Discrete Probability Distributions

❖ Random Variable

Random variable is a quantity resulting from an experiment that, by chance, can assume different values.

A random variable, denoted by a capital letter such as X, is a "rule" which assigns a number to each outcome in sample space of a probability experiment. for example:

•Experiment: Tossing a coin three times

•Random variable: the number of heads.

•Experiment: Rolling a dice.

• Random variable: the number appears face up

Types of Random Variables

•**Discrete Random Variable** can assume only certain clearly separated values. It is usually the result of counting something

•**Continuous Random Variable** can assume infinite number of values within a given range. It is usually the result of some type of measurement

Discrete Random Variables - **Examples**

• The number of students in a class.

• The number of children in a family.

• The number of cars entering a carwash in an hour.

Continuous Random Variables - **Examples**

• The distance students travel to class.

• The time it takes an executive to drive to work.

• The length of an afternoon nap.

• The length of time of a particular phone call.

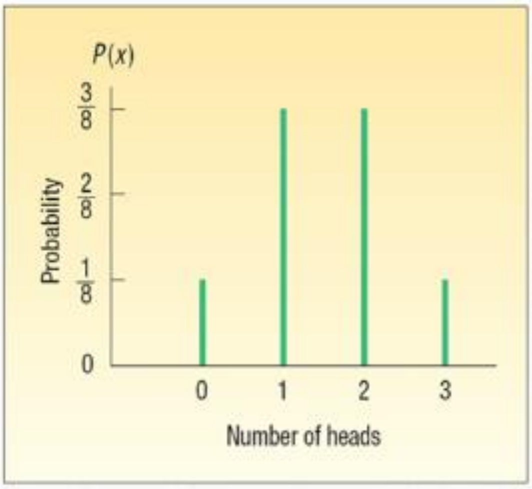
❖ Probability Distribution

PROBABILITY DISTRIBUTION A listing of all the outcomes of an experiment and the probability associated with each outcome.

Experiment: Toss a coin three times. Observe the number of heads. The possible results are: zero heads, one head, two heads, and three heads. What is the probability distribution for the number of heads?

Possible Result	Coin Toss First	Coin Toss Second	Coin Toss Third	Number of Heads
1	T	T	T	0
2	T	T	H	1
3	T	H	T	1
4	T	H	H	2
5	H	T	T	1
6	H	T	H	2
7	H	H	T	2
8	H	H	H	3

Number of Heads, x	Probability of Outcome, $P(x)$
0	$\frac{1}{8}$ = .125
1	$\frac{3}{8}$ = .375
2	$\frac{3}{8}$ = .375
3	$\frac{1}{8}$ = .125
Total	$\frac{8}{8}$ = 1.000



The main **features** of a discrete probability distribution are:

• The sum of the probabilities of the various outcomes is 1.00.

• The probability of a particular outcome is between 0 and 1.00.

• The outcomes are mutually exclusive.

The Mean

• The mean is a typical value used to represent the central location of a probability distribution.

• The mean of a probability distribution is also referred to as its expected value.

MEAN OF A PROBABILITY DISTRIBUTION	$\mu = \Sigma[xP(x)]$

The Variance and Standard Deviation

• Measures the amount of spread in a distribution

• The computational steps are:

1. Subtract the mean from each value, and square this difference.
2. Multiply each squared difference by its probability.
3. Sum the resulting products to arrive at the variance.

• The standard deviation is found by taking the positive square root of the variance.

VARIANCE OF A PROBABILITY DISTRIBUTION	$\sigma^2 = \Sigma[(x - \mu)^2 P(x)]$

- $\sum P(x) = 1, \quad 0 \leq P(x) \leq 1$
- *expected value, mean:* $\mu_X = \sum [x \cdot P(x)]$
- *variance:* $\sigma_X^2 = \sum [(x - \mu)^2 P(x)] = \sum [x^2 \cdot P(x)] - \mu_X^2$
- *standard deviation:* $\sigma_X = \sqrt{\sigma_X^2}$

Example 1:

John Ragsdale sells new cars for Pelican Ford. John usually sells the largest number of cars on Saturday. He has developed the following probability distribution for the number of cars he expects to sell on a particular Saturday.

1. What type of distribution is this?
2. On a typical Saturday, how many cars does John expect to sell?
3. What is the variance and the standard deviation of the distribution?

Number of Cars Sold, x	Probability, $P(x)$
0	.10
1	.20
2	.30
3	.30
4	.10
Total	1.00

1. This is a discrete probability distribution.
2. To get how many cars does John expect to sell on a typical Saturday? We need to calculate the mean (the expected value).

$$\mu = \sum [XP(X)]$$

$$\mu = 0(0.1) + 1(0.2) + 2(0.3) + 3(0.3) + 4(0.1) = 2.1\ cars$$

- To calculate the variance:

$$\sigma^2 = \sum[(X-\mu)^2 P(X)]$$

$$\sigma^2 = [(0-2.1)^2(0.1)] + [(1-2.1)^2(0.2)] + [(2-2.1)^2(0.3)] + [(3-2.1)^2(0.3)] + [(4-2.1)^2(0.1)]$$

$$\sigma^2 = 1.29$$

- To calculate the standard deviation:

$$\sigma = \sqrt{1.29} = 1.136 \text{ cars}$$

Example 2 :

A quiz has 3 multiple choice questions. Each question has 5 choices, and only one of them is correct. What is the probability that a student will guess all the correct choices?

For each question we have 0.2 chance of guessing the correct answer and 0.8 chance of guessing the wrong answer.

Therefore, the probability that a student will guess all the correct choices is:

$$0.2 * 0.2 * 0.2 = 0.2^3 = 0.008$$

Now, if we define a random variable X to be the number of guessing the correct answers.

X can take values 0, 1, 2, and 3.

We answered $P(X = 3)$ or $P(3)$ and it is 0.008

What is $P(0)$?

We can see that $P(0) = 0.8 * 0.8 * 0.8 = 0.8^3 = 0.512$

What is $P(1)$?

So, the probability of guessing one correct answer is

$0.2 * 0.8 * 0.8 = 0.2^1 * 0.8^2 = 0.128$

What is $P(2)$?

So, the probability of guessing two correct answers is

$0.2 * 0.2 * 0.8 = 0.2^2 * 0.8^1 = 0.032$

• Therefore, the probability distribution of X is

X	$P(X)$
0	0.512
1	0.384
2	0.096
3	0.008

$$\mu = ?, \sigma^2 = ?$$

❖ The Binomial Probability Distribution

Probability distribution is binomial if

•n trials, where n is fixed in advance،

•trials have two possible outcomes: success or failure،

•trials independent of one another،

•probability of success same for each trial.

Probability function for binomial is

BINOMIAL PROBABILITY FORMULA $P(x) = {}_nC_x \, \pi^x (1 - \pi)^{n-x}$

where:

C denotes a combination.
n is the number of trials.
x is the random variable defined as the number of successes.
π is the probability of a success on each trial.

with expected value, variance and standard deviation

$$\mu_X = np; \quad \sigma_X^2 = np(1-p); \quad \sigma_X = \sqrt{np(1-p)}.$$

Example 1: number of airplane engine failures.

Each engine of four (n = 4) on an airplane fails 11% (p = 0.11) of the time.

Assume this problem obeys conditions of a binomial experiment.

(a) Chance two engines fail is
$P(2) = {}_4C_2 \times (0.11)^2 \times (1-0.11)^{4-2} \approx$ (circle one) **0.005 / 0.011 / 0.058.**

(StatCrunch: Stat, Calculators, Binomial, n: 4, p: 0.11, Prob(X = 2) (not Prob(X ≤ 2)!) Compute.)

(b) Chance three engines fail is
$P(3) = {}_4C_3 \times (0.11)^3 \times (1-0.11)^{4-3} \approx$ (circle one) **0.005 / 0.011 / 0.040.**

(StatCrunch: Stat, Calculators, Binomial, n: 4, p: 0.11, Prob(X = 3) Compute.)

(c) Chance *at most* two engines fail is
$P(X \le 2) = P(0) + P(1) + P(2) \approx$ (circle one) **0.995 / 0.997 / 0.999.**

(StatCrunch: Stat, Calculators, Binomial, n: 4, p: 0.11, Prob(X ≤ 2) Compute.)

(d) Chance *less than* two engines fail is (circle one)
$P(X < 2) = P(X \le 1) = P(0) + P(1) \approx$ **0.938 / 0.997 / 0.999.**

(StatCrunch: Stat, Calculators, Binomial, n: 4, p: 0.11, Prob(X < 2) Compute.)

(e) *Expected number of failures*
$\mu_X = np = 4(0.11) =$ **0.44 / 0.51 / 0.62.**

(f) *Standard deviation in number of failures*
$\sigma_X = \sqrt{4(0.11)(1-0.11)} \approx$ **0.45 / 0.56 / 0.63.**
So, expect 0.44 "±" 0.63 failures.

Example 2 : number of widget defects.

Each of fourteen randomly chosen widgets are defective 21% of the time.

(a) *Binomial experiment?* Match columns.

binomial conditions	widget example
(a) n trials, n is fixed in advance of experiment.	(A) $p = 0.21$ chance widget is defective.
(b) Trials have possible outcomes: success or failure.	(B) $n = 14$ widgets chosen.
(c) Trials are independent of one another.	(C) each widget is defective or not.
(d) Probability of success same for each trial.	(D) each widget chosen independent of another.

binomial conditions	(a)	(b)	(c)	(D)
widget example				

(b) Chance seven widgets defective

$P(7) = {}_{14}C_7 \times (0.21)^7 \times (1 - 0.21)^{14-7} \approx$ **0.005 / 0.012 / 0.040**.

(StatCrunch: Stat, Calculators, Binomial, n: 14, p: 0.21, Prob(X = 7) Compute.)

(c) Chance *at most* ten widgets defective

$P(X \leq 10) = P(0) + P(1) + \cdots + P(10) \approx$ **0.995 / 0.997 / 0.999**.

(StatCrunch: Stat, Calculators, Binomial, n: 14, p: 0.21, Prob(X ≤ 10) Compute.)

(d) Chance *at least* ten widgets defective

$P(X \geq 10) = P(10) + P(11) + P(12) + P(13) + P(14) = 1 - P(X \leq 9) \approx$
(circle one) **0.000072 / 0.00072 / 0.0072**.

(StatCrunch: Stat, Calculators, Binomial, n: 14, p: 0.21, Prob(X ≥ 10) Compute.)

(e) Chance *between* 7 and 10 widgets defective, *inclusive*

$P(7 \leq X \leq 10) = P(7) + P(8) + P(9) + P(10) = P(X \leq 10) - P(X \leq 6) \approx$
(circle one) **0.005 / 0.015 / 0.034**.

(Stat, Calculators, Binomial, Between, n: 14, p: 0.21, Prob(7 ≤ X ≤ 10) Compute.)

Since $P(7 \leq X \leq 10) \approx 0.015 < 0.05$, it **is / is not**
unusual to have 7 and 10 widgets defective, inclusive.

(f) *Expected number of defectives*

$\mu_X = np = 14(0.21) =$
(circle one) **2.44 / 2.51 / 2.94**.

(g) *Standard deviation in number of defectives*:

$\sigma_X = \sqrt{np(1-p)} \approx$ (circle one) **1.52 / 1.63 / 1.76**.
So, expect 2.94 "±" 1.52 defectives.