probability

1. Principle of probability, $1 \geq p \geq 0$
2. Sample space . P(s)=1
3. Events.
4. Properties of probability
5. Binomial distribution.
6. Poisson distribution.

Probability is the branch of mathematics concerning numerical descriptions of how likely an event is to occur, or how likely it is that a proposition is true. ... The higher the **probability** of an event, the more likely it is that the event will occur

$$1.0 \geq P \geq 0$$

Basic Probability Rules:

- Probability Rule One (For any event A, $0 \leq P(A) \leq 1$)
- Probability Rule Two (The sum of the probabilities of all possible outcomes is 1)
- Probability Rule Three (The Complement Rule)
- Probabilities Involving Multiple Events
- Probability Rule Four (Addition Rule for Disjoint Events)
- Finding P(A and B) using Logic
- Probability Rule Five (The General Addition Rule)

The probability of an outcome e in a sample space S is a number P between 1 and 0 that measured the likelihood that e will occur on a single trial of the corresponding random experiment , the value of P =0 correspond to out come e being impossible and the value of P =1 correspond to the out come e being certain

BASIC PROPERTIES OF PROBABILTY:

1. The probability of an event E is defined as P(E) = [Number of favourable outcomes of E]/[total number of possible outcomes of E].

2. The probability of a sure event or certain event is 1.

3. The probability of an impossible event is 0.

4. The probability of an event E is a number P(E) such that $0 \leq P(E) \leq 1$. Probability is always a positive number.

5. If A and B are 2 events that are mutually exclusive, then $P(A \cup B) = P(A) + P(B)$.

6. An elementary event is an event having only one outcome. The sum of the probabilities of such events of an experiment is 1.

7. The sum of probabilities of an event and its complementary event is 1. $P(A) + P(A') = 1$.

8. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

9. $P(A \cap B) = P(A) + P(B) - P(A \cup B)$.

10. If $A_1, A_2, A_3, \ldots, A_n$ are mutually exclusive events, then $P(A_1 \cup A_2 \cup A_3 \ldots \cup A_n) = P(A_1) + P(A_2) + \ldots + P(A_n)$

SOLVED EXAMPLES:

Example 1:

Find the probability of getting an even number when a die is tossed.

Solution:

S = {1, 2, 3, 4, 5, 6}

Favourable events = { 2,4,6}

Number of favourable event = 3

Total number of outcomes = 6

Hence probability, P = 3/6 = ½

Example 2:

A carton consists of 200 watches of which 190 are good, 7 have minor defects and 3 have major defects. A merchant Reena will only accept the watches which are good, but another merchant Seema will only reject the watches which have major defects. One watch is drawn at random from the carton. What is the probability that it is acceptable to Reena? Also, find the probability that it is acceptable to Seema?

Solution:

A watch is drawn at random from 200 watches. So there are 200 equally likely outcomes.

No. of outcomes acceptable to Reena = 190/200 = 0.95

No. of outcomes acceptable to Seema = (190+7)/200 = 197/200 = 0.985

Formula to find the Probability of an Event

Probability denotes the occurrence or non-occurrence of an event. The following formula is used in calculating the probability of an event.

Probability of event A = P(A) = (Number of outcomes favourable) / (Total number of outcomes)

Mutually exclusive events:

Two or more events associated to a random experiment are mutually exclusive if the occurrence of one prevents the occurrence of the other.Two or more events associated with a random experiment are mutually exclusive if there is no elementary event favourable to all events. Thus if two events are mutually exclusive then $P(A \cap B) = 0$

Likewise if A, B, C are mutually exclusive events, then $P(A \cap B \cap C) = 0$.

Mutually exhaustive events:

Two or more events associated to a random experiment are mutually exhaustive if their union is the sample space. I.e events A_1, A_2 ..An are associated to a random experiment with sample space S are exhaustive if $A_1 \cup A_2 \cup \ldots A_n = S$. All elementary events associated with a random experiment form a system of mutually exclusive and exhaustive events.

IMPORTANT POINTS TO REMEMBER:

1.If A and B are two events associated with a random experiment, then

$P(A \cup B) = P(A)+P(B)-P(A \cap B)$

2.If A and B are mutually exclusive events, then $P(A \cap B) = 0$.

$\therefore P(A \cup B) = P(A)+P(B)$

3.If A, B and C are three events associated with a random experiment, then

$P(A \cup B \cup C)= P(A)+P(B)+P(C)- P(A\cap B)- P(B\cap C)- P(A\cap C)+P(A\cap B\cap C)$

If A, B and C are mutually exclusive events, then

$P(A\cap B)= P(B\cap C)= P(A\cap C) = P(A\cap B\cap C) = 0$

$\therefore P(A \cup B \cup C) = P(A)+P(B)+P(C)$

4.Let A and B be the two events associated to a random experiment, then

$$P(\bar{A} \cap B) = P(B) - P(A \cap B)$$

$P(\bar{A} \cap B)$ is called the probability of occurrence of B only.

$$P(A \cap \bar{B}) = P(A) - P(A \cap B)$$

$P(A \cap \bar{B})$ is called probability of occurrence of A only.

$P((A \cap \bar{B}) \cup (\bar{A} \cap B))$ = P(A) +P(B)-2P(A∩ B)

$P((A \cap \bar{B}) \cup (\bar{A} \cap B))$ is called the probability of occurrence of exactly one of two events A and B.

5.For any two events A and B the probability that exactly one of A,B occurs is given by

$P(A) + P(B) - 2P(A \cup B) = P(A \cup B) - P(A \cap B)$

6.If there are 3 events, A,B and C then

$P(\text{Atleast two of A, B, C occur}) = P(A \cap B) + P(B \cap C) + P(C \cap A) - 2P(A \cap B \cap C)$

$P(\text{Exactly two of A, B, C occur}) = P(A \cap B) + P(B \cap C) + P(C \cap A) - 3P(A \cap B \cap C)$

$P(\text{Exactly one of A, B, C occur}) = P(A) + P(B) + P(C) + P(A \cap B) - 2P(A \cap B) - 2P(B \cap C) - 2P(A \cap C) + 3P(A \cap B \cap C)$

7.If A and B are independent events associated with a random experiment, then

$P(A/B) = P(A)$ and $P(B/A) = P(B)$

If A and B are two events two events associated with a random experiment, then

$P(A \cap B) = P(A)\,P(B/A)$, if $P(A) \neq 0$

Or $P(A \cap B) = P(B)\,P(A/B)$, if $P(B) \neq 0$.

8.If A and B are independent events then $P(A/B) = P(A)$ and $P(B/A) = P(B)$

$\therefore P(A \cap B) = P(A)\,P(B)$

$P(A \cup B) = 1 - P(\bar{A})P(\bar{B})$

Example 1: A coin is tossed and a die is rolled. What is the probability that the coin shows the head and the die shows 3?

Solution:

When a coin is tossed, the outcome is either head or a tail. Similarly, when a die is rolled, the outcomes will be 1, 2, 3, 4, 5, 6.

Hence the required probability =(1/2) (1/6)=1/12.

Example 2: 8 nails and 12 nuts are present in a box. Among them, half of the nails and half of the nuts are rusted. If an item is chosen at random, what is the probability that it is rusted or is a nail?

Solution:

Total rusted items = 4 + 6 = 10;

Unrusted nails = 4.

Required probability = 4 + 10/8 + 12 = 14/20 = 7/10.

Example 3: The probability of happening an event A in one trial is 0.5. What is the probability that event A happens at least once in three independent trials?

Solution:

Here P(A)=0.5 and P($\overline{A}$)=0.5

The probability that A does not happen at all =$(0.5)^3$

Thus, required Probability = $1 - (0.5)^3$ =0.875

Example 4: A man and a woman appear in an interview for two vacancies for the same post. The probability of man's selection is 1/3 and that of the woman's selection is 1/2. What is the probability that neither of them will be selected?

Solution:

Let E1 be the event that man will be selected and E2 the event that woman will be selected.

Then P(E1)=1/3 so P $\overline{E_1}$ =1-1/3=2/3 and

P(E2)=1/2

So P($\overline{E_2}$)=1-1/2 = 1/2

Clearly E1 and E2 are independent events.

So, P($\overline{E_1} \cap \overline{E_2}$)=P($\overline{E_1}$)×P($\overline{E_2}$)=2/3×1/2=1/3.

Example 5: Three dice are thrown simultaneously. What is the probability of obtaining a total of 17 or 18?

Solution:

Three dice can be thrown in $6\times6\times6=216$ ways.

A total of 17 can be obtained as (5,6,6),(6,5,6), (6,6,5).

A total of 18 can be obtained as (6,6,6).

Hence the required probability $=4/216=1/54$.

Example 6: In order to get at least once a head with probability ≥ 0.9, what is the number of times a coin needs to be tossed?

Solution:

Probability of getting at least one head in n tosses

$\Rightarrow 1 - (1/2)^n \geq 0.9$

$\Rightarrow (1/2)^n \leq 0.1$

$\Rightarrow 2^n \geq 10$

$\Rightarrow n \geq 3$

Hence the least value of $n = 4$.

Example 7: The three ships namely A, B, and C sail from India to Africa. If the ratio of the ships reaching safely is 2: 5, 3: 7 and 6: 11, then find the probability of all of them arriving safely.

Solution:

We have a ratio of the ships A, B and C for arriving safely are 2: 5, 3: 7 and 6: 11 respectively.

The probability of ship A for arriving safely = 2 / [2 + 5] = 2 / 7

Similarly, for B = 3 / [3 + 7] = 3 / 10 and for C = 6 / [6 + 11] = 6 / 17

∴Probability of all the ships for arriving safely = [2 / 7] × [3 / 10] × [6 / 17] = [18 / 595]

Example 8: If A and B are two events such that P (A) = 0.4 , P (A + B) = 0.7 and P (AB) = 0.2, then P (B) =

Solution:

Since we have P (A + B) = P (A) + P (B) − P (AB)

⇒ 0.7 = 0.4 + P (B) − 0.2

⇒ P (B) = 0.5.

Example 9: If A and B are two independent events such that $P,(A\cap\ B')=\ \frac{3}{25}\ and\ P(A'\cap\ B)=$

$\frac{8}{25}$, then find P(A).

Solution:

Since events are independent.

So, $P(A\cap\ B')=\ P(A)\times\ P(B')=\ \frac{3}{25}$

$\Rightarrow\ P(A)\times\ \{1-\ 2P(B)\}=\ \frac{3}{25}$ .....(i)

Similarly,

$P(B)\times\ \{1-\ P(A)\}=\ \frac{8}{25}$ .....(ii)

On solving (i) and (ii), we get

$P(A)=\ \frac{1}{5}\ and\ \frac{3}{5}.$

Example 10: Consider two events A and B to be independent. The probability of occurrence of both the events A and B is 1 / 6 and the probability that neither of them occurs is 1 / 3. Then find the probability of the two events.

Solution:

$$P(A\cap \ B) = \ P(A).P(B) = \ \frac{1}{6}$$

$$P(\bar{A}\cap \ \bar{B}) = \ \frac{1}{3} = \ 1- \ P(A\cup \ B)$$

$$\Rightarrow \ \frac{1}{3} = \ 1- \ [P(A) + \ P(B)] + \ \frac{1}{6}$$

$$\Rightarrow \ P(A) + \ P(B) = \ \frac{5}{6}.$$

$$P(A) \ and \ P(B) \ are \ \frac{1}{2} \ and \frac{1}{3}.$$

Probability Distributions

Probability distributions are a fundamental concept in statistics. They are used both on a theoretical level and a practical level.Some practical uses of probability distributions are:

•To calculate confidence intervals for parameters and to calculate critical regions for hypothesis tests.

•For univariate data, it is often useful to determine a reasonable distributional model for the data.

•Statistical intervals and hypothesis tests are often based on specific distributional assumptions. Before computing an interval or test based on a distributional assumption, we need to verify that the assumption is justified for the given data set. In this case, the distribution does not need to be the best-fitting distribution for the data, but an adequate enough model so that the statistical technique yields valid conclusions.

•Simulation studies with random numbers generated from using a specific probability distribution are often needed.

Binomial Distribution:

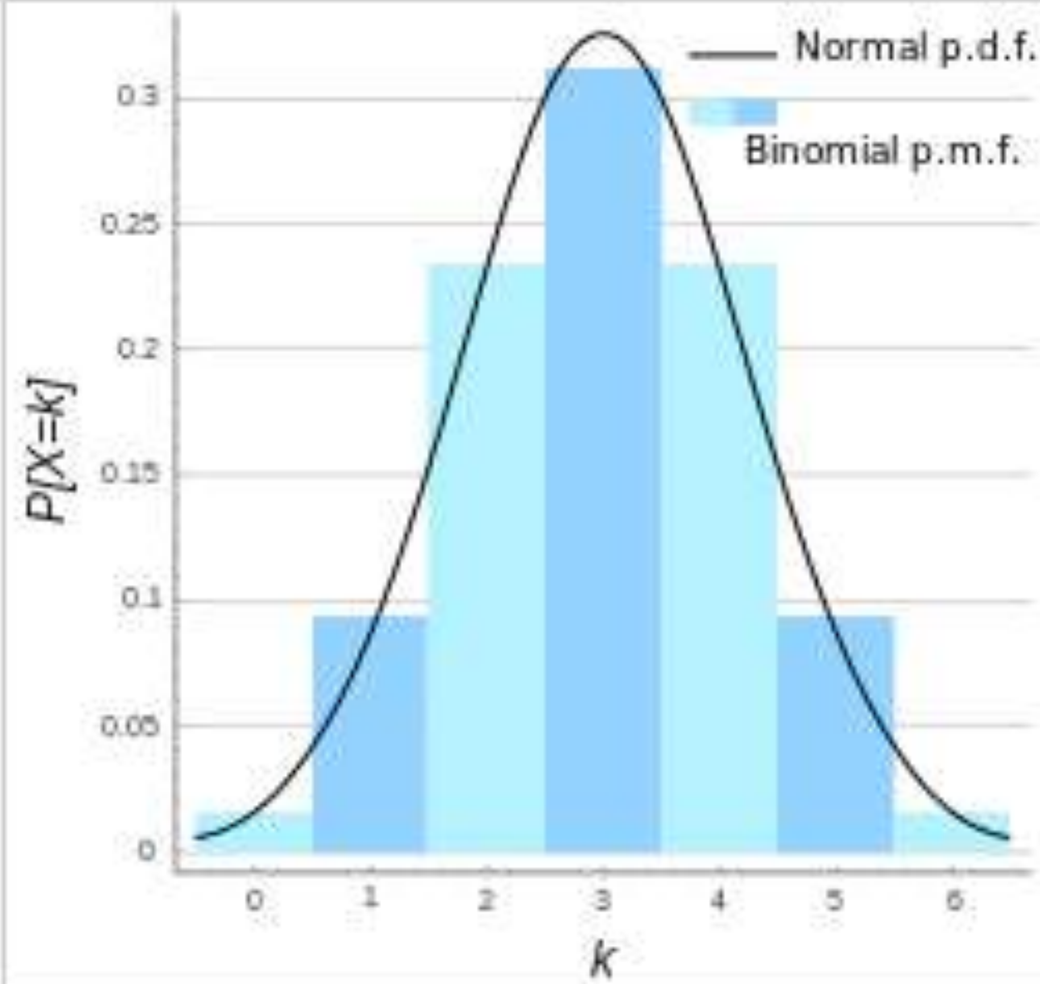
The binomial distribution is used when there are exactly two mutually exclusive outcomes of a trial. These outcomes are appropriately labeled "success" and "failure". The binomial distribution is used to obtain the probability of observing x successes in N trials, with the probability of success on a single trial denoted by p. The binomial distribution assumes that p is fixed for all trials.

The formula for the binomial probability mass function is

$$P(x;p,n) = \binom{n}{x} (p)^x (1-p)^{(n-x)} \qquad \text{for } x = 0, 1, 2, \cdots, n$$

where

$$\binom{n}{x} = \frac{n!}{x!(n-x)!}$$



Binomial probability mass function and normal probability density function approximation for $n = 6$ and $p = 0.5$

Example 1:

Tossing a Coin:

- Did we get Heads (H) or
- Tails (T)

We say the probability of the coin landing **H** is ½
And the probability of the coin landing **T** is ½

Throwing a Die:

- Did we get a four ... ?
- ... or not?

We say the probability of a **four** is 1/6 (one of the six faces is a four)
And the probability of **not four** is 5/6 (five of the six faces are not a four)

Note that a die has 6 sides but here we look at only **two** cases: **"four: yes"** or **"four: no"**

Let's Toss a Coin!

Toss a fair coin **three times** ... what is the chance of getting **two Heads**?

Tossing a coin three times (**H** is for heads, **T** for Tails) can get any of these 8 **outcomes**:

HHH

HHT

HTH

HTT

THH

THT

TTH

TTT

Which outcomes do we want?

"Two Heads" could be in any order: "HHT", "THH" and "HTH" all have two Heads (and one Tail).

So **3 of the outcomes** produce "Two Heads".

What is the probability of each outcome?

Each outcome is equally likely, and there are 8 of them, so each outcome has a probability of 1/8

So the probability of **event** "Two Heads" is:

Number of outcomes we want		Probability of each outcome		
3	×	1/8	=	3/8

So the chance of getting Two Heads is 3/8

> We used special words:
>
> - **Outcome**: any result of three coin tosses (8 different possibilities)
> - **Event**: "Two Heads" out of three coin tosses (3 outcomes have this)

3 Heads, 2 Heads, 1 Head, None

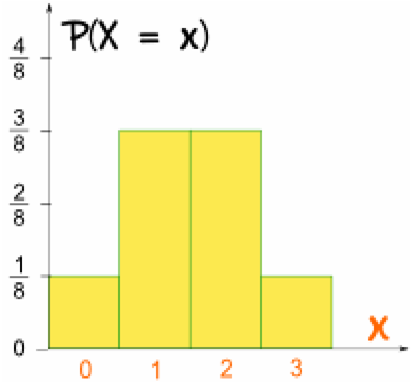
The calculations are (P means "Probability of"):

- P(Three Heads) = P(**HHH**) = **1/8**
- P(Two Heads) = P(**HHT**) + P(**HTH**) + P(**THH**) = 1/8 + 1/8 + 1/8 = **3/8**
- P(One Head) = P(**HTT**) + P(**THT**) + P(**TTH**) = 1/8 + 1/8 + 1/8 = **3/8**
- P(Zero Heads) = P(**TTT**) = **1/8**

We can write this in terms of a Random Variable, X, = "The number of Heads from 3 tosses of a coin":

- P(X = 3) = 1/8
- P(X = 2) = 3/8
- P(X = 1) = 3/8
- P(X = 0) = 1/8

And this is what it looks like as a graph:



It is symmetrical!

Making a Formula

Now imagine we want the chances of **5 heads in 9 tosses**: to list all 512 outcomes will take a long time! So, the formula could be established as follows:

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

It is often called **"n choose k"**

Example 2:

with 3 tosses, what are the chances of 2 Heads?

We have n=3 and k=2:

$$\frac{n!}{k!(n-k)!} = \frac{3!}{2!(3-2)!}$$

$$= \frac{3\times2\times1}{2\times1\times1}$$

$$= 3$$

So there are 3 outcomes that have "2 Heads"

(We knew that already, but now we have a formula for it.)

Example 3 with 9 tosses, what are the chances of 5 Heads?
We have **n=9** and **k=5**:

$$\frac{n!}{k!(n-k)!} = \frac{9!}{5!(9-5)!}$$

$$= \frac{9\times8\times7\times6\times5\times4\times3\times2\times1}{5\times4\times3\times2\times1 \times 4\times3\times2\times1}$$

$$= 126$$

So 126 of the outcomes will have 5 heads

And for 9 tosses there are a total of 2^9 = 512 outcomes, so we get the probability:

Number of outcomes we want		Probability of each outcome		
126	×	$\frac{1}{512}$	=	$\frac{126}{512}$

So:

$$P(X=5) = \frac{126}{512} = 0.24609375$$

About a **25% chance**.

Summary

- The General Binomial Probability Formula:

$$P(k \text{ out of } n) = \frac{n!}{k!(n-k)!} p^k (1-p)^{(n-k)}$$

- Mean value of X: $\mu = np$
- Variance of X: $\sigma^2 = np(1-p)$
- Standard Deviation of X: $\sigma = \sqrt{(np(1-p))}$

Example

Your company makes sports bikes. 90% pass final inspection (and 10% fail and need to be fixed).

What is the expected Mean and Variance of the 4 next inspections?

First, let's calculate all probabilities.

- $n = 4$,
- $p = P(\text{Pass}) = 0.9$

X is the Random Variable "Number of passes from four inspections".

Substitute x = 0 to 4 into the formula:

$$\mathbf{P(k\ out\ of\ n)} = \frac{n!}{k!(n-k)!} p^k(1-p)^{(n-k)}$$

Like this:

- $P(X = 0) = \frac{4!}{0!4!} \times 0.9^0 0.1^4 = 1 \times 1 \times 0.0001 = 0.0001$
- $P(X = 1) = \frac{4!}{1!3!} \times 0.9^1 0.1^3 = 4 \times 0.9 \times 0.001 = 0.0036$
- $P(X = 2) = \frac{4!}{2!2!} \times 0.9^2 0.1^2 = 6 \times 0.81 \times 0.01 = 0.0486$
- $P(X = 3) = \frac{4!}{3!1!} \times 0.9^3 0.1^1 = 4 \times 0.729 \times 0.1 = 0.2916$
- $P(X = 4) = \frac{4!}{4!0!} \times 0.9^4 0.1^0 = 1 \times 0.6561 \times 1 = 0.6561$

Summary: "for the 4 next bikes, there is a tiny 0.01% chance of no passes, 0.36% chance of 1 pass, 5% chance of 2 passes, 29% chance of 3 passes, and a whopping 66% chance they all pass the inspection."

Mean, Variance and Standard Deviation

Let's calculate the Mean, Variance and Standard Deviation for the Sports Bike inspections.

There are (relatively) simple formulas for them. They are a little hard to prove, but they do work!

The mean, or "expected value", is:

$$\mu = np$$

For the sports bikes:

$$\mu = 4 \times 0.9 = 3.6$$

So we can expect 3.6 bikes (out of 4) to pass the inspection.
Makes sense really ... 0.9 chance for each bike times 4 bikes equals 3.6

The formula for Variance is:

$$\text{Variance: } \sigma^2 = np(1-p)$$

And Standard Deviation is the square root of variance:

$$\sigma = \sqrt{np(1-p)}$$

For the sports bikes:

$$\text{Variance: } \sigma^2 = 4 \times 0.9 \times 0.1 = 0.36$$

Standard Deviation is:

$$\sigma = \sqrt{0.36} = 0.6$$

Note: we could also calculate them manually, by making a table like this:

X	P(X)	X × P(X)	X^2 × P(X)
0	0.0001	0	0
1	0.0036	0.0036	0.0036
2	0.0486	0.0972	0.1944
3	0.2916	0.8748	2.6244
4	0.6561	2.6244	10.4976
	SUM:	**3.6**	**13.32**

The mean is the **Sum of (X × P(X))**:

$$\mu = 3.6$$

The variance is the **Sum of (X^2 × P(X))** minus **Mean^2**:

$$\text{Variance: } \sigma^2 = 13.32 - 3.6^2 = 0.36$$

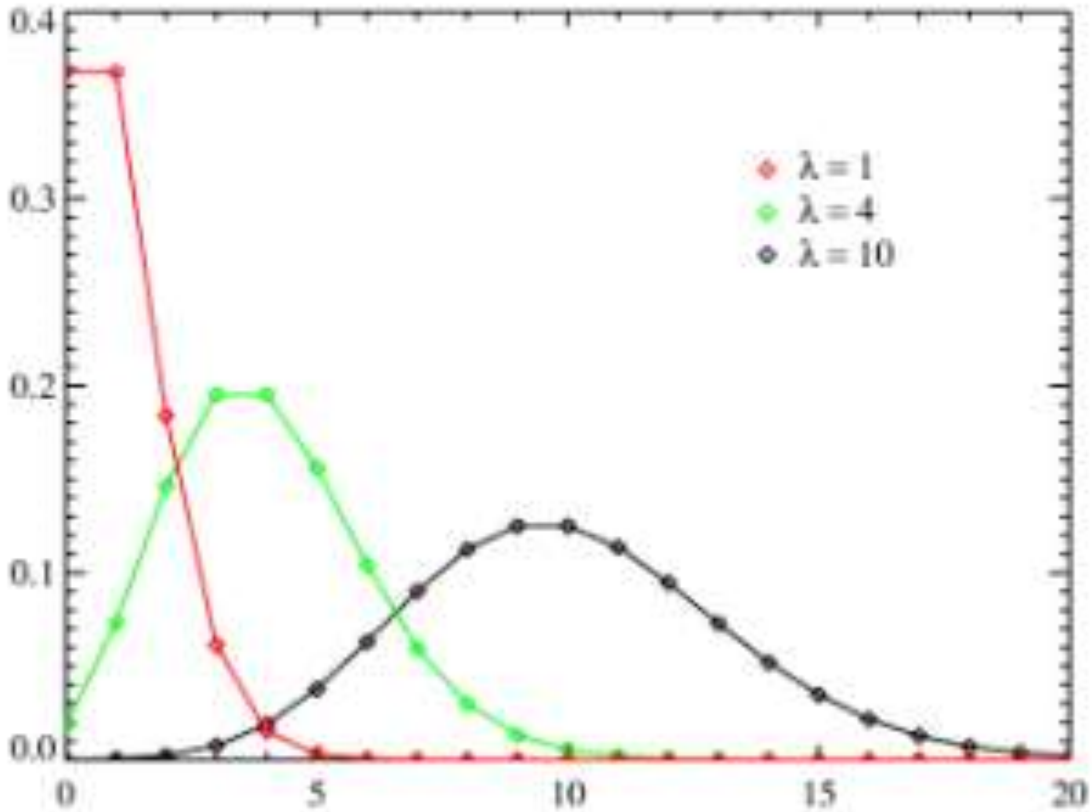
Standard Deviation is:

$$\sigma = \sqrt{(0.36)} = 0.6$$

And we got the same results as before (yay!)

Poisson distribution:

A Poisson distribution is a tool that helps to predict the probability of certain events from happening when you know how often the event has occurred. It gives us the [probability](#) of a given number of events happening in a fixed interval of time.



Poisson distributions, valid only for integers on the horizontal axis.
λ (also written as μ) is the expected number of event occurrences.

Practical Uses of the Poisson Distribution

A textbook store rents an average of 200 books every Saturday night. Using this data, you can **predict the probability that more books will sell** (perhaps 300 or 400) on the following Saturday nights. Another example is the number of diners in a certain restaurant every day. If the average number of diners for seven days is 500, you can predict the probability of a certain day having more customers.

Because of this application, Poisson distributions are used by businessmen to make **forecasts** about the number of customers or sales on certain days or seasons of the year. In business, overstocking will sometimes mean losses if the goods are not sold. Likewise, having too few stocks would still mean a lost business opportunity because you were not able to maximize your sales due to a shortage of stock. By using this tool, businessmen are able to estimate the time when demand is unusually higher, so they can purchase more stock. Hotels and restaurants could prepare for an influx of customers, they could hire extra temporary workers in advance, purchase more supplies, or make contingency plans just in case they cannot accommodate their guests coming to the area.
With the Poisson distribution, companies can adjust supply to demand in order to keep their business earning good profit. In addition, waste of resources is prevented.

Calculating the Poisson Distribution

The Poisson Distribution pmf is: $P(x; \mu) = (e^{-\mu} * \mu^{x}) / x!$

Where:

- The symbol "!" is a factorial.
- μ (the expected number of occurrences) is sometimes written as λ. Sometimes called the **event rate** or rate parameter.

Example question

The average number of major storms in your city is 2 per year. What is the probability that exactly 3 storms will hit your city next year?

- μ = 2 (average number of storms per year, historically)
- x = 3 (the number of storms we think might hit next year)
- e = 2.71828 (e is Euler's number, a constant)

- $P(x; \mu) = (e^{-\mu}) (\mu^x) / x!$
- $= (2.71828^{-2}) (2^3) / 3!$
- $= (0.13534) (8) / 6$
- $= 0.180$

The probability of 3 storms happening next year is 0.180, or 18%

As you can probably tell, you can calculate the Poisson distribution manually but that would take an extraordinary amount of time unless you have a simple set of data. The usual way to calculate a Poisson distribution in real life situations is with software like IBM SPSS.

Poisson distribution vs. Binomial

The above example was over-simplified to show you how to work through a problem. However, it can be challenging to figure out if you should use a binomial distributionor a Poisson distribution. If you aren't given a specific guideline from your instructor, use the following general guideline.

- If your question has an **average probability** of an event happening per unit (i.e. per unit of time, cycle, event) **and** you want to find probability of a certain number of events happening in a period of time (or number of events), then use the Poisson Distribution.
- If you are given an **exact probability** and you want to find the probability of the event happening a certain number out times out of x (i.e. 10 times out of 100, or 99 times out of 1000), use the Binomial Distribution formula.

Solved problems of probability

Sample Space

A **sample space** is the set of all possible outcomes in an experiment.

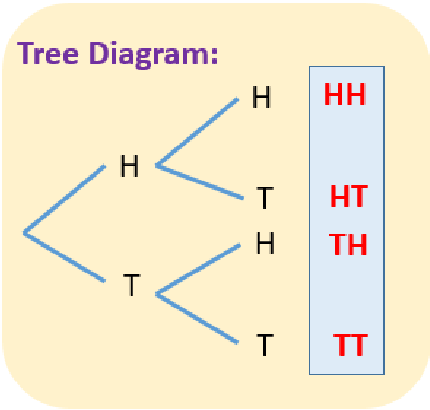
Example:
Two coins are tossed. Represent the sample space for this experiment by making a list, a table, and a tree diagram.
(H – Head, T – Tail)

List:

HH HT TH TT

Table:

	H	T
H	**HH**	**HT**
T	**TH**	**TT**



The sample space is {HH, HT, TH, TT}

Example 1: A coin is thrown 3 times .what is the probability that at least one head is obtained?

Sol: Sample space = [HHH, HHT, HTH, THH, TTH, THT, HTT, TTT]
Total number of ways = 2 × 2 × 2 = 8. Fav. Cases = 7
P (A) = 7/8
OR
P (of getting at least one head) = 1 – P (no head)⇒ 1 – (1/8) = 7/8

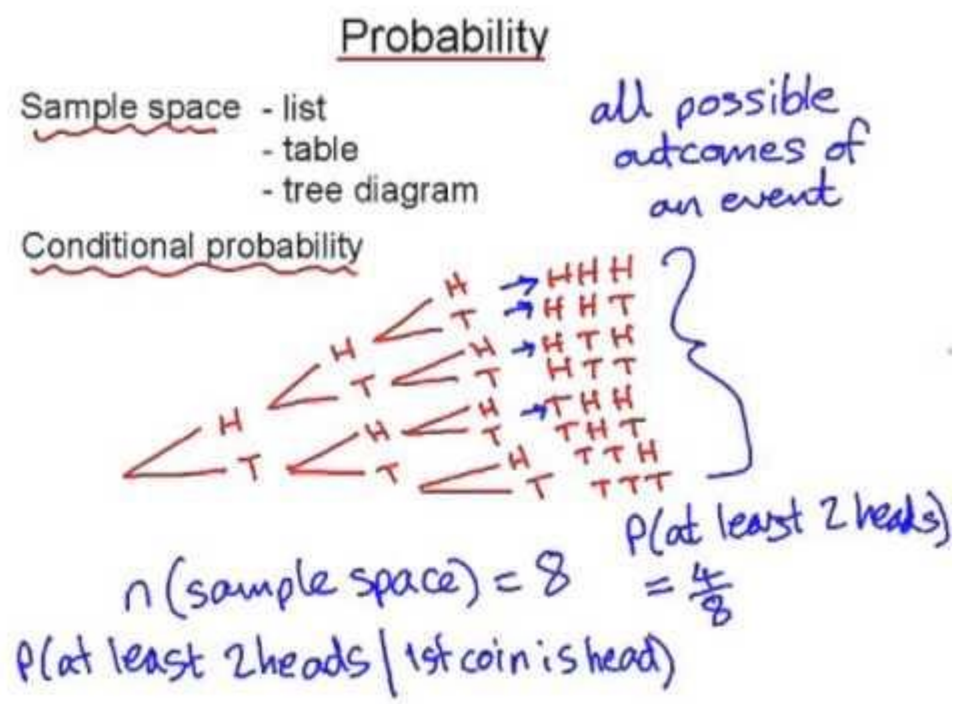
EXAMPLE: 2 Use die to the right answer the following questions:

The sample space is (1,2,3,4,5,6)

2. $P(5) = \frac{1}{6} = 0.1\overline{6} = 16.\overline{6}\%$
3. $P(\text{Even}) = \frac{3}{6} = \frac{1}{2} = 0.5 = 50\%$
4. $P(\text{Prime}) = \frac{4}{6} = \frac{2}{3} = 0.\overline{6} = 66.\overline{6}\%$
5. $P(7) = \frac{0}{6} = 0\%$

Example 3: There are 5 green 7 red balls. Two balls are selected one by one without replacement. Find the probability that first is green and second is red.
Sol: P (G) × P (R) = (5/12) x (7/11) = 35/132

Example 4:



Example 5:

A problem is given to three persons P, Q, R whose respective chances of solving it are 2/7, 4/7, 4/9 respectively. What is the probability that the problem is solved?
Sol: Probability of the problem getting solved = 1 – (Probability of none of them solving the problem)

$$P(P)=\frac{2}{7}\Rightarrow P(\overline{P})=1-\frac{2}{7}=\frac{5}{7},\ P(Q)=\frac{4}{7}\Rightarrow P(\overline{Q})=1-\frac{4}{7}=\frac{3}{7},\ P(R)=\frac{4}{9}\Rightarrow P(\overline{R})=1-\frac{4}{9}=\frac{5}{9}$$

Probability of problem getting solved = 1 – (5/7) x (3/7) x (5/9) = (122/147)

Probability Examples

Examples 1 :

A class of 40 student in Almustaqbal university college classified according to their hobbies as follows:

Solution:

total	Not sport hobbies	Sport hobbies	hobbies
14	10	4	Music hobbies
26	6	20	Not music hobbies

If A is an events of music hobbies, and

B is an events of sport hobbies.

If we choose student randomly, calculate:

$P(A)$, $P(B)$, $P(A \cap B)$, $P(A \cup B)$, $P (A^{''})$, $P (A/B)$, $P(B/A)$, $P(B^{''})$

وان قوانين المطلوب اعلاه كما يلي:

1- $P(A)$ = Probability of music hobbies = 14/40 = 7/20= 0.35
2- $P(B)$ = Probability of sport hobbies = 24/40 = 3/5 = 0.6
3- $P(A \cap B)$= means music hobbies intersect with sport hobbies= 4/40 = 1/10= 0.1
4- $P(A \cup B)$= means union of = $P(A) + P(B) – P(A \cap B)$ = 0.35 + 0.6 - 0.1= 0.85
5- $P(A^{''})$ =means not from music hobbies=1- P(A) =1- 0.35=0.65
6- $P(B^{''})$= means not from sport hobbies= 1- P(B) = 1-0.6= 0.4
7- $P(A/B) = P (A \cap B)/P(B)$ = 0.1/0.6= 0.167 من هواة الموسيقى علما انه من هواة الرياضه
8- $P(B/A) = P (A \cap B)/P(A)$ = 0.1/0.35 = 0.285 من هواة الرياضه علما انه من هواة الموسيقى

EXAMPLE 2 :

A container has 6 red marbles and 4 black marbles. Two marbles are drawn without replacement from it. What is the probability that both of the marbles are black?

CLARFICATIONS:

The probability that two events A and B both occur is the probability of the intersection of A and B. It is denoted by A⋂B.

1. When A and B are independent, the following equation gives the probability of A intersection B. ***P(A⋂B) = P(A).P(B)***

2. When A and B are mutually exclusive events, then ***P(A⋂B) = 0.***

3. The probability that events A and B both occur is equal to the probability that event A occurs times the probability that event B occurs, given that A has already happened.

P(A⋂B) = P(A).P(B/A)

Solution:

Let A denotes the event that the first marble is black and B denotes the event that the second marble is black.

No. of black marbles = 4

Total no. of marbles = 10

P(A) = 4/10

Now we select from remaining marbles.

So P(B/A) = 3/9.

P(A⋂B) = P(A).P(B/A)

= (4/10).(3/9)

= 4/30

= 0.133

Notes : IMPORTANT

Dependent and independent event in Probability

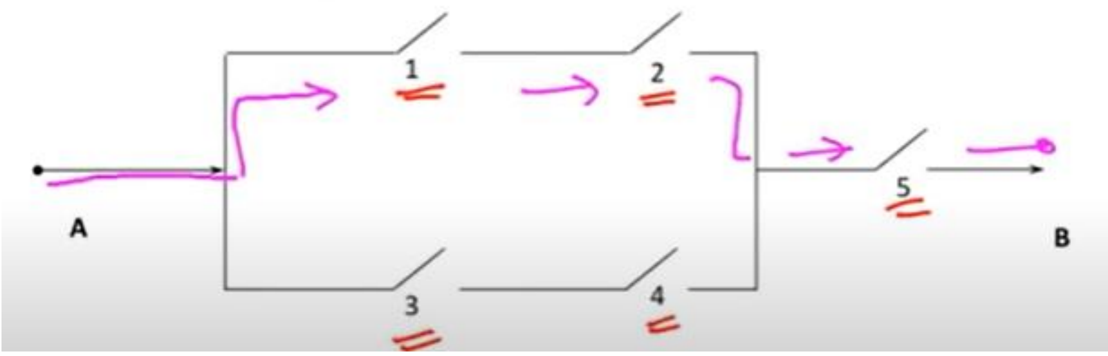
In a probability notation, events A and B are independent if $P(B \mid A) = P(B)$. Events A and B are independent if and only if $P(A \cap B) = P(A) \times P(B)$.

If A and B are dependent events, then $P(A \cap B) = P(B \mid A) \times P(A)$.

Example 3:

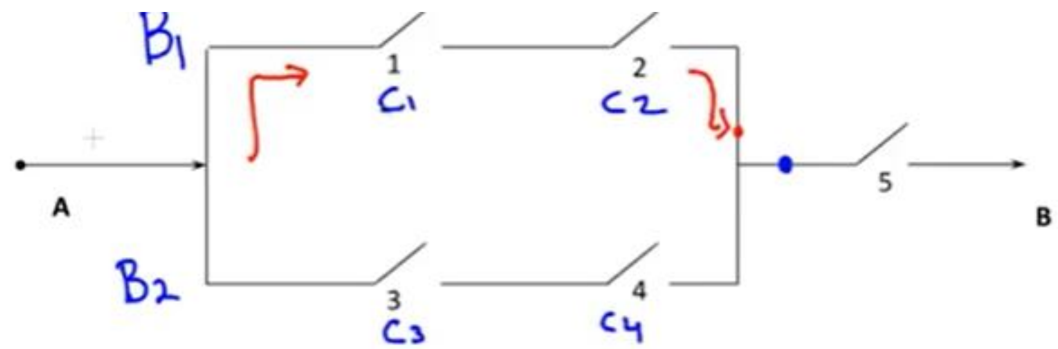
The probabilities of closing the ith relay in the circuit shown below are

Circuit	1	2	3	4	5
P(closure)	.70	.60	.65	.65	.97



If all relays function independently, what is the probability

That a current flows between A and B



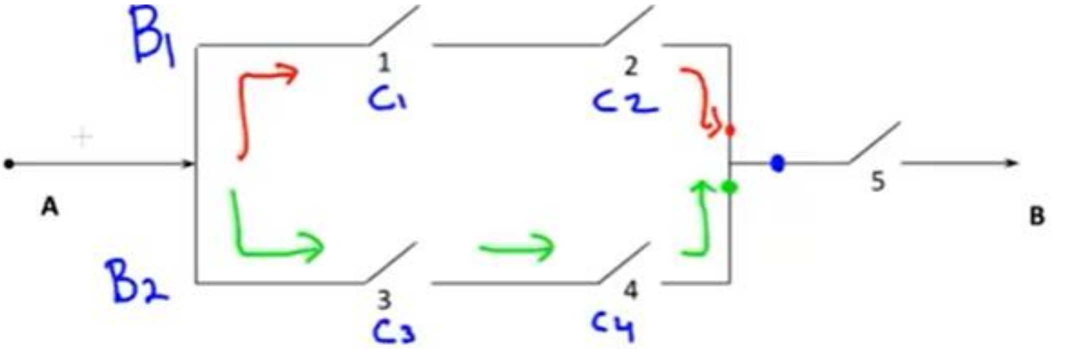
P(upper branch works) = P(C1 **and** C2)

P(upper branch works)= P(C1).P(C2) = 0.7 x 0.6 = 0.42

For the lower branch as follows:

P(upper branch **or** lower branch **or bot**

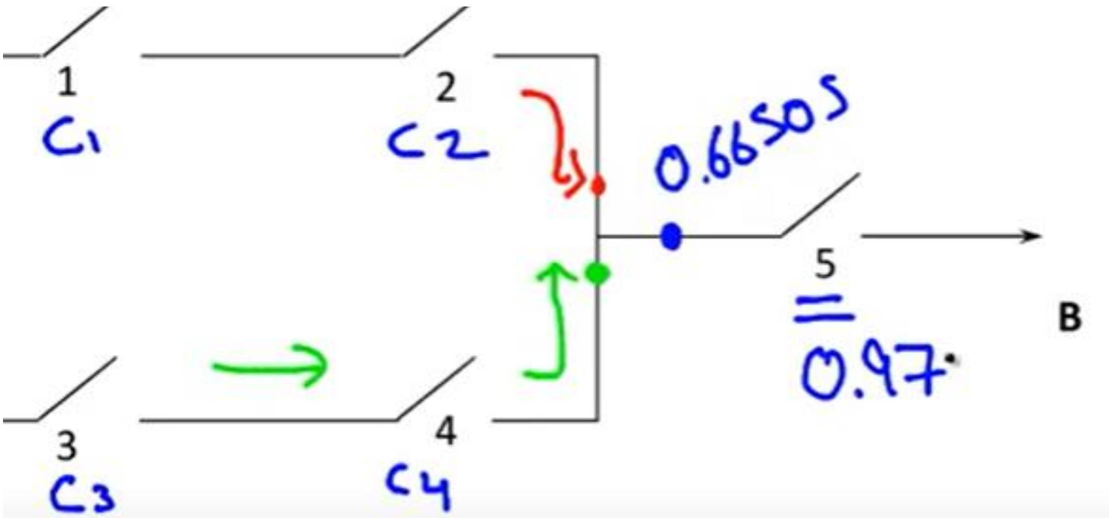
$P(B1 \cup B2) = P(B1) + P(B2) - P(B1 \cap B2)$



Probability of lower branch = 0.65 x 0.65 = 0.4225

=0.42 + 0.4225 – (0.42 x 0.4225) =0.66505

As independent event then P(B1∩B2) then this will equal to (P (B1) x P(B2).

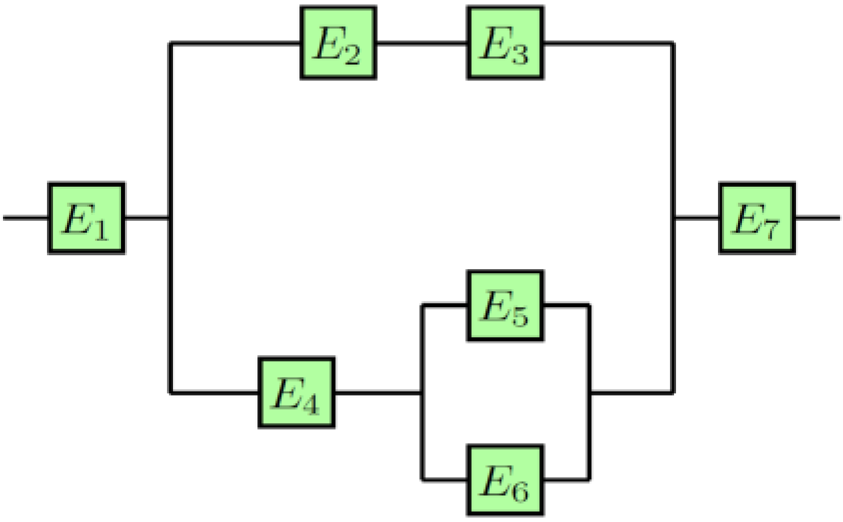


So the probability for the circuit to work is

P(C5 ∩(B1 U B2) = 0.97 x 0.66505 =0.645 .

Example 4:

The following circuit as shown below with their items probability find the probability that the circuit operate functionally well.



Component	E_1	E_2	E_3	E_4	E_5	E_6	E_7
Probability of functioning well	0.9	0.5	0.3	0.1	0.4	0.5	0.8

Solution:

It is clear the series treat by intersection ∩ and the parallel treated by union U

E_2 and E_3 connecting in serious so, (intersection)

$E_2 \cap E_3 = P(E_2) * P(E_3) = 0.5 * 0.3 = 0.15$

E_5 and E_6 connecting in parallel so, (union)

$P(E_5) \cup P(E_6) = P(E_5) + P(E_6) - P(E_5) \cap P(E_6) =$

$0.5 + 0.4 - 0.5 * 0.4 = 0.9 - 0.2 = 0.7$

This result (probability of 0.7) connecting in serious with E_4 so,

$P(0.7) \cap P(E_4) = 0.7 * 0.1 = 0.07$

0.07 connected in parallel with 0.15 so,

$P(0.07) \cup P(0.15) = 0.07 + 0.15 - 0.07*0.15=0.2095$

This will be connected with E_1 in serious so, intersection yield,

$P(0.2095) \cap P(E_1) = 0.2095 * 0.9= 0.18855$

This will be connected with E_7 in serious so, intersection yield,

$P(0.18855) \cap P(E_7) = 0.18855 * 0.8 = 0.15084$

So, the probability for the above circuit to work is

0.15084

Basic definitions and work sheet probability

We often specify a set by listing its members, or **elements**, in parentheses like this $\{\}$.

For example $A = \{2, 4, 6, 8\}$ means that A is the set consisting of numbers 2,4,6,8.

We could also write $A =$\{even numbers less than 9\}.

The **union** of A and B is the set of elements which belong to A or to B (or both) and can be written as $A \cup B$.

The **intersection** of A and B is the set of elements which belong to both A and B, and can be written as $A \cap B$.

The **complement** of A, frequently denoted by $\overline{A}$, is the set of all elements which do not belong to A. In making this definition we assume that all elements we are thinking about belong to some larger set U, which we call the **universal** set.

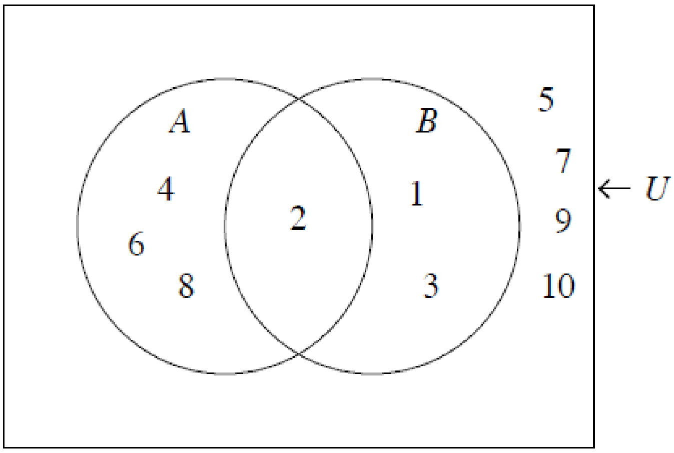
The **empty set**, written $\emptyset$ or $\{\}$, means the set with no elements in it.

A set C is a **subset** of A if **all** the elements in C are also in A.

For example, let

$$\begin{aligned} U &= \{\text{all positive numbers} \leq 10\} \\ A &= \{2, 4, 6, 8\} \\ B &= \{1, 2, 3\} \\ C &= \{6, 8\} \end{aligned}$$

Sets A, B and U may be represented in a Venn Diagram as follows:



A intersection B, $A \cap B$, is shown in the Venn diagram by the overlap of the sets A and B, $A \cap B = \{2\}$.

The union of the sets A and B, $A \cup B$, is the set of elements that are in $A = \{2, 4, 6, 8\}$ together with the elements that are in $B = \{1, 2, 3\}$ including each element once only.

So, $A \cup B = \{1, 2, 3, 4, 6, 8\}$.

The complement of A is the set $\overline{A}$ is contains all the elements in U which are not in A. So, $\overline{A} = \{1, 3, 5, 7, 9, 10\}$.

C is a subset of A as every element in $C = \{6, 8\}$ is also in $A = \{2, 4, 6, 8\}$.

Finite Equiprobable Spaces

In loose terms, we say that the probability of something happening is $\frac{1}{4}$, if, when the experiment is repeated often under the same conditions, the stated result occurs 25% of the time.

For the moment, we will confine our discussion to cases where there are a finite number of equally likely outcomes.

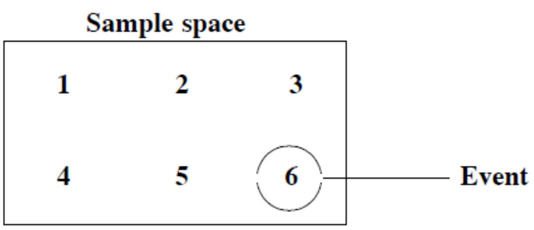
For example, if a coin is tossed, there are two equally likely outcomes: heads (H) or tails (T). If a die is tossed, there are six equally likely outcomes: 1,2,3,4,5,6.

Some Notation

The set of all possible outcomes of the given experiment is called the **sample space**. An **event** is a subset of a sample space.

Calculating Probabilities

Look again at the example of rolling a six faced die. The possible outcomes in this experiment are 1,2,3,4,5,6, so the sample space is the set {1,2,3,4,5,6}. The 'event' of 'getting a 6' is the subset {6}. We represent this in the following diagram.



There are six possibilities in the sample space and only one of these corresponds to getting a 6, so the probability of getting a 6 when you roll a die is $\frac{1}{6}$.

We say that the probability of an event A occuring is

$$P(A) = \frac{\text{Number of elements in } A}{\text{Total number of elements in the sample space}}$$

Example

If a fair coin is tossed, it is clear from our definition of probability above that P (obtaining a head) $= \frac{1}{2}$.

Example

A card is selected at random from a pack of 52 cards. Let $A =$ 'the card is a heart' and $B =$ 'the card is an ace'.
Find $P(A),\ P(B)$.

Solution

$P(A) = \frac{13}{52}$ since there are 13 hearts in the pack. $P(B) = \frac{4}{52}$ since there are 4 aces in the pack.

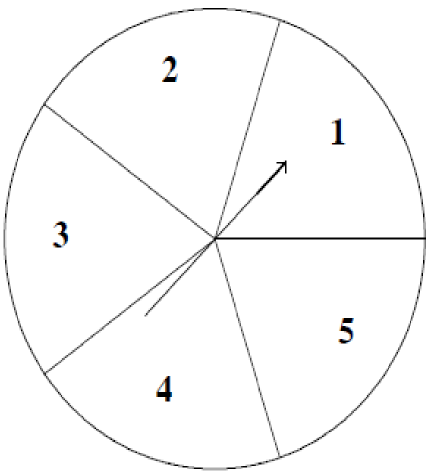
To calculate the probability of an event, we simply need to find out the total number of possible outcomes of an experiment and the number of outcomes which correspond to the **given** event.

Exercise 1

What are your chances of winning a raffle in which 325 tickets have been sold, if you have one ticket?

Exercise 2

A cursor is spun on a disc divided into five equal sectors as shown below. The position of the pointer is noted. (If it is on a line the cursor is spun again.)



Let A be the event 'pointer is in the first sector' and B the event 'pointer is in the 2nd or 4th sector'.
Find $P(A)$, $P(B)$.

Example

Consider the following problem. Two coins are tossed. Let A be the event 'two heads are obtained', and, B be the event 'one head and one tail is obtained'.

Find $P(A),\ P(B)$.

Solution

The sample space={HH, HT, TH, TT}

A={HH}

B={HT, TH}.

Since there are 4 outcomes in the sample space.

$P(A) = \frac{1}{4}$

$P(B) = \frac{2}{4} = \frac{1}{2}$

Notice that HT and TH must be regarded as **different** outcomes.

Often we can conveniently represent the possible outcomes on a diagram and count directly. We will also develop some techniques and rules to assist in our calculations.

Now let us see what happens in reality. Try the following experiment:

Roll a die 50 times and record the number of each of the outcomes 1,2,3,4,5,6.

Continue rolling and record the number of each outcome after 100 rolls. Now record the number after 200 rolls. Find the **relative frequency** of each outcome after 50, 100 and 200 rolls.

For example calculate $\frac{\text{the number of times '1' occurs}}{\text{total number of rolls}}$.

Does it get closer to $\frac{1}{6}$, i.e. 0.17?

Complementary Events

If an event is a **certainty**, then its probability is one.

In common language we often say it is 100% certain (which is the same thing).

For example, in the coin tossing experiment, let C be the event 'obtaining a head or a tail'.

The sample space is {H, T}. The event is {H, T}.

So $P(C) = \frac{2}{2} = 1$.

Example

If a normal die is rolled, what is the probability that the number showing is less than 7?

Solution

Sample space = {1,2,3,4,5,6}

Event = {1,2,3,4,5,6}

Hence the probability (number is less than 7) $= \frac{6}{6} = 1$.

If an event is **impossible**, then its probability is zero.

Example

Find the probability of throwing an 8 on a normal die.

Here there are **no** possible outcomes in the event.

i.e. Sample space = {1,2,3,4,5,6}

Event = {}, i.e. the empty set.

Hence the probability of throwing an 8 is $\frac{0}{6} = 0$.

If the event is neither impossible nor certain, then clearly its **probability is between 0 and 1**.

Two events are **complementary** if they cannot occur at the same time and they make up the whole sample space.

Example

When a coin is tossed, the sample space is $\{H, T\}$ and the events H = 'obtain a head' and T = 'obtain a tail' are complementary.

If we calculate the probabilities we find that

$P(H) = \frac{1}{2}$, $P(T) = \frac{1}{2}$ and $P(H) + P(T) = 1$.

Example

A die is rolled. Let A be the event 'a number less than 3 is obtained' and let B be the event 'a number of 3 or more is obtained'.

Then $P(A) = \frac{2}{6}$, and $P(B) = \frac{4}{6}$.

So that $P(A) + P(B) = 1$.

We have illustrated following law.

If two events are **complementary**, then their **probabilities add up to 1**.

Example
A marble is drawn at random from a bag containing 3 red, 2 blue, 5 green and 1 yellow marble. What is the probability that it is not green?

Solution
There are two ways of doing this problem.

Method A:

We can work out the probability that the marble is green:

$P(G) = \frac{5}{11}$.

Since a marble is either green or not green, the probability that it is not green,

$P(\overline{G}) = 1 - \frac{5}{11} = \frac{6}{11}$.

Method B:

Alternatively, we can find the probability that the marble is red, blue or yellow which is $\frac{6}{11}$.

Example

What is the probability of drawing a heart or spade from a pack of 52 cards when one card is drawn at random?

Solution

$P(\text{heart}) = \frac{13}{52}$

$P(\text{spade}) = \frac{13}{52}$

$P(\text{heart or spade}) = \frac{26}{52}$ since 26 of the cards are either hearts or spades.

Notice $P(\text{heart or spade}) = P(\text{heart}) + P\ (\text{spade})$.

We may now state the **addition law** for **mutually exclusive events**.

If two events A and B are mutually exclusive, the probability of A or B happening, denoted $P(A \cup B)$, is:

$$P(A \cup B) = P(A) + P(B).$$

Conditional Probability

A lecture on a topic of public health is held and 300 people attend. They are classified in the following way:

Gender	Doctors	Nurses	Total
Female	90	90	180
Male	100	20	120
Total	190	110	300

If one person is selected at random, find the following probabilities:

(a) P(a doctor is chosen);

(b) P(a female is chosen);

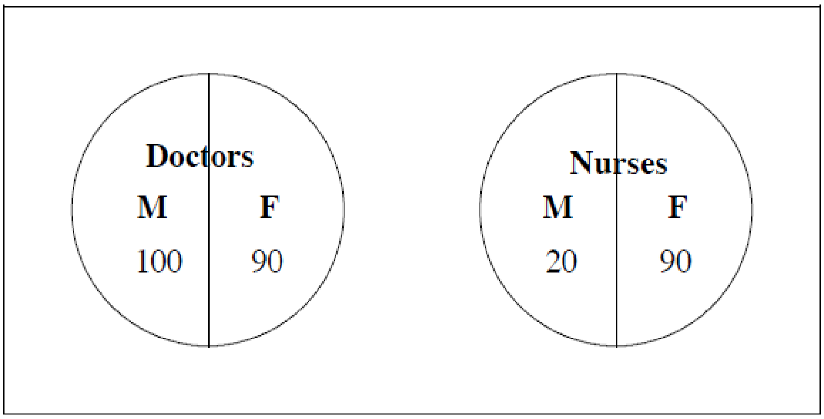
(c) P(a nurse is chosen);

(d) P(a male is chosen);

(e) P(a female nurse is chosen);

(f) P(a male doctor is chosen).

Solution:



(a) The number of doctors is 190 and the total number of people is 300, so $P(\text{doctor}) = \frac{190}{300}$

(b) $P(\text{female}) = \frac{180}{300}$

(c) $P(\text{male}) = \frac{120}{300}$

(d) $P(\text{nurse}) = \frac{110}{300}$

(e) There are 90 female nurses, therefore $P(\text{female} \cap \text{nurse}) = \frac{90}{300}$

(f) $P(\text{male doctor}) = P(\text{male} \cap \text{doctor}) = \frac{100}{300}$.

Now suppose you are given the information that a female is chosen and you wish to find the probability that she is a nurse. This is what we call **conditional probability**. We want the probability that the person chosen is a nurse, subject to the condition that we know she is female. The notation used for this is:

$$P(\text{nurse} \mid \text{female})$$

Read this as 'the probability of the person chosen being a nurse, **given** that she is female'. Since there are 180 females and of these 90 are nurses, the required probability is $\frac{90}{180} = \frac{1}{2}$.

We can see that
$$\begin{aligned} P(\text{nurse} \mid \text{female}) &= \frac{90}{180} \\ &= \frac{90/300}{180/300} \\ &= \frac{P(\text{nurse} \cap \text{female})}{P(\text{female})} \end{aligned}$$

Summary

1. If there are a finite number of equally likely outcomes of an experiment, the probability of an event A is

$$P(A) = \frac{\text{Number of possible outcomes in } A}{\text{Total number of possible outcomes}}$$

2. The probability of an event happening lies between zero and one. If the event cannot happen, its probability is zero and if it is certain to happen, its probability is one.

3. If two events are **complementary**, i.e. they are mutually exclusive (can't happen together) and make up the whole sample space, then their probabilities add up to 1.

4. For two events A and B the probability of A or B (or both) happening is

   $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

In particular if A and B are **mutually exclusive**, $P(A \cup B) = P(A) + P(B)$. That is, the chance that **at least one of them** will happen equals the sum of their probabilities.

5. The **conditional probability** of A given B is $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$, provided that $P(B) \neq 0$.

   So $P(A \cap B) = P(A \mid B).P(B)$.

6. A and B are defined to be independent events if $P(A) = P(A \mid B)$. That is, knowing the outcome of one event does not change the probability of the outcome of the other. From (5.) above we see that in this case $P(A \cap B) = P(A).P(B)$, that is the probability of **both** events happening is the **product** of the individual probabilities.