

General solution of homogenous S.O.D.E

$$y'' + ay' + by = 0 \quad a, b = \text{constants}$$

Case.1// Real Roots $m_1 \neq m_2$

$$y(x) = C_1 e^{m_1 x} + C_2 e^{m_2 x}$$

Case.2// Double Roots $m_1 = m_2$

$$y(x) = C_1 e^{m_1 x} + C_2 x e^{m_2 x}$$

Case.3// Conjugate, complex Roots

$$m_1 = p + qi, \quad m_2 = p - qi$$

$$y(x) = e^{px} (C_1 \cos qx + C_2 \sin qx)$$

Ex.1/ solve $y'' + y' - 2y = 0$?

Sol:

$$m^2 + m - 2 = 0$$

$$(m + 2)(m - 1)$$

$$\therefore m_1 = -2 \quad \text{and} \quad m_2 = 1$$

$$\therefore y(x) = C_1 e^{-2x} + C_2 e^x$$

Ex.2/ solve $y'' - 4y' + y = 0$?

sol/

$$m^2 - 4m + 1 = 0$$

$$m_{1,2} = \frac{4 \pm \sqrt{16 - 4 \times 1 \times 1}}{2 \times 1} \rightarrow = 2 \pm \sqrt{3}$$

$$y(x) = C_1 e^{(2+\sqrt{3})x} + C_2 e^{(2-\sqrt{3})x}$$

Ex.3/ $y'' - 7y' = 0$; Solve the D.E?

sol/

$$m^2 - 7m = 0 \rightarrow m(m-7) = 0$$

$$m_1 = 0, m_2 = 7$$

$$y(x) = C_1 e^{0x} + C_2 e^{7x}$$

$$y(x) = C_1 + C_2 e^{7x}$$

Ex.4// Solve; $y'' - 6y' + 25y = 0$?

$$m^2 - 6m + 25 = 0$$

$$m_{1,2} = \frac{6 \pm \sqrt{36 - (4 \times 1 \times 25)}}{2 \times 1} \rightarrow m_{1,2} = 3 \pm 4i$$

$$y(x) = e^{3x} (C_1 \cos 4x + C_2 \sin 4x)$$