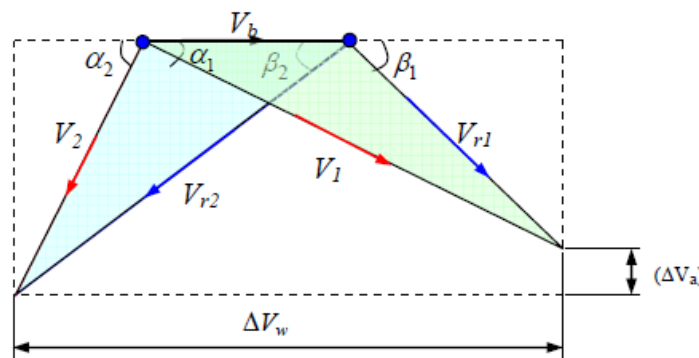


Example (5.5) the first stage of a turbine is a two-row velocity-compounded impulse wheel. The steam velocity at inlet is 600 m/s, the mean blade velocity is 120 m/s, and the blade velocity coefficient for all blades is 0.9. The nozzle angle is 16° and the exit angles for; (the first row of moving blades, the fixed blades, and the second row of moving blades) are 18° , 21° , and 35° respectively. Calculate:

- (1) The blade inlet angles for each row.
- (2) The driving force for each row of moving blades and the axial thrust on the wheel, for a mass flow rate of 1 kg/s.
- (3) The diagram power per kg/s steam flow, and the diagram efficiency for the wheel.

Solution: given;

$$V_1 = 600 \text{ m/s}, V_b = 120 \text{ m/s}, K_1 = K_2 = 0.9, \alpha_1 = 16^\circ, \beta_2 = 18^\circ, \\ \alpha_3 = 21^\circ, \beta_4 = 35^\circ, m = 1 \text{ kg/s}$$



Velocity Diagram for First and Second Rows Impulse Turbine.

First row calculations:

From velocity triangle

$$V_{r1} \times \cos \beta_1 = V_1 \times \cos \alpha_1 - V_b = 600 \times \cos 16 - 120$$

$$V_{r1} \times \cos \beta_1 = 457 \quad \dots (a)$$

$$V_{r1} \times \sin \beta_1 = V_1 \times \sin \alpha_1 = 600 \times \sin 16$$

$$V_{r1} \times \sin \beta_1 = 165.4 \quad \dots (b)$$

Divide eq.(b) by eq.(a), it gets;

$$\frac{V_{r1} \times \sin \beta_1}{V_{r1} \times \cos \beta_1} = \frac{165.4}{457} = \tan \beta_1 \rightarrow \beta_1 = \tan^{-1} \left[\frac{165.4}{457} \right]$$

$$\beta_1 = 20^\circ \text{ sub. into eq.(a);}$$

$$V_{r1} = \frac{457}{\cos 20} \rightarrow V_{r1} = 486.3 \text{ m/s}$$

$$K = \frac{V_{r2}}{V_{r1}} \rightarrow V_{r2} = K \times V_{r1} = 0.9 \times 486.3 = 437.7 \text{ m/s}$$

$$V_2 \times \sin \alpha_2 = V_{r2} \times \sin \beta_2 = 437.3 \times \sin 18$$

$$V_2 \times \sin \alpha_2 = 135.3 \dots (c)$$

$$V_2 \times \cos \alpha_2 = V_{r2} \times \cos \beta_2 - V_b = 437.7 \times \cos 18 - 120$$

$$V_2 \times \cos \alpha_2 = 296.3 \dots (d)$$

Divide eq.(c) by eq.(d), it gets;

$$\frac{V_2 \times \sin \alpha_2}{V_2 \times \cos \alpha_2} = \frac{135.3}{296.3} = \tan \alpha_2 \rightarrow \alpha_2 = \tan^{-1} \left[\frac{135.3}{296.3} \right]$$

$$\alpha_2 = 24.5^\circ \text{ sub. into (c)}$$

$$V_2 = \frac{135.3}{\sin 24.5} \rightarrow V_2 = 326.3 \text{ m/s}$$

$$\Delta V_w = V_{w1} + V_{w2}$$

$$\Delta V_w = (V_1 \times \cos \alpha_1 + V_2 \times \cos \alpha_2) = 600 \times \cos(16) + 326.3 \times \cos(24.5)$$

$$\Delta V_w = 873.7 \text{ m/s}$$

$$\Delta V_f = V_{f1} - V_{f2} = V_1 \times \sin \alpha_1 - V_2 \times \sin \alpha_2 = 600 \times \sin 16 - 326.3 \times \sin 24.5$$

$$\Delta V_f = 30 \text{ m/s}$$

Second row calculations:

$$K = \frac{V_3}{V_2} \rightarrow V_3 = K \times V_2 = 0.9 \times 326.3 = 293.7 \text{ m/s}$$

From velocity triangle

$$V_{r3} \times \cos \beta_3 = V_3 \times \cos \alpha_3 - V_b = 293.7 \times \cos 21 - 120$$

$$V_{r3} \times \cos \beta_3 = 154.2 \dots (e)$$

$$V_{r3} \times \sin \beta_3 = V_3 \times \sin \alpha_3 = 293.7 \times \sin 21$$

$$V_{r3} \times \sin \beta_3 = 105.3 \dots (f)$$

Divide eq.(e) by eq.(f), it gets;

$$\frac{V_{r3} \times \sin \beta_3}{V_{r3} \times \cos \beta_3} = \frac{105.3}{154.2} = \tan \beta_3 \rightarrow \beta_3 = \tan^{-1} \left[\frac{105.3}{154.2} \right]$$

$$\beta_3 = 34.3^\circ \text{ sub. into eq.(e);}$$

$$V_{r3} = \frac{154.2}{\cos 34.3} \rightarrow V_{r3} = 186.7 \text{ m/s}$$

$$K = \frac{V_{r4}}{V_{r2}} \rightarrow V_{r4} = K \times V_{r3} = 0.9 \times 186.7 = 168 \text{ m/s}$$

$$V_4 \times \sin \alpha_4 = V_{r4} \times \sin \beta_4 = 168 \times \sin 35$$

$$V_4 \times \sin \alpha_4 = 96.4 \dots (g)$$

$$V_4 \times \cos \alpha_4 = V_{r4} \times \cos \beta_4 - V_b = 168 \times \cos 35 - 120$$

$$V_4 \times \cos \alpha_4 = 17.6 \dots (h)$$

Divide eq.(g) by eq.(h), it gets;

$$\frac{V_4 \times \sin \alpha_4}{V_4 \times \cos \alpha_4} = \frac{96.4}{17.6} = \tan \alpha_4 \rightarrow \alpha_4 = \tan^{-1} \left[\frac{96.4}{17.6} \right]$$

$$\alpha_4 = 80^\circ \text{ sub. into (g)}$$

$$V_4 = \frac{96.4}{\sin 80} \rightarrow V_4 = 98 \text{ m/s}$$

$$\Delta V_w = V_{w3} + V_{w4}$$

$$\Delta V_w = (V_3 \times \cos \alpha_3 + V_4 \times \cos \alpha_4) = 293.7 \times \cos(21) + 98 \times \cos(80)$$

$$\Delta V_w = 291.2 \text{ m/s}$$

$$\Delta V_f = V_{f3} - V_{f4} = V_3 \times \sin \alpha_3 - V_4 \times \sin \alpha_4 = 293.7 \times \sin 21 - 98 \times \sin 80$$

$$\Delta V_f = 9 \text{ m/s}$$

For the two rows

$$\text{Total Driving Force} = m^o \times \Delta V_w = [(V_{w1} + V_{w2}) + (V_{w3} + V_{w4})]$$

$$\text{Total Driving Force} = 1 \times [(873.7) + (291.2)]$$

$$\text{Total Driving Force} = 1165 \text{ N}$$

$$\text{Total thrust} = m^o \times \Delta V_f = m^o \times [(V_{f1} - V_{f2}) + (V_{f3} - V_{f4})]$$

$$\text{Total thrust} = 1 \times [(30) + (9)]$$

$$\text{Total thrust} = 39 \text{ N}$$

$$\text{Power} = m^o \times V_b \times \Delta V_w = 1 \times 120 \times [(873.7) + (291.2)]$$

$$\text{Power} = 139.8 \text{ kW}$$

$$\eta_{\text{diagram}} = \eta_{\text{blade}} = \frac{2 \times V_b \times \Delta V_w}{V_1^2} = \frac{2 \times 120 \times 1165}{600^2}$$

$$\eta_{\text{diagram}} = 77.7 \%$$