ALMUSTAQBAL UNIVERSITY

DEPARTMENT OF BUILDING & CONSTRUCTION ENGINEERING TECHNOLOGY

ANALYSIS AND DESIGN OF REINFORCED CONCRETE STRUCTURES II

YIELD LINE THEORY SOLVED EXAMPLES

I

EXAMPLE ONE: by using the yield line theory, determine the moment for a reinforced concrete one-way slab with a length of 6m and width of 2m and is supported by two brick walls at its width subjected to a distributed load of (w).

SOLUTION:

$W_E = W_I$

$W_E = w \times A \times \delta$

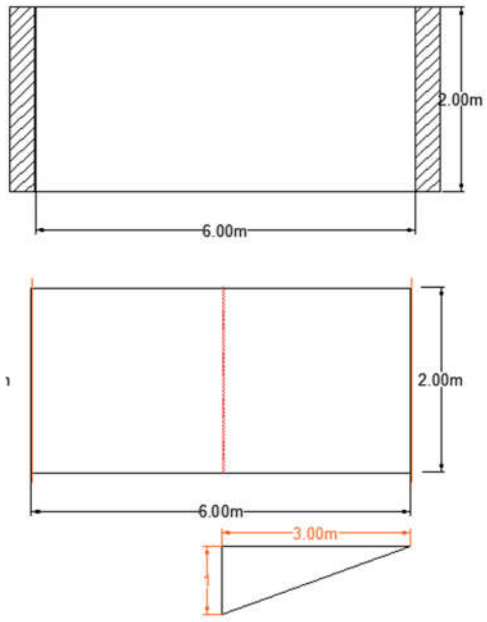
$= w \times (3 \times 2) \times \frac{1}{2} \times 2 = 6w.$

$W_I = m \times l \times \theta$

$= m \times 2 \times \frac{1}{3} \times 2 = \frac{4m}{3}$

$W_E = W_I$

$6w = \frac{4m}{3} \rightarrow \therefore m = \frac{18w}{4}$



EXAMPLE TWO: by using the yield line theory, determine the moment for a reinforced concrete one-way slab with a length of 6m and width of 2m and is supported by two concrete walls at its width subjected to a distributed load of (w).

SOLUTION:

$W_E = w \times A \times \delta$

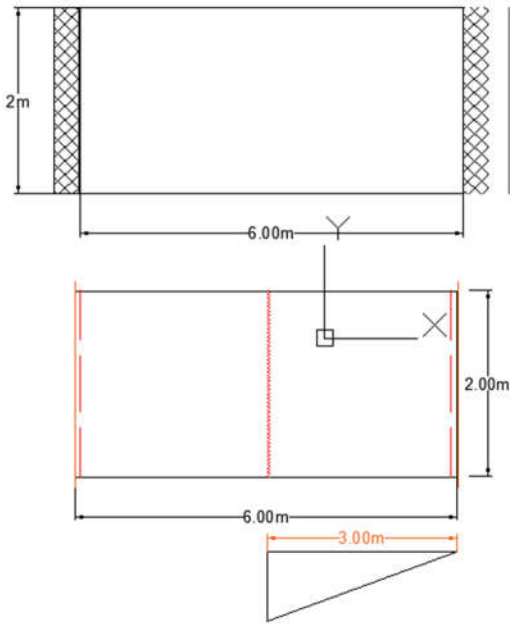
$= w \times (3 \times 2) \times \frac{1}{2} \times 2 = 6w.$

$W_I = m \times l \times \theta = (m \times 2 \times \frac{1}{3} + m \times 2 \times \frac{1}{3}) \times 2$

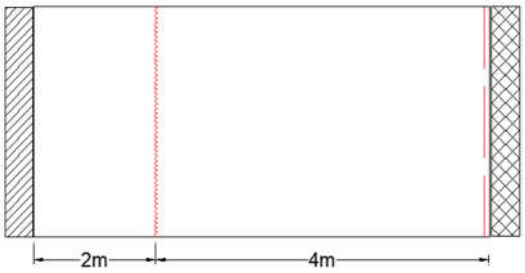
$= \frac{8}{3} m.$

$W_E = W_I$

$6w = \frac{8}{3} m \rightarrow \quad m = \frac{18}{8} w$



EXAMPLE THREE: by using the yield line theory, determine the moment for a reinforced concrete one-way slab with a length of 6m and width of 2m and is supported by one concrete wall and one brick wall at its width subjected to a distributed load of (w).



SOLUTION:

$$W_E = w \times (2 \times 2) \times \frac{1}{2} + w \times (4 \times 2) \times \frac{1}{2}$$

$$= 6w.$$

$$W_I = \left(m^{+ve} \times 2 \times \frac{1}{2} + m^{+ve} \times 2 \times \frac{1}{4} + m^{-ve} \times 2 \times \frac{1}{4}\right)$$

$(m^{+ve} = m^{-ve})$

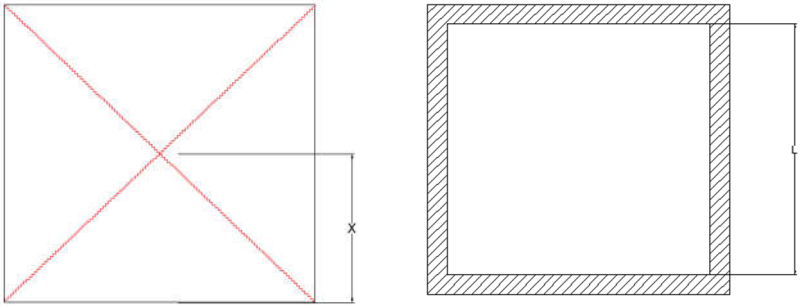
$$= 2m.$$

$$W_E = W_I$$

$$6w = 2m$$

$$\therefore m = 3w.$$

EXAMPLE 4: Using the yield line theory, determine the ultimate resisting moment per linear meter (m) for the isotropic simply supported reinforced concrete two-way square slab subjected to a uniformly distributed load (w).



SOLUTION:

$$W_E = w \times \frac{1}{2} \times l \times \frac{l}{2} \times \frac{1}{3} \times 4 = \frac{wl^2}{3}$$

$$W_I = m \times l \times \frac{1}{\frac{l}{2}} \times 4 = 8m$$

$$W_E = W_I$$

$$\frac{wl^2}{3} = 8m$$

$$m = \frac{wl^2}{24}$$

EXAMPLE 5: Using the yield line theory, determine the ultimate resisting moment per linear meter (m) for the isotropic simply supported reinforced concrete two-way square slab subjected to a concentrated load (P) at the centre of the slab.

SOLUTION:

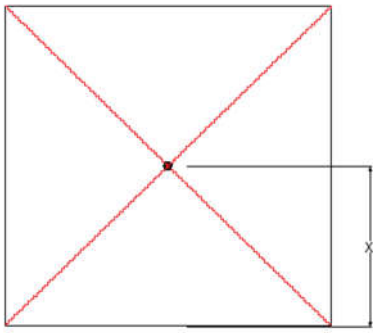
$$W_E = P \times 1 = P$$

$$W_I = m \times l \times \frac{1}{\frac{l}{2}} \times 4 = 8m.$$

$$W_E = W_I$$

$$P = 8m$$

$$\therefore m = \frac{P}{8}$$

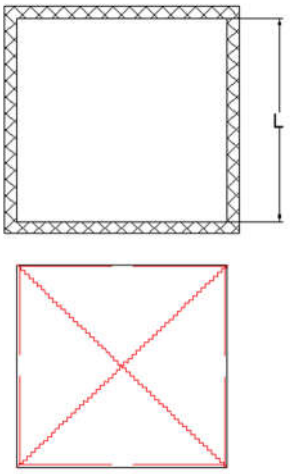


EXAMPLE 6: Using the yield line theory, determine the ultimate resisting moment per linear meter (m) for the isotropic two-way square slab supported by four concrete walls subjected to a uniformly distributed load (w).

SOLUTION:

$$W_E = w \times \frac{1}{2} \times l \times \frac{l}{2} \times \frac{1}{3} \times 4 = \frac{wl^2}{3}$$

$$W_I = (m^{+ve} \times l \times \frac{1}{\frac{l}{2}} + m^{-ve} \times l \times \frac{1}{\frac{l}{2}}) \times 4 = 16m.$$



$$W_E = W_I$$

$$\frac{wl^2}{3} = 16m$$

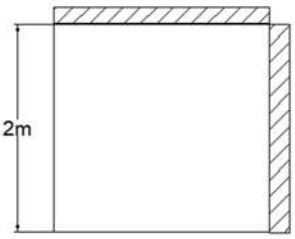
$$\therefore m = \frac{wl^2}{48}$$

EXAMPLE 7: using the yield line theory, determine the moment (m) for the isotropic reinforced concrete two-way slab shown. The slab is subjected to a uniformly distributed load (w).

SOLUTION:

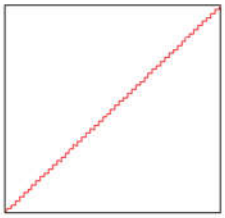
$$W_E = w \times 2 \times 2 \times \frac{1}{2} \times \frac{1}{3} \times 2 = \frac{4}{3}w.$$

$$W_I = \left(m \times 2 \times \frac{1}{2}\right) \times 2 = 2m.$$



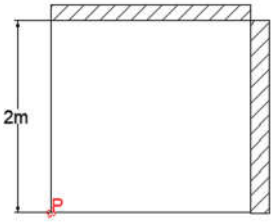
$$W_E = W_I$$

$$\frac{4}{3}w = 2m \rightarrow \therefore m = \frac{4}{6}w.$$



EXAMPLE 8: using the yield line theory, determine the moment (m) for the isotropic reinforced concrete two-way slab shown. The slab is subjected to a concentrated load at the corner (P).

SOLUTION:

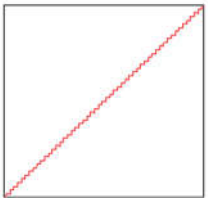


$$W_E = P \times 1 = P$$

$$W_I = \left(m \times 2 \times \frac{1}{2}\right) \times 2 = 2m.$$

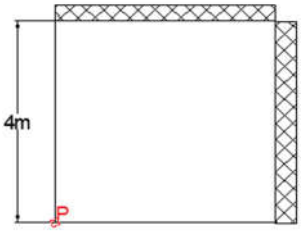
$$W_E = W_I$$

$$\therefore m = \frac{P}{2}$$



EXAMPLE 9: using the yield line theory, determine the moment (m) for the isotropic reinforced concrete two-way slab shown. The slab is subjected to a uniformly distributed load (w).

SOLUTION:



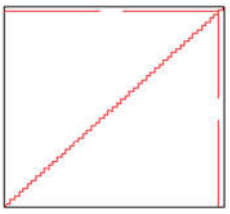
$$W_E = \left(w \times 4 \times 4 \times \frac{1}{2} \times \frac{1}{3}\right) \times 2 = \frac{16}{3}w.$$

$$W_I = \left(m \times 4 \times \frac{1}{4} + m \times 4 \times \frac{1}{4}\right) \times 2 = 4m.$$

$$W_E = W_I$$

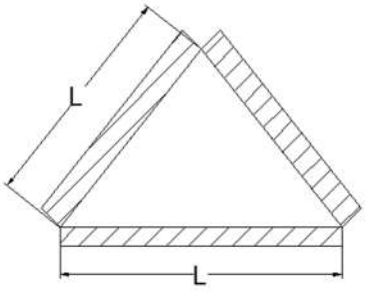
$$\frac{16}{3}w = 4m$$

$$\therefore m = \frac{4}{3}w.$$



EXAMPLE 10: using the yield line theory, determine the ultimate resisting moment per linear meter (m) for an isotropic reinforced concrete slab simply supported at all edges and subjected to uniformly distributed load (w). (assume $x = \frac{l}{3}$).

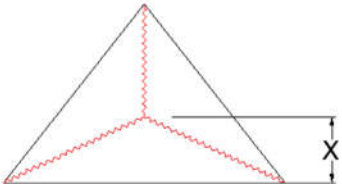
SOLUTION:



$$W_E = \left(w \times \frac{1}{2} \times l \times \frac{l}{3} \times \frac{1}{3}\right) \times 3 = \frac{wl^2}{6}$$

$$W_I = \left(m \times l \times \frac{1}{\frac{l}{3}}\right) \times 3 = 9m.$$

$$W_E = W_I \rightarrow \frac{wl^2}{6} = 9m \rightarrow \therefore m = \frac{wl^2}{54}$$



EXAMPLE 11: using the yield line theory, determine the ultimate resisting moment per linear meter (m) for an isotropic reinforced concrete slab simply supported at all edges and subjected to uniformly distributed load (w).

SOLUTION:

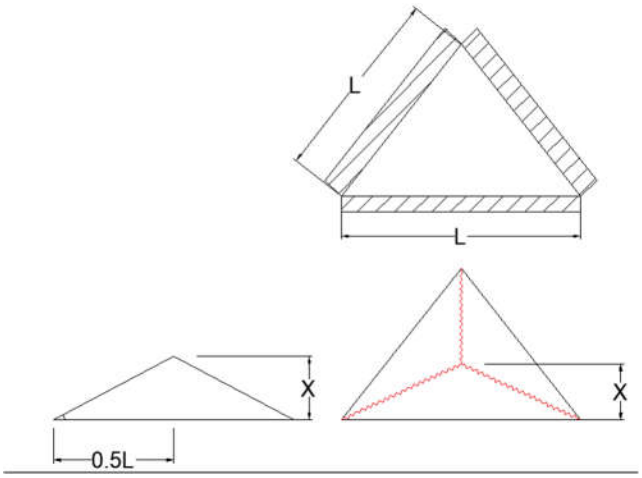
$$W_E = w\left(l \times 0.2886l \times \frac{1}{2} \times \frac{1}{3}\right) \times 3 = 0.1443wl^2$$

$$W_I = \left(m \times \frac{1}{0.2886l}\right) \times 3 = 10.39m.$$

$$W_E = W_I$$

$$0.1443wl^2 = 10.39m$$

$$\therefore m = 0.013wl^2$$



EXAMPLE 12: using the yield line theory, determine the moment (m) for the isotropic reinforced concrete slab subjected to a concentrated load (P).

SOLUTION:

$$W_E = P \times 1 = P$$

$$W_I = m \times 2 \times \frac{1}{2} \times 2 + m \times 4 \times \frac{1}{2} = 4m.$$

$$W_E = W_I$$

$$\therefore m = \frac{P}{4}$$