



عنوان المحاضرة:

Polar Coordinates الأحداثيات القطبية

Polar Coordinates

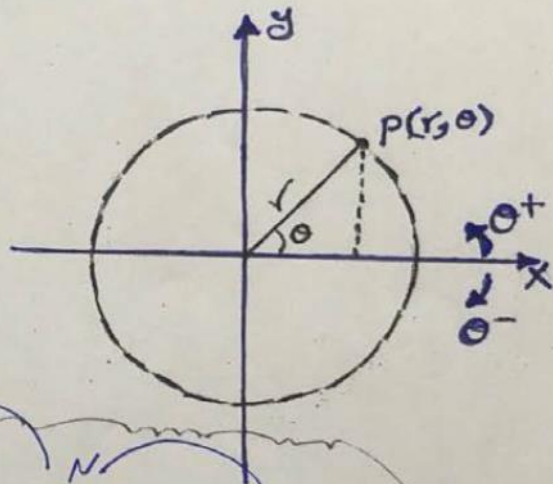
We can define the polar coordinates for each point in the plane as the pair  $(r, \theta)$ , where  $r$  is the distance of point from origin and  $\theta$  is the angle between the line segment from the origin the point and the X-axis.

$$x = r \cos \theta$$

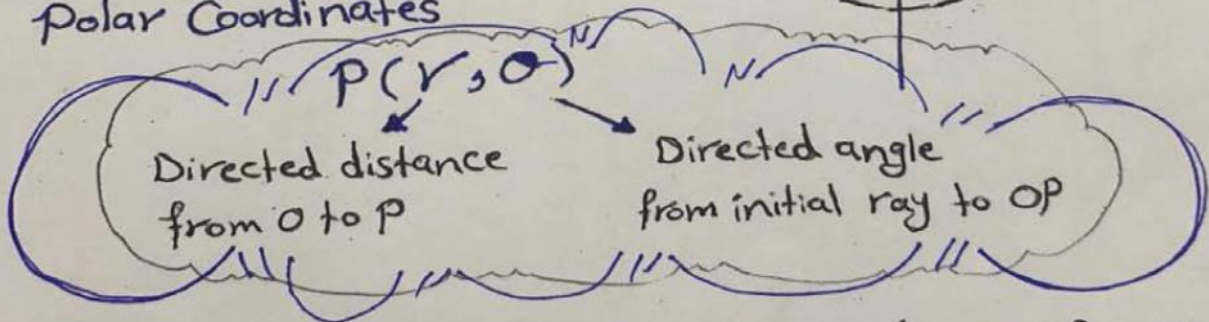
$$y = r \sin \theta$$

$$r = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1} \frac{y}{x}$$



Polar Coordinates



There are many choices for the representation of any point in Polar Coordinates  $P(r, \theta)$

For  $r = r$  then  $\theta = \theta + 2\pi n$  where  $n = 0, \pm 1, \pm 2, \dots$

If  $r = -r$  then  $\theta = (\pi - \theta) + 2\pi n$  where  $n = 0, \pm 1, \pm 2, \dots$

i.e  $P(r, \theta) = P(r, \theta + 2\pi n)$

$P(-r, \theta) = P(-r, (\pi - \theta) + 2\pi n)$



Ex: Find all the polar coordinates of the point  $P(2, \pi/6)$ ?

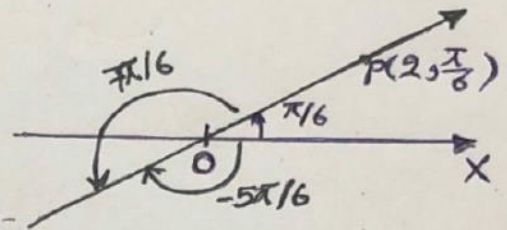
Solution:

For  $r=2$

$$\theta = \theta + 2\pi n \quad n=0, \pm 1, \pm 2, \dots$$

$$n=0 \Rightarrow \theta = \frac{\pi}{6}$$

$$n=1 \Rightarrow \theta = \frac{\pi}{6} + 2\pi = 13\pi/6$$



For  $r=-2$

$$\theta = (\pi - \theta) + 2\pi n \quad n=0, \pm 1, \pm 2, \dots$$

$$n=0 \Rightarrow \theta = \pi - \frac{\pi}{6} = 5\pi/6$$

$$n=1 \Rightarrow \theta = (\pi - \frac{\pi}{6}) + 2\pi = 17\pi/6$$

$$\therefore P(2, \frac{\pi}{6}) = P(2, \frac{13\pi}{6}) = P(-2, \frac{5\pi}{6}) = P(-2, \frac{17\pi}{6})$$

Ex: Convert  $P(-1, -1)$  into polar coordinate.

Solution:

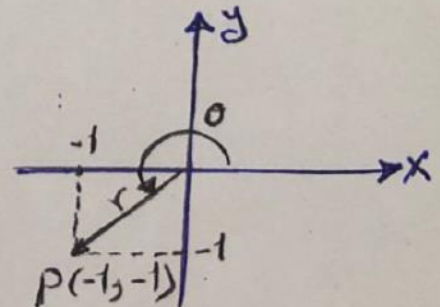
$$r = \sqrt{(-1)^2 + (-1)^2} = \sqrt{2}$$

$$\theta = \tan^{-1} \frac{-1}{-1} \Rightarrow \theta = \pi/4 \quad \text{This is not correct angle}$$

because  $P(-1, -1)$  is in the third quadrant So,

$$\theta = \frac{\pi}{4} + \pi = 5\pi/4$$

$$\therefore P(-1, -1) = P(\sqrt{2}, \frac{5\pi}{4})$$





Ex: - Convert  $2x - 5x^3 = 1 + xy$  into polar coordinates

Solution: -

$$x = r \cos \theta$$

$$y = r \sin \theta$$

$$\theta = \tan^{-1} \frac{y}{x}$$

$$2x - 5x^3 = 1 + xy$$

$$2r \cos \theta - 5(r \cos \theta)^3 = 1 + (r \sin \theta)(r \cos \theta)$$

$$= 2r \cos \theta - 5r^3 \cos^3 \theta = 1 + r^2 \sin \theta \cos \theta$$

Ex: - Convert the polar equation  $4r \cos \theta + r \sin \theta = 8$  into cartesian coordinates equation that express  $y$  in term of  $x$ .

Solution: -

$$x = r \cos \theta \quad \& \quad y = r \sin \theta$$

$$\theta = \tan^{-1} \frac{y}{x}$$

$$4(r \cos \theta) + r \sin \theta = 8$$

$$4x + y = 8$$

$$y = 8 - 4x$$

Ex: - Convert  $r = -8 \cos \theta$  into Cartesian coordinate.

Solution: -

$$x = r \cos \theta \quad \& \quad y = r \sin \theta$$

$$r = \sqrt{x^2 + y^2} \Rightarrow r^2 = x^2 + y^2$$

$$r = -8 \cos \theta \Rightarrow r^2 = -8r \cos \theta$$

$$x^2 + y^2 = -8x$$



اسم المادة : رياضيات-2  
اسم التدريسي : د حسين كاظم حلواص  
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----- نهاية محاضرة " الأحداثيات القطبية Polar Coordinates " -----