then the root between 1.25 and 1.5

x_3 = (1.25 + 1.5) /2 = 1.375

f(1.375) = 0.22461 > 0 then the root between 1.25 and 1.375

Third iteration

x_4 = (1.25 + 1.375)/2 = 1.3125

f(1.3125) = =-0.05151 then root lies between 1.3125 and 1.375

Forth iteration

x_5 = (1.3125 + 1.375)/2= 1.34375

f(1.34375) = .08261> 0

Then the root lies between 1.3125 and 1.34375

Fifth iteration:

x_6= (1.3125 + 1.34375)/2 = 1.32812

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Then the new root lies between 1.3125 and 1.32812
then ,
Sixth iteration
x_7 = (1.3125 + 1.32812)/2 = 1.32031
f(1.32031) = --0.01871 < 0
Then the new root lies between 1.32812 and 1.32031
Then
Eight iteration
x_8 = (1.32812 + 1.32031)/2 = 1.32422
f(1.32422) = -0.00213 $<$ 0
The new root lies between 1.32422 and 1.32812
Nine iteration
x_9= (1.32422 + 1.32812)/2 = 1.32617
f(1.32617) =0.00621> 0
Ten iteration
The new root lies between 1.32617 and 1.32422
x_{10}= (1.32617 + 1.32422)/2 = 1.3252
f(1.3252)= 0.00204> 0
The new root lies between 1.3252 and 1.32422
The new root = x_{11}= 1.32471 and f(1.32471) = -0.00005< 0
Then the approximate root = - 1.32471

home work

Example : find the root of the equation

x^3 - 4 x = 9