Karnaugh Map (K-Map)

The second method that used to simplify the Boolean function is the Karnaugh map. K-map basically deals with the technique of inserting the values of the output variable in cells within a rectangle or square grid according to a definite pattern. The number of cells in the K-map is determined by the number of input variables and is mathematically expressed as two raised to the power of the number of input variables, i.e., 2^n, where the number of input variables is n.

Thus, to simplify a logical expression with **two inputs**, we require a Kmap with ($2^2 = 4$) cells. A **four-input** logical expression would lead to a ($2^4 = 16$) celled-K-map, and so on.

K-mapping & Minimization Steps

Step 1: generate K-map based on the number of input variables n

- Put a 1 in all specified minterms
- Put a 0 in all other boxes (optional)

Step 2: group all adjacent 1s without including any 0s. All groups must be rectangular and contain a "power-of-2" number of 1s 1, 2, 4, 8, 16, 32, …

Step 3: define product terms using variables common to all minterms in group

Step 4: sum all essential groups plus a minimal set of remaining groups to obtain a minimum SOP.

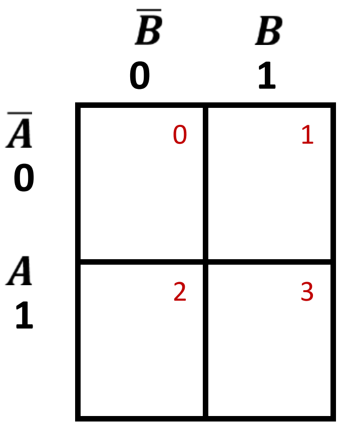
1- Two variables K-Map

Number of input variables are 2

Hence the number of squares = $2^n = 2^2 = 4$

Inputs A B	Decimal equivalent	Minterms		Output F
0 0	0	m_0	$\bar{A}\bar{B}$	
0 1	1	m_1	$\bar{A}B$	
1 0	2	m_2	$A\bar{B}$	
1 1	3	m_3	AB	

And K-Map of two variables is:



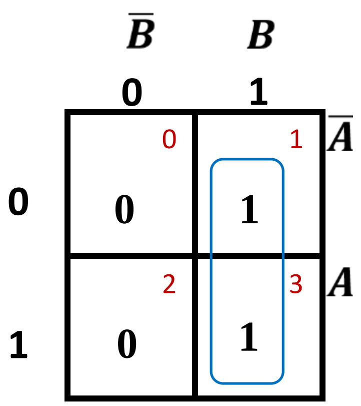
Example: simplify the Boolean expression by using K-Map

$$F = \bar{A}B + AB$$

Solution:

Number of input variables are 2

Hence the number of squares = $2^n = 2^2 = 4$



$$F = B$$

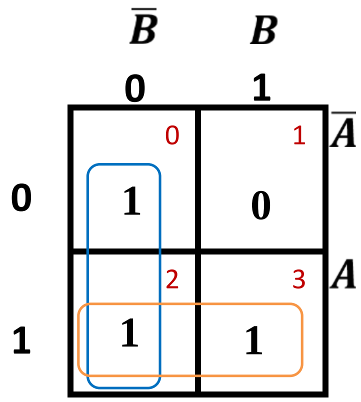
Example: simplify the Boolean expression by using K-Map

$$F(A, B) = \sum m(2, 0, 3)$$

Solution:

Number of input variables are 2

Hence the number of squares = $2^n = 2^2 = 4$



$$F(A, B) = \overline{B} + A$$

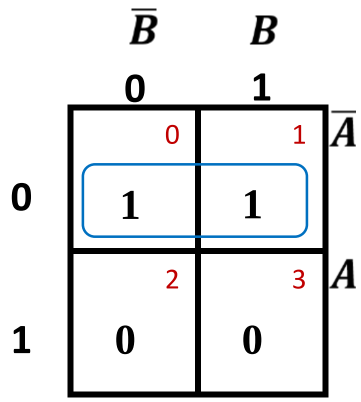
Example: simplify the Boolean expression by using K-Map

$$F = \overline{A}B + \overline{A}\overline{B}$$

Solution:

Number of input variables are 2

Hence the number of squares = $2^n = 2^2 = 4$



$$F = \overline{A}$$

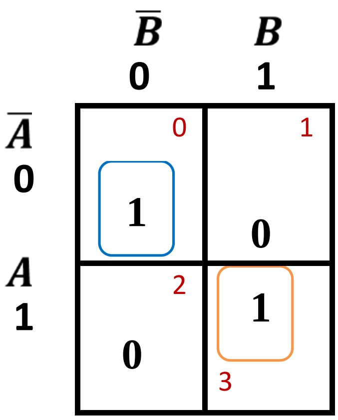
Example: simplify the Boolean expression by using K-Map

$$\boldsymbol{F(A,B)=\sum m(0,3)}$$

Solution:

Number of input variables are 2

Hence the number of squares = $2^n = 2^2 = 4$



$$\boldsymbol{F(A,B)=\sum m(0,3)=\bar{A}\bar{B}+AB}$$

2- Three Variables K-Map

Number of input variables are 3

Hence the number of squares = $2^n = 2^3 = 8$

The truth table is

Inputs A B C	Decimal equivalent	Minterms		Output F
0 0 0	0	m_0	$\bar{A}\bar{B}\bar{C}$	
0 0 1	1	m_1	$\bar{A}\bar{B}C$	
0 1 0	2	m_2	$\bar{A}B\bar{C}$	
0 1 1	3	m_3	$\bar{A}BC$	
1 0 0	4	m_4	$A\bar{B}\bar{C}$	
1 0 1	5	m_5	$A\bar{B}C$	
1 1 0	6	m_6	$AB\bar{C}$	
1 1 1	7	m_7	ABC	

And the K-Map of three variables is:

	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0	0	1	3	2
A 1	4	5	7	6

Example: simplify the Boolean expression by using K-Map

$$F(A,B,C) = \bar{A}\bar{B}\bar{C} + \bar{A}BC + \bar{A}B\bar{C}$$

Solution:

Number of input variables are 3

Hence the number of squares = $2^n = 2^3 = 8$

	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10	
0	(0) 1	(1) 0	(3) 1	(2) 1	$\bar{A}$
1	(4) 0	(5) 0	(7) 0	(6) 0	A

$$F(A,B,C) = \bar{A}\bar{C} + \bar{A}B$$

Example: simplify the Boolean expression by using K-Map

$$F(A,B,C) = \sum m(0,3,7,6)$$

Solution:

Number of input variables are 3

Hence the number of squares = $2^n = 2^3 = 8$

		$\overline{B}\,\overline{C}$ 00	$\overline{B}C$ 01	BC 11	$B\overline{C}$ 10
$\overline{A}$	0	0: $\boxed{1}$	1: 0	3: 1	2: 0
A	1	4: 0	5: 0	7: 1	6: 1

$$F(A,B,C) = \overline{A}\,\overline{B}\,\overline{C} + BC + AB$$

3- Four Variables K-map

Number of input variables are 4

Hence the number of squares = $2^n = 2^4 = 16$

The truth table is

Inputs A	B	C	D	Decimal equivalent	Minterms		Output F
0	0	0	0	0	m_0	$\overline{A}\,\overline{B}\,\overline{C}\,\overline{D}$	
0	0	0	1	1	m_1	$\overline{A}\,\overline{B}\,\overline{C}D$	
0	0	1	0	2	m_2	$\overline{A}\,\overline{B}C\overline{D}$	
0	0	1	1	3	m_3	$\overline{A}\,\overline{B}CD$	
0	1	0	0	4	m_4	$\overline{A}B\overline{C}\,\overline{D}$	
0	1	0	1	5	m_5	$\overline{A}B\overline{C}D$	
0	1	1	0	6	m_6	$\overline{A}BC\overline{D}$	
0	1	1	1	7	m_7	$\overline{A}BCD$	
1	0	0	0	8	m_8	$A\overline{B}\,\overline{C}\,\overline{D}$	
1	0	0	1	9	m_9	$A\overline{B}\,\overline{C}D$	
1	0	1	0	10	m_{10}	$A\overline{B}C\overline{D}$	
1	0	1	1	11	m_{11}	$A\overline{B}CD$	
1	1	0	0	12	m_{12}	$AB\overline{C}\,\overline{D}$	
1	1	0	1	13	m_{13}	$AB\overline{C}D$	
1	1	1	0	14	m_{14}	$ABC\overline{D}$	
1	1	1	1	15	m_{15}	$ABCD$	

And the K-Map of four variables is:

	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	0	1	3	2
$\bar{A}B$ 01	4	5	7	6
AB 11	12	13	15	14
$A\bar{B}$ 10	8	9	11	10

Example: simplify the Boolean expression by using K-Map

$$F(A,B,C,D) = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}C\bar{D} + A\bar{B}\bar{C}\bar{D} + A\bar{B}C\bar{D} + \bar{A}BCD + AB\bar{C}D$$

Solution: Number of input variables are 4

Hence the number of squares = $2^n = 2^4 = 16$

		$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
00	$\bar{A}\bar{B}$	1 (0)	0	0 (3)	1 (2)
01	$\bar{A}B$	0 (4)	0 (5)	$\boxed{1}$ (7)	0 (6)
11	AB	0 (12)	$\boxed{1}$ (13)	0 (15)	0 (14)
10	$A\bar{B}$	1 (8)	0 (9)	0 (11)	1 (10)

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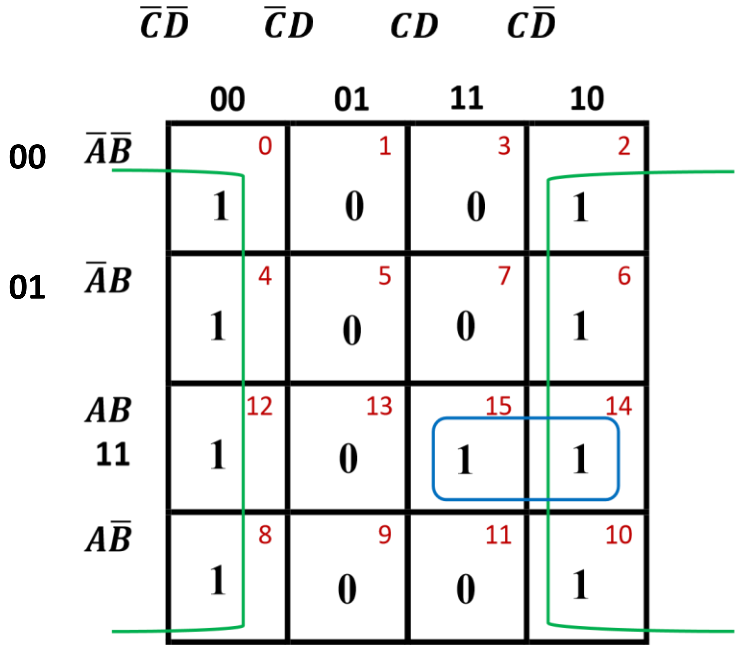
$$F(A,B,C,D) = \bar{B}\bar{D} + \bar{A}BCD + AB\bar{C}D$$

Example: simplify the Boolean expression by using K-Map

$$F(A,B,C,D) = \sum m(0,2,4,6,12,14,15,8,10)$$

Solution: Number of input variables are 4

Hence the number of squares = $2^n = 2^4 = 16$



$$F(A,B,C,D) = \bar{D} + ABC$$