Inverse laplace transform:

$f(t) = L^{-1}\{F(s)\}$

Theorm1: $L^{-1}\{C_1 F_1(s) + C_2 F(s)\}$

$= C_1 L^{-1}\{F_1(s)\} + C_2 L^{-1}\{F_2(s)\}$

$= C_1 f_1(t) + C_2 f_2(t)$

Theorm2: $L^{-1}\{F(s-a)\} = e^{at} \cdot f(t)$

Theorm 3: $L^{-1}\left\{\int_s^{\infty} F(u)\,du\right\} = \frac{f(t)}{t}$

Theorm 4: $L^{-1}\left\{\frac{F(s)}{s}\right\} = \int_0^t f(u)\,du$

EX-1/ Find $L^{-1}\left\{\frac{1}{s^2+4}\right\}$

sol:

$\frac{1}{2} L^{-1}\left\{\frac{2}{s^2+4}\right\}$

$\therefore \frac{1}{2}\sin 2t$

Ex.2/ Find $L^{-1}\left\{\frac{1}{s^2-2s+5}\right\}$

Sol:

$$L^{-1}\left\{\frac{1}{s^2-2s+1+4}\right\} \Rightarrow = L^{-1}\left\{\frac{1}{(s-1)^2+4}\right\}$$

$$\therefore f(t) = \frac{1}{2} e^{t} \sin 2t$$

Ex.3/ Find $L^{-1}\left\{\frac{4}{(s-1)^4}\right\}$

Sol:

$$f(t) = \frac{4}{3!} e^{t} t^3$$

Ex.4/ Find $L^{-1}\left\{\frac{s+1}{s^2+2s+10}\right\}$

$$= L^{-1}\left\{\frac{s+1}{(s+1)^2+9}\right\} \rightarrow = e^{-t}\cos 3t$$

Ex.5/ Find $L^{-1}\left\{\frac{s-4}{s^2+2s-10}\right\}$

sol:

$$= L^{-1}\left\{\frac{s-4+1-1}{(s+1)^2+9}\right\} \rightarrow = L^{-1}\left\{\frac{s+1}{(s+1)^2+9} - \frac{5}{(s+1)^2+9}\right\}$$

$$= e^{-t}\cos 3t - \frac{5}{3} e^{-t}\sin 3t$$

Ex.6/ Find $L^{-1}\left\{\frac{1}{s^2+4s+8}\right\}$

$$= L^{-1}\left\{\frac{1}{(s+2)^2+4}\right\} \rightarrow = e^{-2t} L^{-1}\left\{\frac{1}{s^2+4}\right\}$$

$$f(t) = \frac{1}{2} e^{-2t} \sin 2t$$

Ex 7/ Find $L^{-1}\left\{\frac{2s+16}{s^2+4s+13}\right\}$

Sol:

$$=2L^{-1}\left\{\frac{s+8}{s^2+4s+4+9}\right\} \rightarrow =2L^{-1}\left\{\frac{s+8}{(s+2)^2+9}\right\}$$

$$=2L^{-1}\left\{\frac{s+8+2-2}{(s+2)^2+9}\right\}$$

$$=2L^{-1}\left\{\frac{s+2}{(s+2)^2+9}\right\}+2L^{-1}\left\{\frac{6}{(s+2)^2+9}\right\}$$

$$=2e^{-2t}\cos 3t+4e^{-2t}\sin 3t$$

Note: في حالة وجود exp. نوخر منشة exp. بعكس الإشارة

Ex: Find $L^{-1}\left\{\frac{e^{-2s}}{s^2+4}\right\}$

sol:

$$\frac{1}{2}L^{-1}\left\{\frac{2e^{-2s}}{s^2+4}\right\}$$

$$=\frac{1}{2}u(t-2)\,L^{-1}\left\{\frac{2}{s^2+4}\right\}$$

$$=\frac{1}{2}u(t-2)\sin 2t$$

Ex. Find $L^{-1}\left\{\frac{e^{-3s}}{s^2-9}\right\}$

Sol:

$$\frac{1}{3} L^{-1}\left\{\frac{3e^{-3s}}{s^2-9}\right\}$$

$$= \frac{1}{3} u(t-3) \sinh 3t$$

Ex. Find $L^{-1}\left\{\frac{e^{-s}}{(s-3)^3}\right\}$

$$= u(t-1) L^{-1}\left\{\frac{1}{(s-3)^3}\right\}$$

$$= u(t-1) e^{+3t}\left\{\frac{1}{s^3}\right\}$$

$$= \frac{1}{2} u(t-1) e^{3t} t^2$$