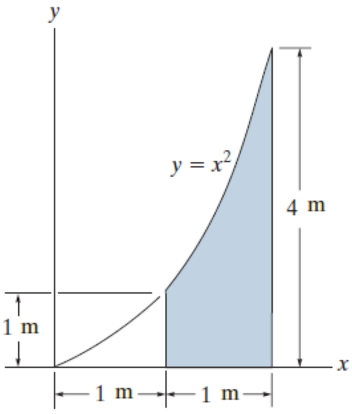
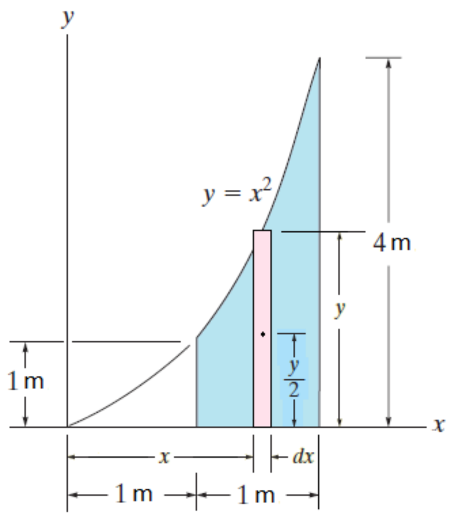
Example No. 2: Determine the centroid $(\bar{x}, \bar{y})$ of the shaded area.



Solution:

$$\bar{x} = \frac{My}{A} = \frac{\int_A x_c \cdot dA}{\int_A dA}, \qquad \bar{y} = \frac{Mx}{A} = \frac{\int_A y_c \cdot dA}{\int_A dA}$$

By vertical strip



$$x_c = x, \qquad y_c = \frac{y}{2}$$

$$dA = y\,dx, \quad y = x^2$$

$$dA = x^2 dx$$

$$A = \int_A dA = \int_1^2 x^2 dx = \frac{x^3}{3}\Big]_1^2 = \left(\frac{2^3}{3} - \frac{1^3}{3}\right) = \frac{7}{3}\ m^2$$

$$My = \int_A x_c \cdot dA = \int_1^2 x \cdot x^2 dx = \int_1^2 x^3 dx = \frac{x^4}{4}\Big]_1^2 = \left(\frac{2^4}{4} - \frac{1^4}{4}\right) = 3.75\ m^3$$

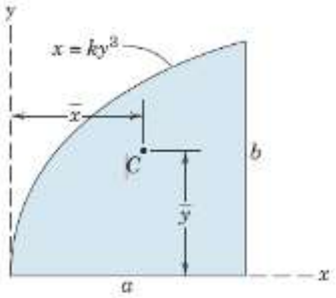
$$Mx = \int_A y_c \cdot dA = \int_1^2 \frac{y}{2} \cdot x^2 dx = \int_1^2 \frac{x^2}{2} \cdot x^2 dx = \frac{1}{2}\int_1^2 x^4\, dx$$

$$= \frac{1}{2} \times \frac{x^5}{5}\Big]_1^2 = \left(\frac{2^5}{10} - \frac{1^5}{10}\right) = 3.1\ m^3$$

$$\bar{x} = \frac{My}{A} = \frac{3.75}{7/3} = 1.607\ m$$

$$\bar{y} = \frac{Mx}{A} = \frac{3.1}{7/3} = 1.329\ m$$

Example No. 3: Locate the centroid of the area under the curve $x = ky^3$ if $a = 8\ m$, and $b = 12\ m$.



Solution:

$x = ky^3$

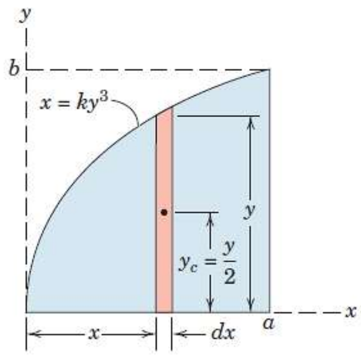
Substituting point (a,b) to find k:

$$a = kb^3 \quad \rightarrow \quad k = \frac{a}{b^3} = \frac{8}{1728}$$

$$x = \frac{8}{1728}\, y^3$$

$$\bar{x} = \frac{My}{A} = \frac{\int_A x_c \cdot dA}{\int_A dA}, \qquad \bar{y} = \frac{Mx}{A} = \frac{\int_A y_c \cdot dA}{\int_A dA}$$

Method I: by vertical strip



$x_c = x$

$$y_c = \frac{y}{2}$$

$$dA = y\,dx, \quad y = \left(\frac{1728}{8}x\right)^{1/3} = 6\,x^{1/3}$$

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$dA = 6\,x^{1/3}\,dx$

$$A = \int_A dA = 6\int_0^8 x^{1/3}dx = 6\,\frac{x^{4/3}}{4/3}\Big]_0^8 = 72\,m^2$$

$$My = \int_A x_c \cdot dA = \int_0^8 x \cdot 6\,x^{1/3}dx = 6\int_0^8 x^{4/3}dx = 6\,\frac{x^{7/3}}{7/3}\Big]_0^8 = 329.143\,m^3$$

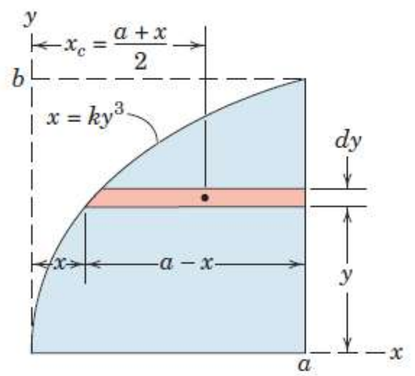
$$Mx = \int_A y_c \cdot dA = \int_0^8 \frac{y}{2} \cdot 6\,x^{1/3}dx = \int_0^8 \frac{6\,x^{1/3}}{2} \cdot 6x^{1/3}\,dx = 18\int_0^8 x^{2/3}dx$$

$$= 18\,\frac{x^{5/3}}{5/3}\Big]_0^8 = 345.6\,m^3$$

$$\bar{x} = \frac{329.143}{72} = 4.571\,m$$

$$\bar{y} = \frac{345.6}{72} = 4.8\,m$$

H.W: Method II: **by horizontal strip**



$$x_c = x + \frac{(8-x)}{2} = \frac{(8+x)}{2}$$

$y_c = y,$

$dA = (8-x)\,dy, \qquad x = \frac{8}{1728}\,y^3$

$$dA = \left(8 - \frac{8}{1728}\,y^3\right)dy = \frac{8}{1728}(1728 - y^3)\,dy$$

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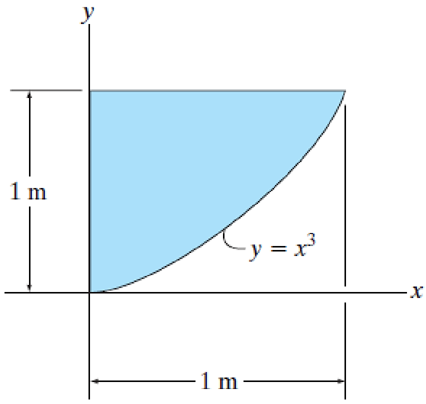
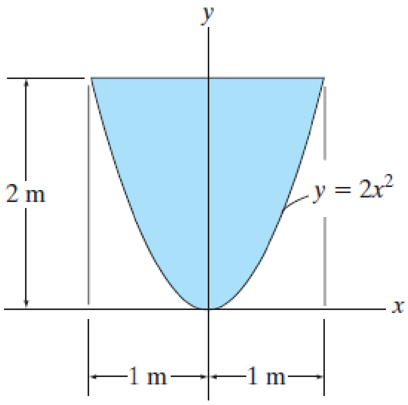
Problems:

1. Determine the centroid $(\bar{x}, \bar{y})$ of the shaded area by using vertical and horizontal strip.



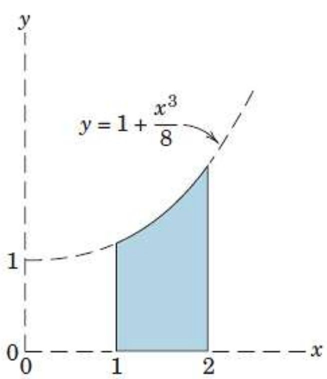
Answer: $\bar{x} = 0.4\ m,\ \ \bar{y} = 0.571\ m$

2. Determine the centroid y of the shaded area.



Answer: $\bar{y} = 1.2\ m$

3. Determine the x- and y-coordinates of the centroid of the shaded area.



Answer: $\bar{x} = 1.549,\ \bar{y} = 0.756$

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