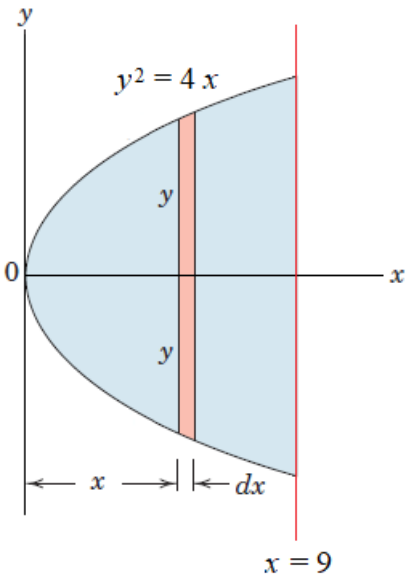
Example No. 3: Determine the moments of inertia of the area bounded by the line $x = 9$ and the parabola $y^2 = 4\,x$ with respect to the $x - axis$ and $y - axis$.

Solution:



$$y^2 = 4\,x, \quad y = 2\,x^{1/2}$$

$$I_x = \int dI_x = \int \frac{bh^3}{12}$$

$$I_x = \int_0^9 \frac{(2y)^3 dx}{12} = \int_0^9 \frac{\left(2 \times 2\,x^{1/2}\right)^3}{12} dx$$

$$= \frac{64}{12}\int_0^9 x^{3/2}\,dx = \frac{64}{12}\left[\frac{x^{5/2}}{5/2}\right]_0^9 = 518.4\,m^4$$

$$I_y = \int dI_y = \int I_C + A\,d^2$$

$$I_y = \int_0^9 \left(\frac{bh^3}{12} + A\,d^2\right)$$

$$I_y = \int_0^9 \left(\frac{(2y)\,dx^3}{12} + (2y)\,dx \cdot x^2\right)$$

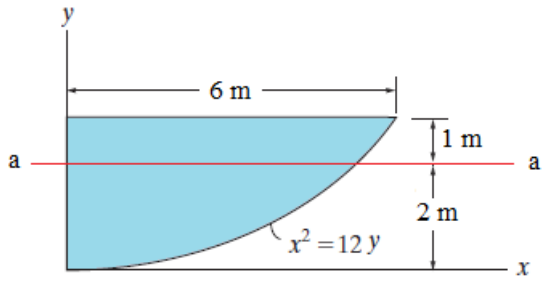
$$\frac{y dx^3}{12} \approx 0, \quad y = 2\,x^{1/2}$$

ملاحظة: dx هو بعد جدا صغير لذلك فان تربيعه او تكعيبه يساوي صفر.

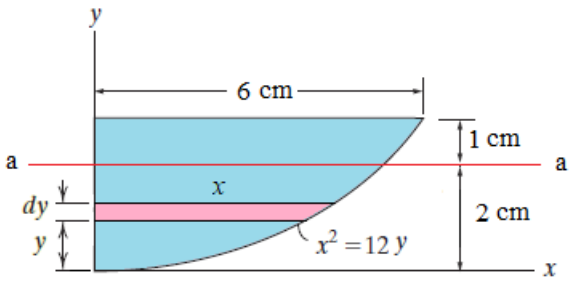
$$I_y = \int_0^9 4\,x^{1/2} \cdot x^2\,dx = 4\int_0^9 x^{5/2}\,dx = 4\left[\frac{x^{7/2}}{7/2}\right]_0^9$$

$$I_y = 2499.43\,m^4$$

Example No. 4: Determine the radius of gyration of the shaded area shown in figure with respect to the $a-axis$.



Solution:



$$r_a = \sqrt{\frac{I_a}{A}}$$

$$I_a = \int dI_a = \int I_C + A\,d^2$$

$$I_a = \int_0^3 \left(\frac{bh^3}{12} + A\,d^2\right)$$

$$I_a = \int_0^3 \left(\frac{x\,dy^3}{12} + x\,dy\cdot(2-y)^2\right)$$

$$\frac{x\,dy^3}{12} \approx 0,\quad x = \sqrt{12}\,y^{1/2}$$

$$I_a = \int_0^3 \sqrt{12}\,y^{1/2}\cdot(4-4y+y^2)\,dy = \sqrt{12}\int_0^3 \left(4\,y^{1/2} - 4y^{3/2} + y^{5/2}\right)dy$$

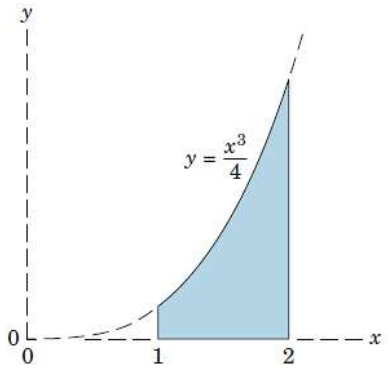
$$I_a = \sqrt{12}\left[\frac{y^{3/2}}{3/2} - \frac{4y^{5/2}}{5/2} + \frac{y^{7/2}}{7/2}\right]_0^3 = 7.886\ cm^4$$

$$A = \int dA = \int x\cdot dy = \int_0^3 \sqrt{12}\,y^{1/2}\cdot dy = \sqrt{12}\left[\frac{y^{3/2}}{3/2}\right]_0^3 = 12\ cm^2$$

$$r_a = \sqrt{\frac{I_a}{A}} = \sqrt{\frac{7.886}{12}} = 0.811\ cm$$

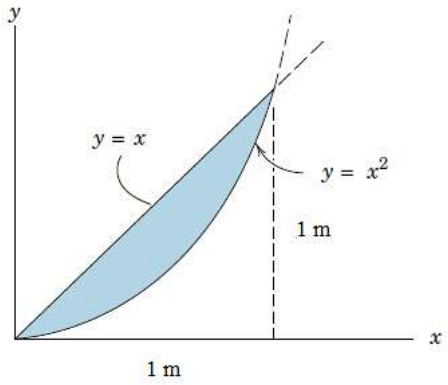
Problems:

1. Determine the rectangular and polar radii of gyration of the shaded area about the axes shown.



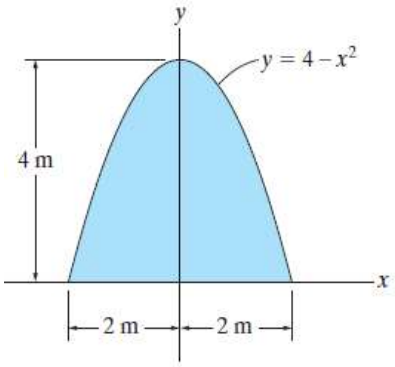
Answer: $r_x = 0.754,\ \ r_y = 1.673,\ \ r_o = 1.835$

2. Determine the moments of inertia of the shaded area about the *x*- and *y*-axes.



Answer: $Ix = \frac{1}{28}\ m^4,\ \ Iy = \frac{1}{20}\ m^4$

3. Determine the moment of inertia of the area about the *y* axis.



Answer: $Iy\ = \ 8.53\ m^4$