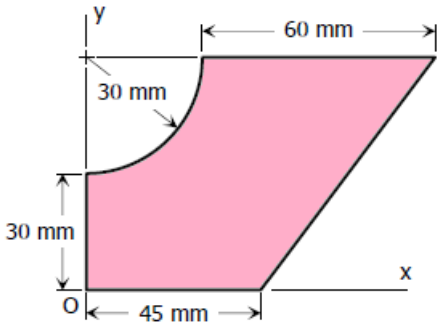
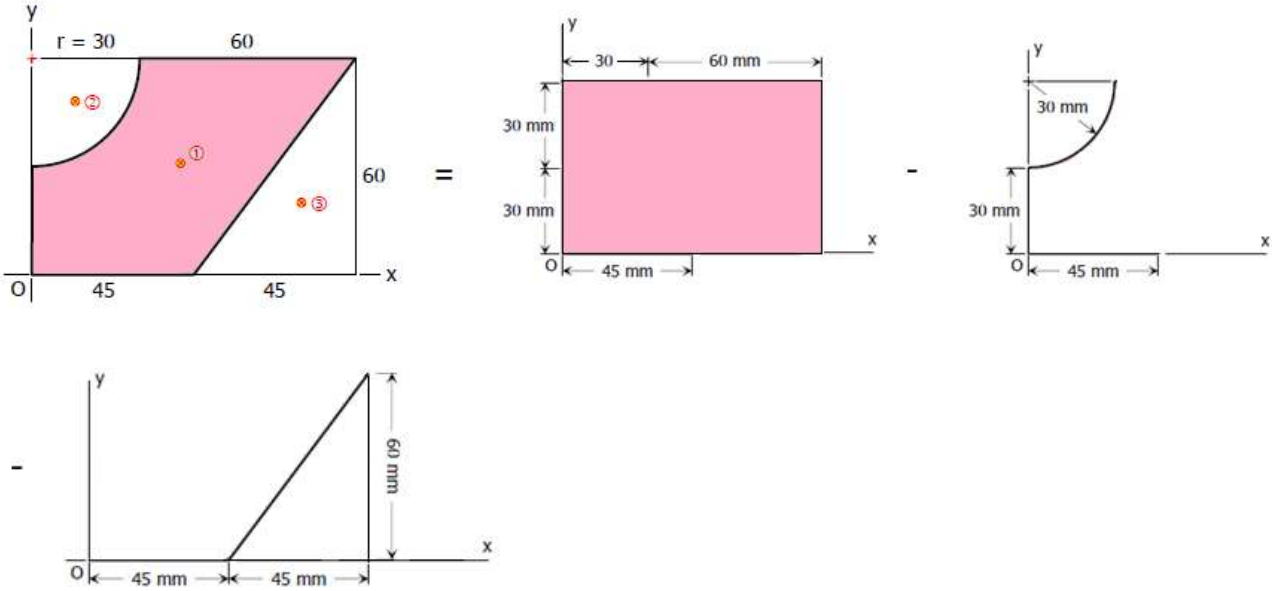
Example No. 2: Locate the centroid of the shaded area in Figure.



Solution:



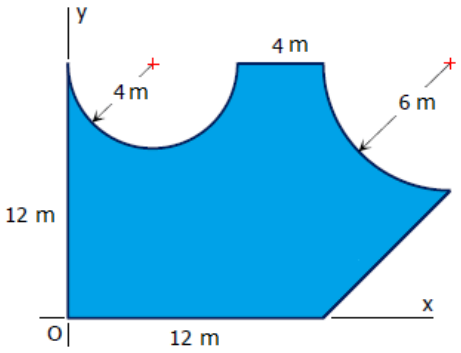
Shape	A	$\bar{x}$	$\bar{y}$	$A \cdot \bar{x}$	$A \cdot \bar{y}$
Rectangular	$90 \times 60 = 5400$	$1/2 \times 90 = 45$	$\frac{1}{2} \times 60 = 30$	243000	162000
quarter circle	$-\frac{\pi}{4} \times 30^2 = -706.86$	$4r/3\pi = 12.73$	$60 - \frac{4r}{3\pi} = 47.27$	-8998.3	-33413.3
Triangle	$-\frac{1}{2} \times 45 \times 60 = -1350$	$45 + \frac{2}{3} \times 45 = 75$	$\frac{1}{3} \times 60 = 20$	-101250	-27000
Sum	$3343.14\ mm^2$			$132751.7\ m^3$	$101586.7\ m^3$

For the shaded region:

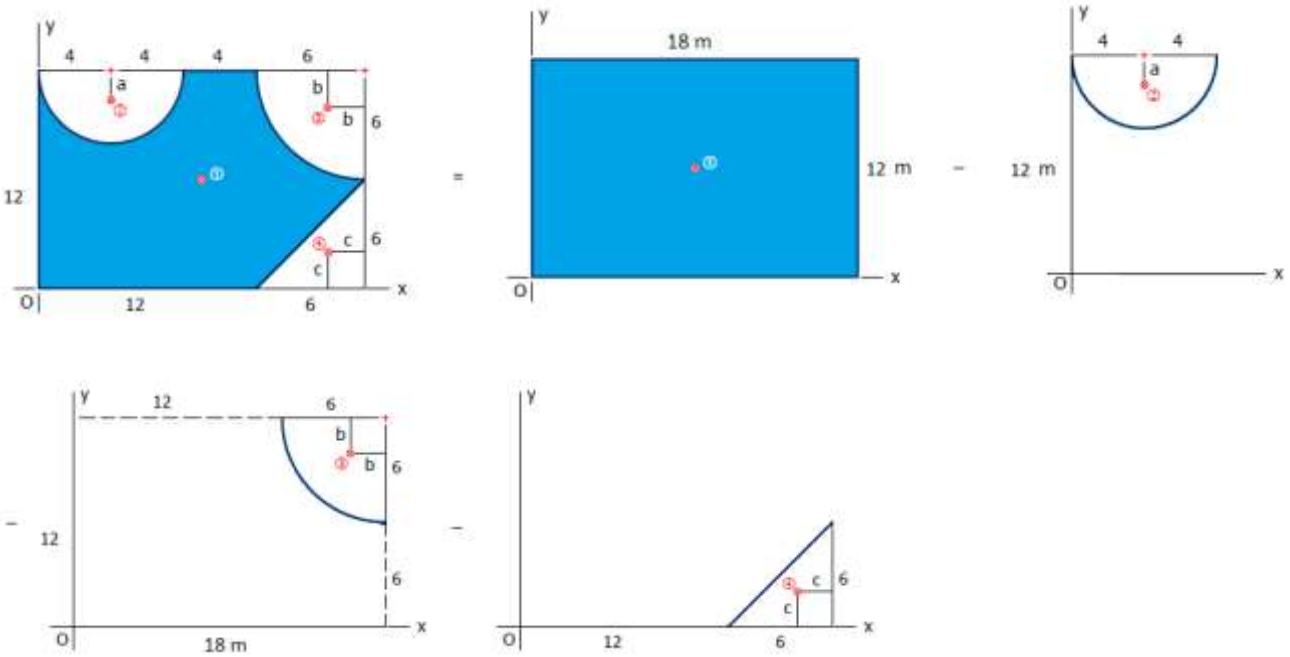
$$\bar{x} = \frac{\sum_{i=1}^{n} A_i \cdot x_i}{\sum_{i=1}^{n} A_i} = \frac{132751.7}{3343.14} = 39.71\ mm$$

$$\bar{y} = \frac{\sum_{i=1}^{n} A_i \cdot y_i}{\sum_{i=1}^{n} A_i} = \frac{101586.7}{3343.14} = 30.39\ mm$$

Example No. 3: Find the coordinates of the centroid of the shaded area shown in Fig.



Solution:



$$a = \frac{4r}{3\pi} = \frac{4 \times 4}{3\pi} = 1.698\ m, \qquad b = \frac{4r}{3\pi} = \frac{4 \times 6}{3\pi} = 2.546\ m$$

$$c = \frac{1}{3} \times 5 = 2\ m$$

Shape	A	$\bar{x}$	$\bar{y}$	$A \cdot \bar{x}$	$A \cdot \bar{y}$
Rectangular	$18 \times 12 = 216$	$\frac{1}{2} \times 18 = 9$	$\frac{1}{2} \times 12 = 6$	1944	1296
Semicircle	$-\frac{\pi}{2} \times 4^2 = -25.133$	$r = 4$	$12 - a = 10.302$	-100.53	-258.92
quarter circle	$-\frac{\pi}{4} \times 6^2 = -28.274$	$18 - b = 15.454$	$12 - b = 9.454$	-436.95	-267.30
Triangle	$-\frac{1}{2} \times 6 \times 6 = -18$	$18 - c = 16$	$c = 2$	-288	-36
Sum	$144.593\ m^2$			$1123.61\ m^3$	$733.777\ m^3$

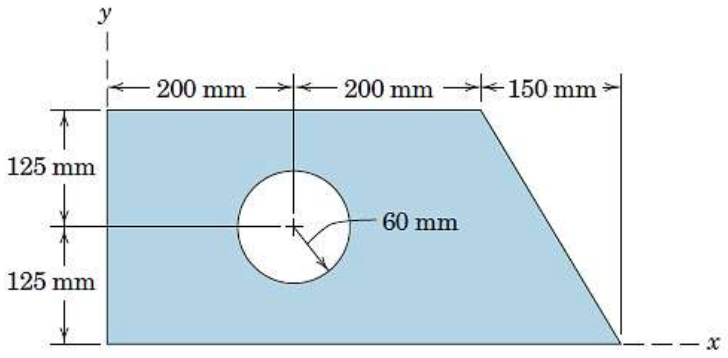
For the shaded region:

$$\bar{x} = \frac{\sum_{i=1}^{n} A_i \cdot x_i}{\sum_{i=1}^{n} A_i} = \frac{1123.61}{144.593} = 7.736\ m$$

$$\bar{y} = \frac{\sum_{i=1}^{n} A_i \cdot y_i}{\sum_{i=1}^{n} A_i} = \frac{733.777}{144.593} = 5.075\ m$$

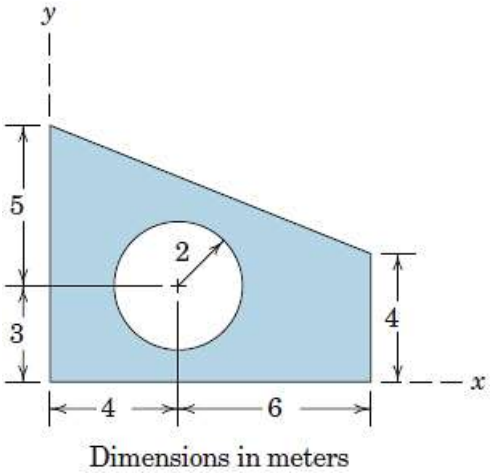
Problems:

1. Determine the coordinates of the centroid of the shaded area.



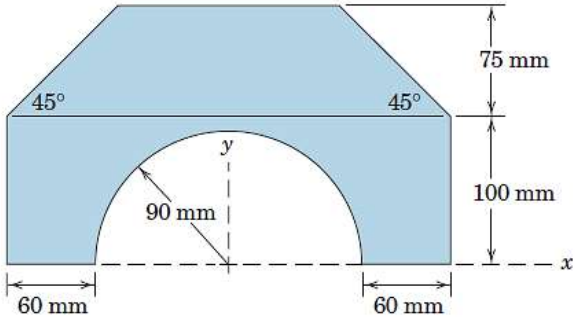
Answer: $\bar{x} = 244\ mm, \bar{y} = 117.7\ mm$

2. Determine the *x*- and *y*-coordinates of the centroid of the shaded area.



Answer: $\bar{x} = 4.56\ m, \bar{y} = 3.14\ m$

3. Determine the *y*-coordinate of the centroid of the shaded area.



Answer: $\bar{y} = 95.6\ mm$