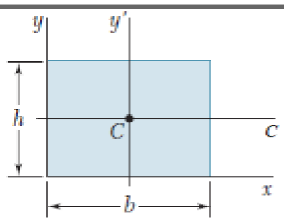
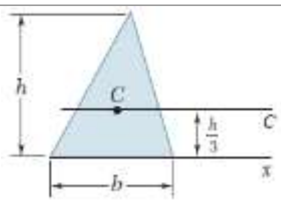
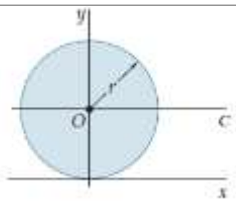
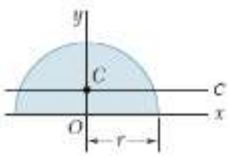
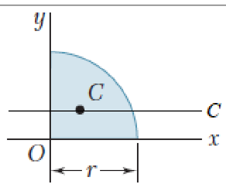
7.4 Moments of Inertia of Composite Areas

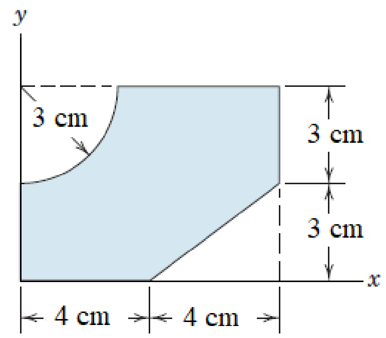
Consider a composite area A made of several component areas $A1$, $A2$, $A3$, …., such as rectangles, triangles, and circles. Therefore, we can obtain the moment of inertia for the composite area A with respect to a given axis by the *algebraic sum* of the moments of inertia of the areas $A1$, $A2$, $A3$, . . . with respect to the same axis.

The Moment of Inertia of Common geometrical shapes can be determined in a like manner and the results are summarized below in tabular form.

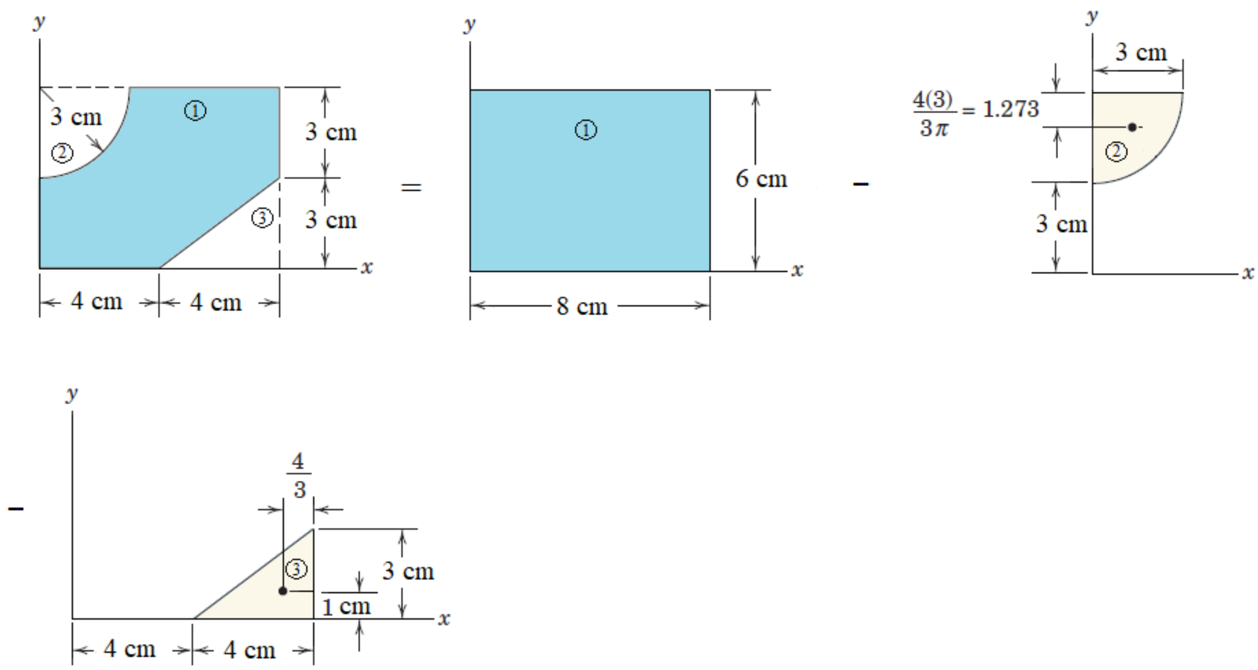
Table 7.1 Moment of Inertia of Common Shapes

No.	***Shape***	***Figure***	I_C	I_x
1	Rectangle		$I_C = \frac{bh^3}{12}$	$I_x = \frac{bh^3}{3}$
2	Triangle		$I_C = \frac{bh^3}{36}$	$I_x = \frac{bh^3}{12}$
3	Circle		$I_C = \frac{\pi r^4}{4}$	$I_x = \frac{5\pi r^4}{4}$
4	Semicircle		$I_C = 0.11\, r^4$	$I_x = \frac{\pi r^4}{8}$
5	Quarter Circle		$I_C = 0.055\, r^4$	$I_x = \frac{\pi r^4}{16}$

Example No. 1: Determine the moments of inertia and radius of gyration about the *x*- axes for the shaded area shown.



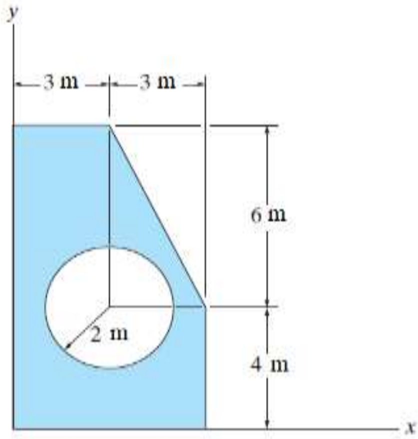
Solution:



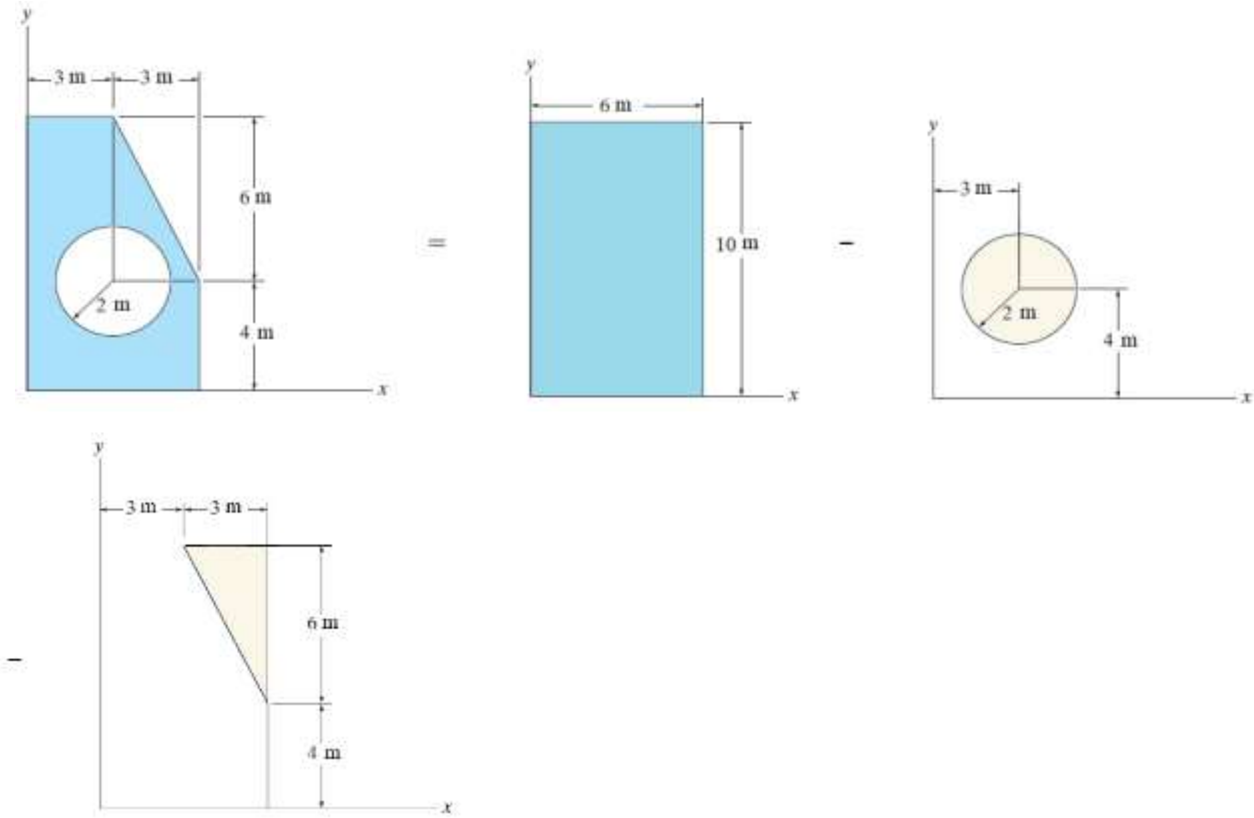
Shape	I_C	A	d	Ad^2	$I_x = I_C + Ad^2$
Rectangular	$\frac{8\times 6^3}{12} = 144$	$6\times 8 = 48$	3	432	576
Quarter Circle	$-0.055\times 3^4 = -4.455$	$-\frac{\pi}{4}\times 3^2 = -7.069$	$6 - 1.273 = 4.727$	-157.944	-162.4
Triangle	$-\frac{4\times 3^3}{36} = -3$	$-\frac{1}{2}\times 3\times 4 = -6$	1	-6	-9
Sum		34.931			404.6

$$\therefore I_x = 404.6\ cm^4, \qquad r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{404.6}{34.931}} = 3.403\ cm$$

Example No. 2: Determine the moments of inertia about the *y*- axes for the shaded area shown.



Solution:

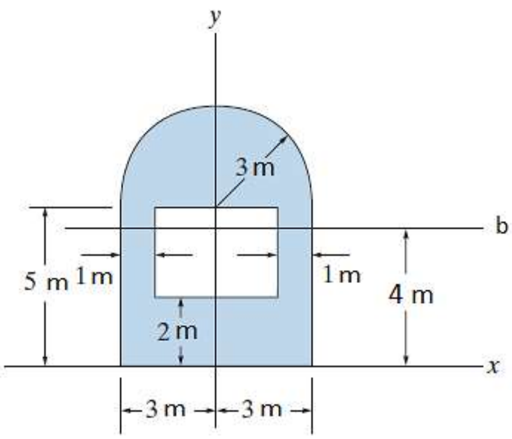


Shape	I_C	A	d	Ad^2	$I_y = I_C + Ad^2$
Rectangular	$\frac{10 \times 6^3}{12} = 180$	$6 \times 10 = 60$	3	540	720
Circle	$-\frac{\pi\ 2^4}{4} = -12.566$	$-\pi \times 2^2$ $= -12.566$	3	-113.094	-125.66

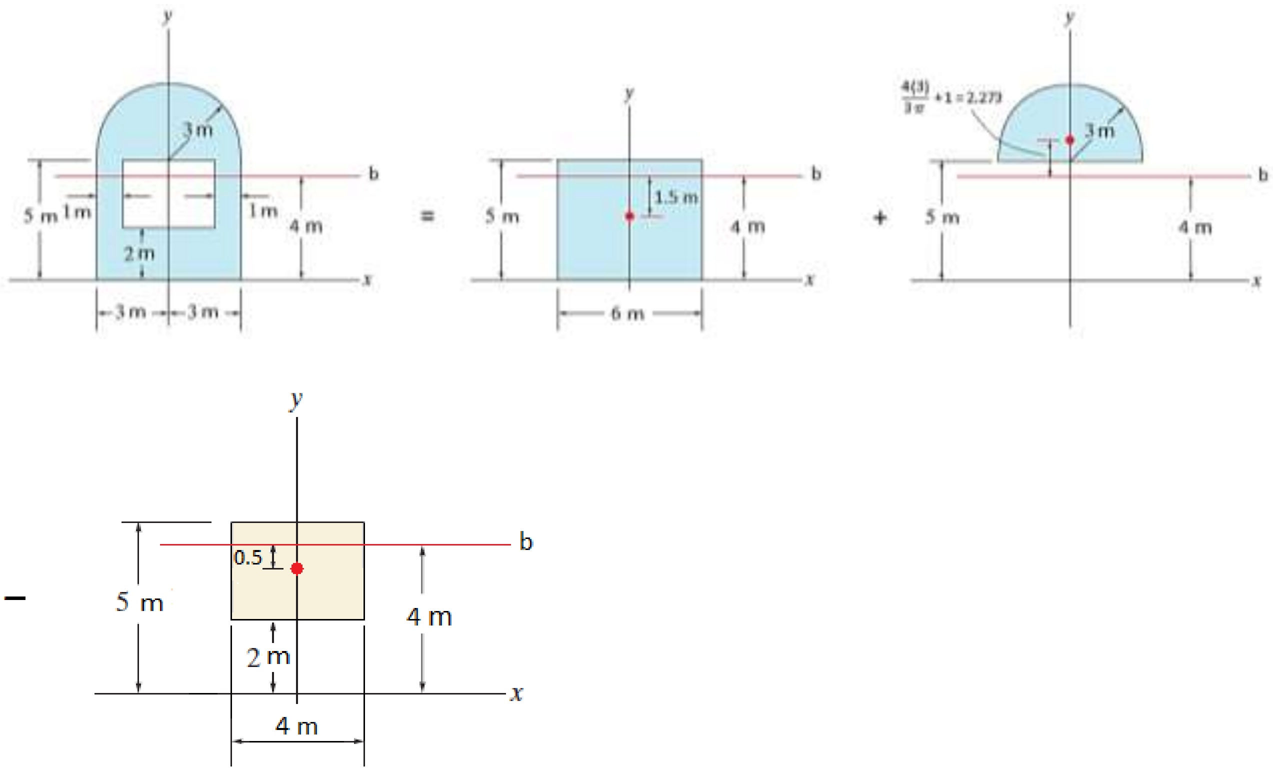
Triangle	$-\dfrac{6\times 3^3}{36}=-4.5$	$-\dfrac{1}{2}\times 3\times 6$ $=-9$	5	-225	-229.5
Sum					364.84

$\therefore I_y \;=\; 364.84\; m^4$

Example No. 3: Locate the centroid $\bar{y}$ of the composite area, then determine the moment of inertia of this area about the centroid b axis.



Solution:

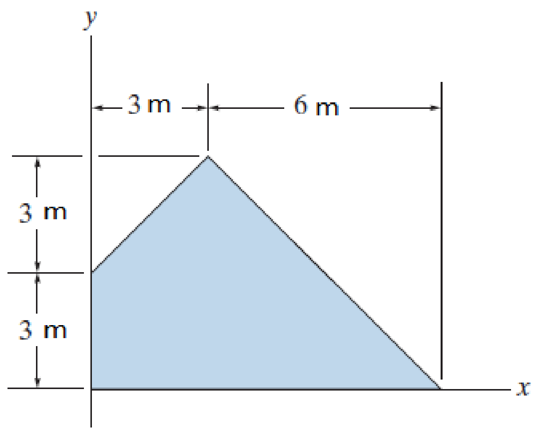


Shape	I_C	A	d	Ad^2	$I_b = I_C + Ad^2$
Rectangular 1	$\frac{6 \times 5^3}{12} = 62.5$	$6 \times 5 = 30$	-1.5	67.5	130
Semicircle	$0.11 \times 3^4 = 8.91$	$\frac{\pi}{2} \times 3^2 = 14.137$	2.273	73.04	81.95
Rectangular 2	$-\frac{4 \times 3^3}{12} = -9$	$-4 \times 3 = -12$	-0.5	-3	-12
Sum					200

$\therefore I_b = 200\, m^4$

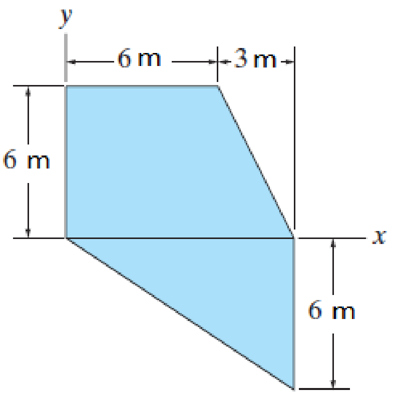
Problems:

1. Determine the moment of inertia of the composite area about the x and y axis.



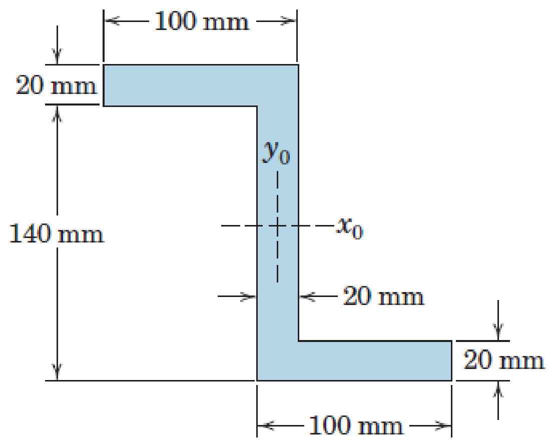
Answer: $Ix = 209\, m^4,\ \ Iy = 533\, m^4$

2. Determine the moment of inertia Iy of the shaded area about the y axis.



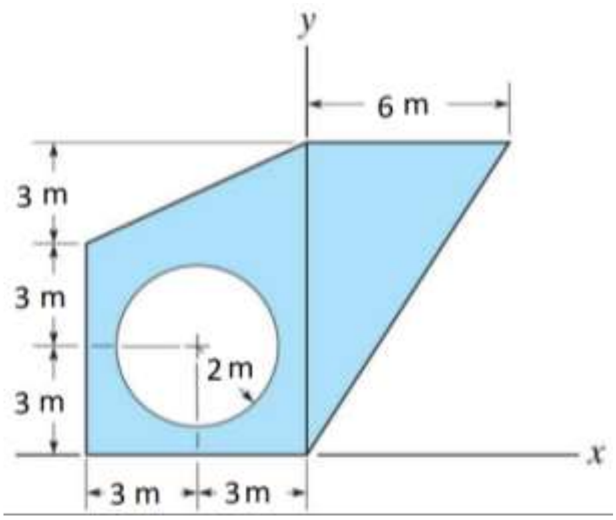
Answer: $Iy = 1971\ m^4$

3. Determine the moments of inertia of the Z-section about its centroid x and y axes.



Answer: $Ix = 22.6 \times 10^6\ mm^4,\ \ Iy = 9.81 \times 10^6\ mm^4$

4. Determine the moment of inertia of the composite area about the x and y axis.



Answer: $Ix = 1845\ m^4,\ \ Iy = 522\ m^4$