3.3 Runge-Kutta Method

تعتبر هذه الطريقة أكثر دقة من الطرق السابقة وتوجد لها عدة صيغ من أشهرها الصيغة ذات المرتبة الرابعة

والتي يكون القانون العام :

$$y_{i+1} = y_i + \frac{1}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right)$$

where:

$$k_1 = h \cdot f(x_i\ , y_i)$$

$$k_2 = h.f\left(xi + \frac{h}{2}, yi + \frac{k_1}{2}\right)$$

$$k_3 = h.f\left(xi + \frac{h}{2}, yi + \frac{k_2}{2}\right)$$

$$k_4 = h.f\left(xi + h, yi + k_3\right)$$

$$f(x,y) = \dot{y}$$

Example (3): Find the solution of differential equation by using Runge-Kutta Method at $x = 0.1$, $\frac{dy}{dx} = \dot{y} = x + y$, use $y(0) = 1$, $h = 0.1$.

Solve:

$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

i	x_i	y_i (Num.)
0	0	1
1	0.1	1.11034

$For\ i = 0$: $x_o = 0$, $y_o = 1$, $h = 0.1$

$$y_1 = y_o + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$f(x,y) = \dot{y} = x + y$$

$$k_1 = h \cdot f(x_o\ , y_o) = h \cdot f(0\ , 1) = 0.1 \times (0 + 1) = 0.1$$

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$$k_2 = h.f\left(x_0+\frac{h}{2}, y_0+\frac{k_1}{2}\right) = k_2 = h.f\left(0+\frac{0.1}{2}, 1+\frac{0.1}{2}\right)$$

$$=0.1*(0.05+1.05)=0.11$$

$$k_3 = h.f\left(x_0+\frac{h}{2}, y_0+\frac{k_2}{2}\right) = k_3 = h.f\left(0+\frac{0.1}{2}, 1+\frac{0.11}{2}\right)$$

$$=0.1*(0.05+1.055)=0.1105$$

$$k_4 = h.f\left(x_0+h, y_0+k_3\right) = k_4 = h.f\left(0+0.1, 1+0.1105\right)$$

$$=0.1*(0.1+1.155)=0.12105$$

$$y_1 = y_1+\frac{1}{6}\left(k_1+2k_2+2k_3+k_4\right)$$

$$y_1 = 1+\frac{1}{6}\left(0.1+2*0.11+2*0.1105+0.12105\right)$$

$$for\ i=1: x_1 = x_0+h = 0.1,\ y_1 = 1.11034$$

Example (4): By using Runge-Kutta Method, solve the differential equation at $x = 1.2$, $\dot{y} = x\,e^{(y-x^2)}$, use $y(1) = 2$, $h = 0.2$. Answer up to 3 decimal.

Solve:

i	x_i	y_i
0	1	2
1	1.2	2.661

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$$y_{i+1} = y_i + \frac{1}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right)$$

For i=0, x0=1 ,y0=2 , h=0.2

$$y_1 = y_1 + \frac{1}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right)$$

$$k_2 = h.f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right) = k_2 = h.f\left(0 + \frac{0.2}{2}, 2 + \frac{0.0.544}{2}\right)$$

$$= k_2 = 0.2 * 1.1 * e^{\left(2.272 - 1.1^2\right)} = 0.636$$

$$k_3 = h.f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right) = k_3 = h.f\left(1 + \frac{0.2}{2}, 2 + \frac{0.544}{2}\right)$$

$$k_2 = 0.2 * 1.1 * e^{\left(2.318 - 1.1^2\right)} = 0.666$$

$$k_4 = h.f\left(x_0 + h, y_0 + k_3\right) = k_4 = h.f\left(1 + 0.2, 2 + 0.666\right)$$

$$k_2 = 0.2 * 1.2 * e^{\left(2.666 - 1.2^2\right)} = 0.818$$

$$y_1 = 2 + \frac{1}{6}\left(0.544 + 2 * 0.636 + 2 * 0.666 + 0.818\right)$$

$$for\ i = 1 : x_1 = x_0 + h = 1.2,\ y_1 = 2.661$$

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Problem:

1. By using Runge-Kutta Method, solve the differential equation at $x = 0.8$, $\dot{y} = 1.3e^{-x} - 2y$, use $y(0) = 5$, $h = 0.4$. Answer up to 3 decimal.
2. Find the solution of differential equation $\dot{y} = \frac{x-y}{2}$ at $x = 0.2$, use $y(0) = 1$, $h = 0.1$, by using Runge-Kutta Method.
3. Find the solution of differential equation $\dot{y} = -2x - y$ at $x = 0.4$, by using Runge-Kutta Method. Use $y(0) = -1$, $h = 0.2$.

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