Lecture3

Shell model

Advantages of the shell model:

1. The **shell model** was able to explain the **existence of magic numbers** (**2, 4, 20, 28, 50, 82, 126**).
2. The **shell model** was able to provide correct values for the **angular momentum** of the nuclei in their ground state.

Disadvantages of the shell model:

1. The **shell model** failed to predict the **excitation states** of the nucleus.
2. The **shell model** failed to explain the **spherical asymmetry** of manynuclei.

Theory of The Nuclear Shell Model

The **nucleons** inside the nucleus are **linked** in the form of **orbits** similar to theorbits of electrons of the atom, and this has been called the **shell structure** orthe **structure of levels** in which some of the shells are **closed** due to the **stability** of some nucleons.

The scientists **W. Elasser** and **M. Mayer** proposed the first idea of the **closednuclear shell**, where the **total angular momentum of the nucleus** was understood through this model, so that the **total angular momentum** of any nucleus was formed from the **spin of its components**:

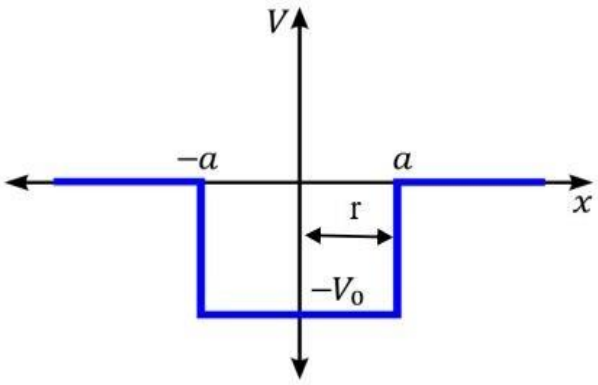
1. The nuclear spin of **protons** equal to $(\frac{1}{2}\hbar)$.
2. The nuclear spin of **neutrons** equal to $(\frac{1}{2}\hbar)$.
3. **Orbital Angular Momentum** of nucleons is due to its motion in the nucleus.

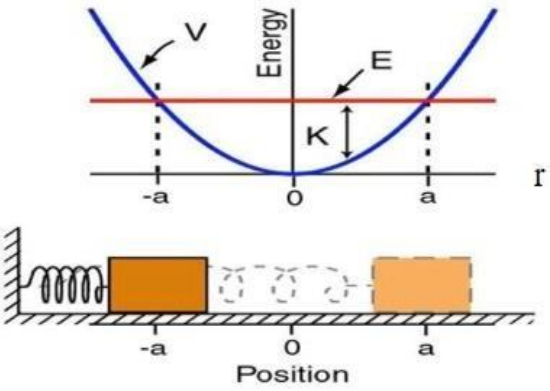
The **structure of the nuclear shell** is not easily reached, due to our lack of knowledge of the **nuclear potential**. So, through the **characteristics** of the **nuclear force**, the **nuclear potential** can be imposed, which **depends on** twobasic **assumptions**:

1. Each **nucleon** moves **freely** and **fluently** in the **field** which is called **potential** which is the diagonal distance from the center of the nucleus.
2. The **energy levels**, or **shells**, are **filled** relation to the Pauli Exclusion Principle.

Based on these two assumptions, many solutions and calculations have been made according to quantum mechanics to develop a **general model** that includes the presence of the shell or nuclear levels, using the **potential models**as:

1. **Square Well Potential** (the nucleus has **spherical** shape).
2. **Harmonic Oscillator Potential**.

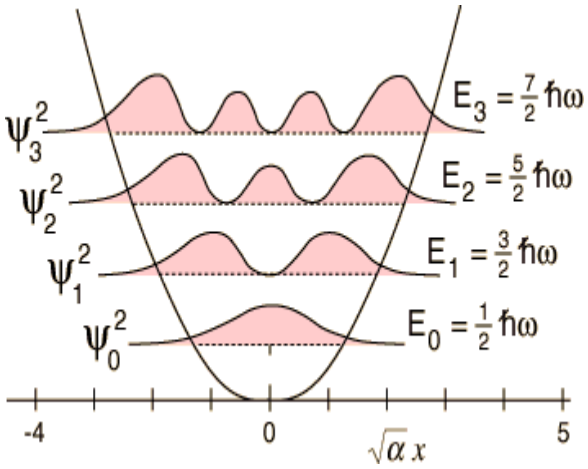


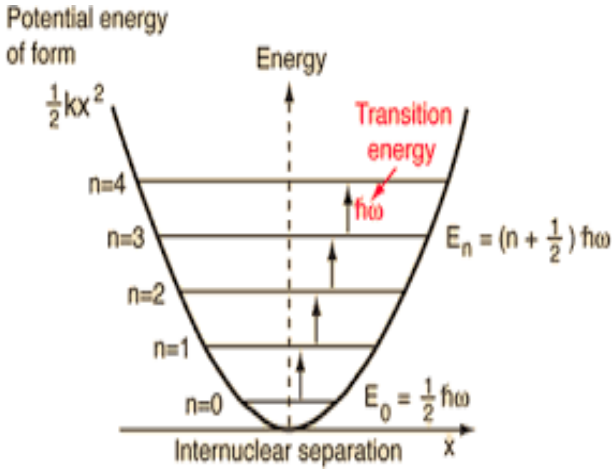


Square Well Potential

$$\begin{cases} x \le r \quad V(x) = -V_0 \\ x > r \quad V(x) = 0 \end{cases}$$ حاجز الجهد

$$\begin{cases} x < r \quad V(x) = 0 \\ x > r \quad V(x) = \infty \end{cases}$$ حاجز غير محدود





For **Harmonic Oscillator Potential**:

$$V(r) = -V_0 + \frac{1}{2}Kr^2 \qquad , K = m_0 w^2, \quad w = \sqrt{\frac{k}{m}}$$

Where w is frequency and m_0 is mass of particle and K is wave number

There is a **model** linking between **Orbital Angular Momentum** (**L**) and **Spin Angular Momentum** (**S**) for each nucleon and it is called **Spin- Orbital Coupling Model**. In this **model** the nucleons move inside the nucleus in the potential field of the nucleus and there is no possibility of collision with eachother according to **Pauli's rule**.

The **Nuclear orbits** of the nucleons are split according to the followingequation:

$$J = l \pm \frac{1}{2}$$

Where: l is the **number of orbitals** (0, 1, 2, 3, 4, 5…), and J is the **angular momentum**.

The **Nuclear orbits** are **filled in** according to the following equation:

$$number\ of\ nucleons = 2J + 1$$

- The **total angular momentum** of any nucleus in the ground state thatcontains its nucleons (**even - even**) is equal to **zero**, meaning that:

$$\sum J_n = 0 \quad , \sum J_p = 0$$

Where: J_n *and* J_P are the total angular momentum of neutrons and protons respectively.

- In a nucleus that contains numbers of **even** neutrons and **odd** protons,the **total angular momentum** with respect to the neutrons is equal to

zero $\sum J_n = 0$, ***while*** the **ground state** of the nucleus spin depends on the spin of the last **single proton**.

- In a nucleus that contains numbers of **even** protons and **odd** neutrons, the **total angular momentum** with respect to the protons is equal to **zero** $\sum JP = 0$, ***while*** the **ground state** with respect to the spin of the nucleus depending on the spin of the last **single neutron**.
- In a nucleus that contains **odd** numbers of protons and neutrons, the ground state of the nucleus spin can be calculated from the spin of the last **neutron** and **proton**.

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