ALMUSTAQBAL UNIVERSITY
DEPARTMENT OF BUILDING &
CONSTRUCTION ENGINEERING TECHNOLOGY

ANALYSIS AND DESIGN OF REINFORCED CONCRETE STRUCTURES II

YIELD LINE ANALYSIS OF REINFORCED CONCRETE SLABS

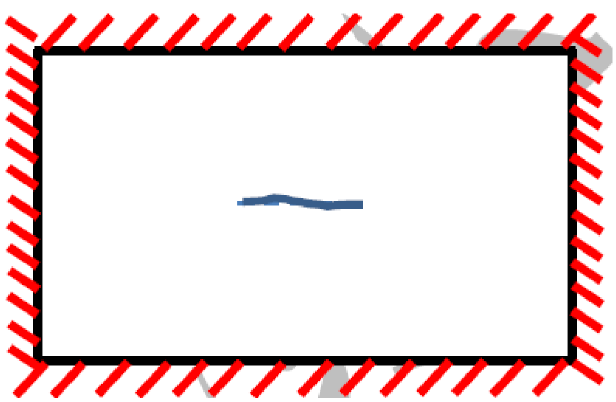
YIELD LINE ANALYSIS OF REINFORCED CONCRETE SLABS

- The **YIELD LINE METHOD** is a method used to analyse slabs, or in other words to determine the moments.
- This method differs from the other methods for analysis that it does not have rules or restrictions as the direct design method has.
- The yield line method which was developed by Johansen in 1940 is a great tool for estimating the required bending resistance and hence, the required reinforcement especially for slabs of non-regular geometry or loading.

WHAT IS A YIELD LINE?

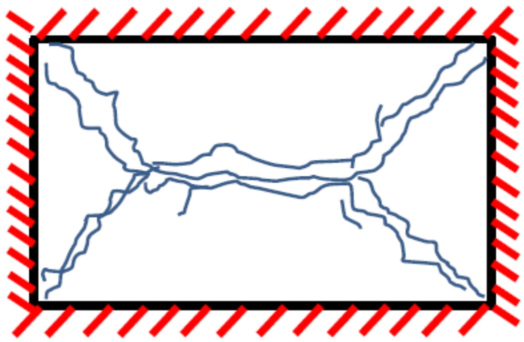
Consider a reinforced concrete slab which supported from all edges and is progressively loaded to failure,

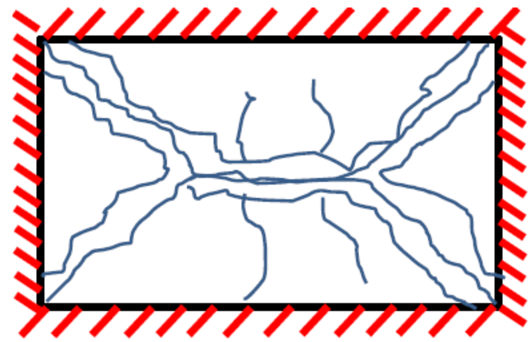
- Prior to cracking, bending moments are distributed according to the linear elastic theory.
- After cracking, the bending moments are redistributed due to the decrease in flexural rigidity of the cracked portions.



- With further loading, the steel reinforcement starts to yield, and the slab undergoes a redistribution of the bending moment.

- As the load slowly increases further, the line where the cracking concentrates (across which the steel reinforcement has yielded) will increase until a collapse mechanism is formed. These lines are called yield lines.



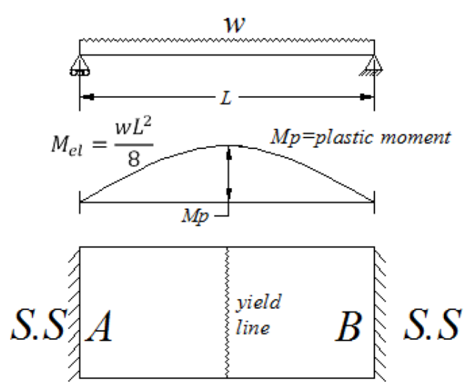


- The yield line distribution is called a yield line pattern.

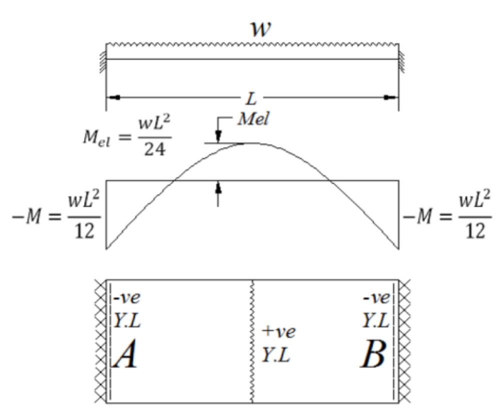
IDEALISATION

Idealisation means how are we going to represent the slabs when using the yield line theory.

- If we have a rectangular slab supported by two brick walls at two sides, the moment distribution will follow the figure below. Where maximum positive moment will be located at mid span. Therefore, the yield line will follow the position of maximum positive moment.



- Otherwise, if we have a rectangular slab that is supported by two concrete walls at two sides, the moment distribution will follow the figure below.



In this case we have two types of moments, positive moment at mis span of the slab, and negative moment at both sides where the slab meets the concrete supports. Therefore, we have three yield lines here, one at mid span where the max. positive moment is and the other two are parallel to the support where maximum negative moment is acting.

- Hence, we use the following notation to draw the yield line.

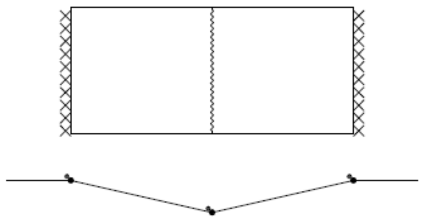
■	Column
//////////////////////	Simply supported
XXXXXXXXXXXXXXXXXXXXXX	Either cautious of fixed end
════════════════	Beam
~~	+ve Y.L [Tension at bottom face]
----------------------------	-ve Y.L [Tension at Top face]
○	Point
— · — · — · — · — · —	Axes of rotation

AXIS OF ROTATION:

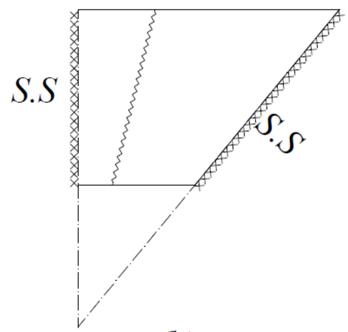
- Axis of rotation are imaginary lines that the cracked portions of the slab rotate on.
- The slab supports will be considered as the rotation axis of the slab's cracked portions.
- Number of cracked portions= number of axis of rotation.
- Column supports have several axes of rotation, the best axis of rotation that is the axis that cuts the biggest number of other axis of rotation.

HOW TO DRAW A YIELD LINE:

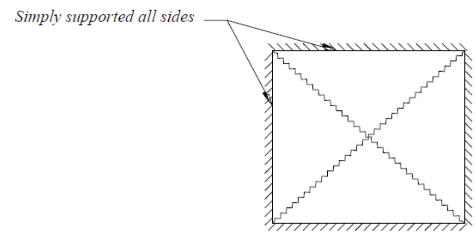
- Draw the axis of rotation.
- Yield lines are drawn as straight lines because they represent the intersection of two planes.
- Yield lines start to form from the intersection of axes of rotation. If there were no intersection points, yield lines are drawn parallel to the axes of rotation.
- After drawing the yield line, cracked portions of the slab are drawn as regular shapes such as triangle, square, or rectangle. Every shape is then defined.
- Look at these examples.



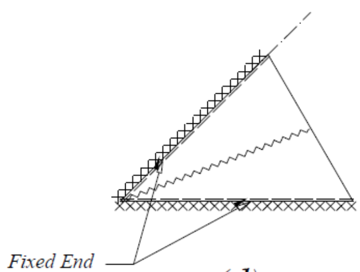
(a)


S.S
S.S

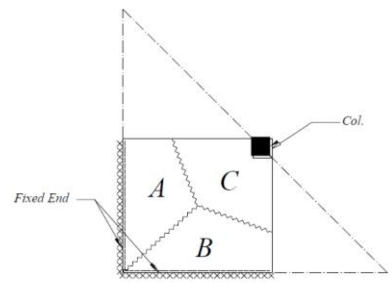
(b)


Simply supported all sides

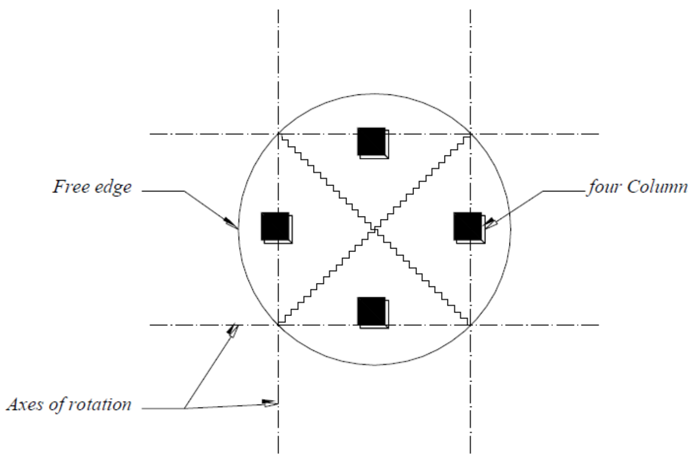
(c)


Fixed End

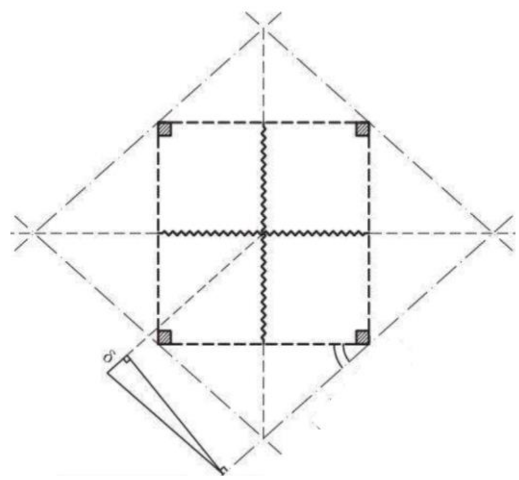
(d)

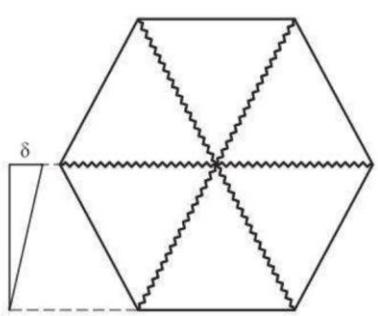

Col.
Fixed End
A
C
B

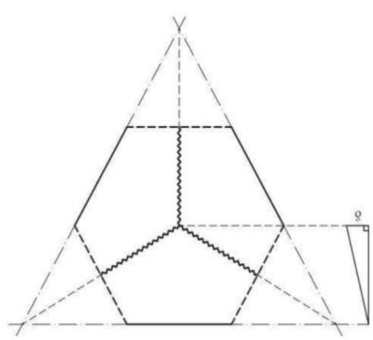
(f)

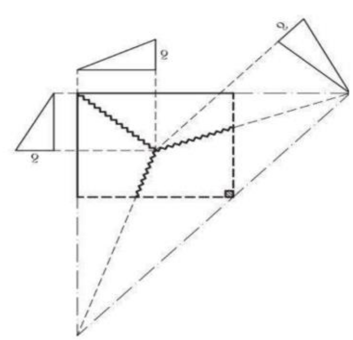

Free edge
four Column
Axes of rotation

(g)




δ





APPROACHES FOR THE YIELD LINE THEORY

1. **ENERGY METHOD:** external work done by the loads during the small virtual displacement of the collapse mechanism is equal to the internal work done by the resisting moments.
2. **EQUILIBRIUM METHOD:** equilibrium of the segment or parts of slab into which the slab is divided by the yield line.

we will use the **ENERGY METHOD** in solving yield line questions, where:

$$W_E = W_I$$

STEPS TO SOLVE YIELD LINE QUESTIONS

- Assume that the maximum deflection at the point where the yield lines intersect (δ) is equal to 1.

$$\delta = 1$$

- External work (**W$_E$**) is calculated depending on the type of loading:
  - **the slab is subjected to a concentrated load.**

$$W_E = P \times \delta$$

  - **the slab is subjected to a distributed load.**

$$W_E = \sum w \times A \times \delta_{max}(maximum\ deflection)$$

    Where:
    W=uniformly distributed load (kN/m^2).
    A= area of shape (m^2)
    δ= if it was a triangle ($\frac{1}{3}\delta_{max}$), if it was a square or rectangle ($\frac{1}{2}\delta_{max}$)

  - **the slab is subjected to a distributed load and a concentrated load.**

$$W_E = \sum w \times area \times \delta_{max}(maximum\ deflection) + P \times \delta(under\ the\ load)$$

- The internal work (**W$_I$**) is calculated without partitions.

$$W_I = \sum m_{+ve} \times \theta \times l_1 + m_{-ve} \times \theta \times l_2$$

  Where:

  m_{+ve}: positive moment (always exists).
  l_1: projection length of the yield line perpendicular to the axis of rotation.
  θ: rotation angle and is calculated from:

$$\theta = \frac{\delta}{y}$$

  m_{-ve}: negative moment and it exists wherever a fixed support is used.
  l_2: true length of the fixed support.

- Then we equate the external work to the internal work to find what is required in the question.