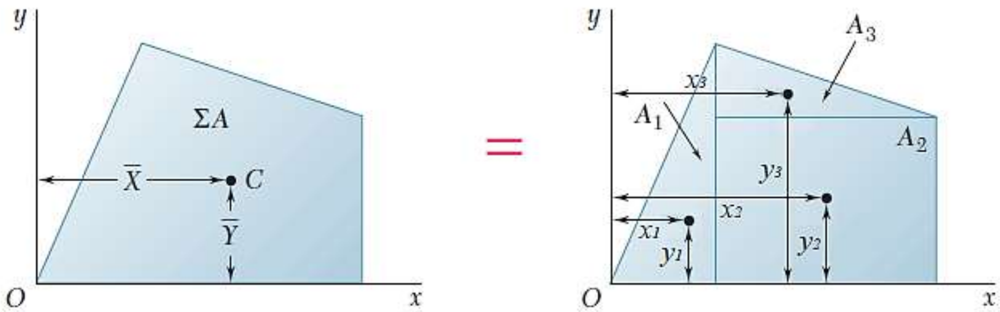
6.2 Centroid of Composite Figure

In engineering work, we frequently need to locate the centroid of a composite area. Such an area may be composed of regular geometric shapes such as rectangle, triangle, circle, semicircle, quarter circle, etc. In such cases, we divide the given area into regular geometric shapes for which the positions of centroids are readily known. Let Ai be the area of an element and $(\bar{x}, \bar{y})$ be the respective centroid coordinates. Then for the composite area,

$$A = A_1 + A_2 + \cdots + A_n$$

$$A = \sum_{i=1}^{n} A_i$$

The moment of the total area with respect to any axis will be sum of the moment of its regular geometric shapes respect to the same axis.



$$My = A_1 \cdot x_1 + A_2 \cdot x_2 + \cdots + A_n \cdot x_n$$

$$My = \sum_{i=1}^{n} A_i \cdot x_i$$

$$Mx = A_1 \cdot y_1 + A_2 \cdot y_2 + \cdots + A_n \cdot y_n$$

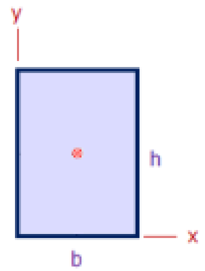
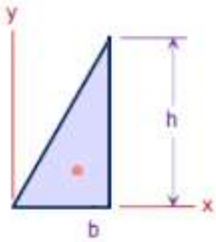
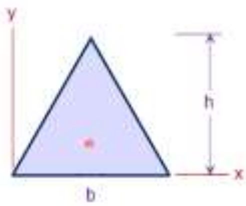
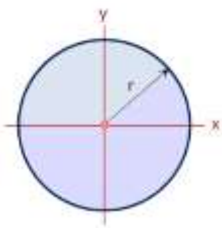
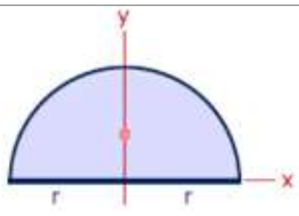
$$Mx = \sum_{i=1}^{n} A_i \cdot y_i$$

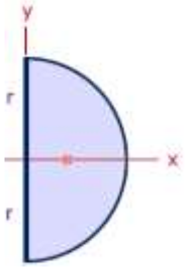
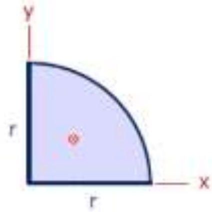
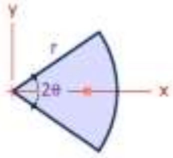
$$\bar{x} = \frac{My}{A} = \frac{\sum_{i=1}^{n} A_i \cdot x_i}{\sum_{i=1}^{n} A_i}$$

$$\bar{y} = \frac{Mx}{A} = \frac{\sum_{i=1}^{n} A_i \cdot y_i}{\sum_{i=1}^{n} A_i}$$

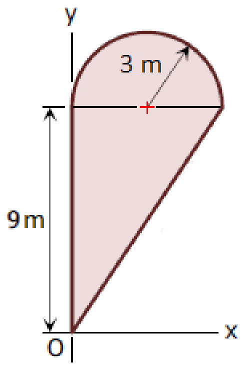
The centroid coordinates of Common geometrical shapes can be determined in a like manner and the results are summarized below in tabular form.

Table 6.1 Centroids of Common geometrical shapes

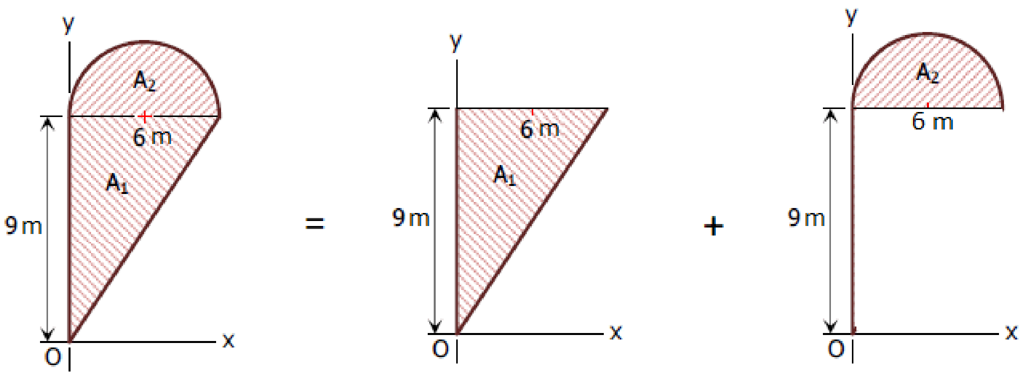
No.	***Shape***	***Figure***	***Area***	$\bar{x}$	$\bar{y}$
1	Rectangle		$A = b.h$	$b/2$	$h/2$
2	Right angled Triangle		$A = \frac{1}{2}\, b.h$	$2b/3$	$h/3$
3	Isosceles Triangle		$A = \frac{1}{2}\, b.h$	$b/2$	$h/3$
4	Circle		$A = \pi r^2$	0	0
5	Semicircle		$A = \frac{\pi}{2} r^2$	0	$\frac{4r}{3\pi}$

6	Semicircle		$A = \frac{\pi}{2} r^2$	$\frac{4r}{3\pi}$	0
7	Quarter Circle		$A = \frac{\pi}{4} r^2$	$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$
8	Circular sector		$A = \theta r^2$ $\theta : rad$	$\frac{2r \sin\theta}{3\theta}$	0

Example No. 1: Determine the coordinates of the centroid of the area shown in Figure with respect to the given axes.



Solution:



Shape	A	$\bar{x}$	$\bar{y}$	$A \cdot \bar{x}$	$A \cdot \bar{y}$
Triangle	$\frac{1}{2}6 \times 9 = 27$	$\frac{1}{3} \times 6 = 2$	$\frac{2}{3} \times 9 = 6$	54	162
Semicircle	$\frac{\pi}{2} \times 3^2 = 14.14$	$r = 3$	$9 + \frac{4r}{3\pi} = 10.27$	42.42	145.218
Sum	$41.14\ m^2$			$96.42\ m^3$	$307.218\ m^3$

For the shaded region:

$$\bar{x} = \frac{\sum_{i=1}^{n} A_i \cdot x_i}{\sum_{i=1}^{n} A_i} = \frac{96.42}{41.14} = 2.34\ m$$

$$\bar{y} = \frac{\sum_{i=1}^{n} A_i \cdot y_i}{\sum_{i=1}^{n} A_i} = \frac{307.218}{41.14} = 7.47\ m$$