3.1. INTRODUCTION

In the two preceding chapters you studied how to calculate the stresses and strains in structural members subjected to axial loads, that is, to forces directed along the axis of the member. In this chapter structural members and machine parts that are in *torsion* will be considered. More specifically, you will analyze the stresses and strains in members of circular cross section subjected to twisting couples, or *torques,* **T** and **T**′ (Fig. 3.1). These couples have a common magnitude T, and opposite senses. They are vector quantities and can be represented either by curved arrows as in Fig. 3.1*a*, or by couple vectors as in Fig. 3.1*b*.

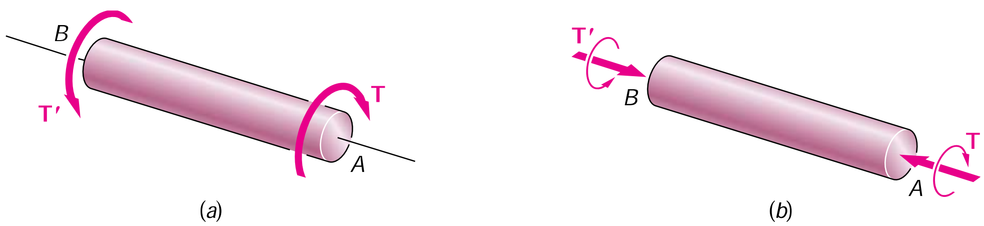


Fig. 3.1

Members in torsion are encountered in many engineering applications. The most common application is provided by *transmission shafts,* which are used to transmit power from one point to another. For example, the shaft shown in Fig. 3.2 is used to transmit power from the engine to the rear wheels of an automobile. These shafts can be either solid, as shown in Fig. 3.1, or hollow.

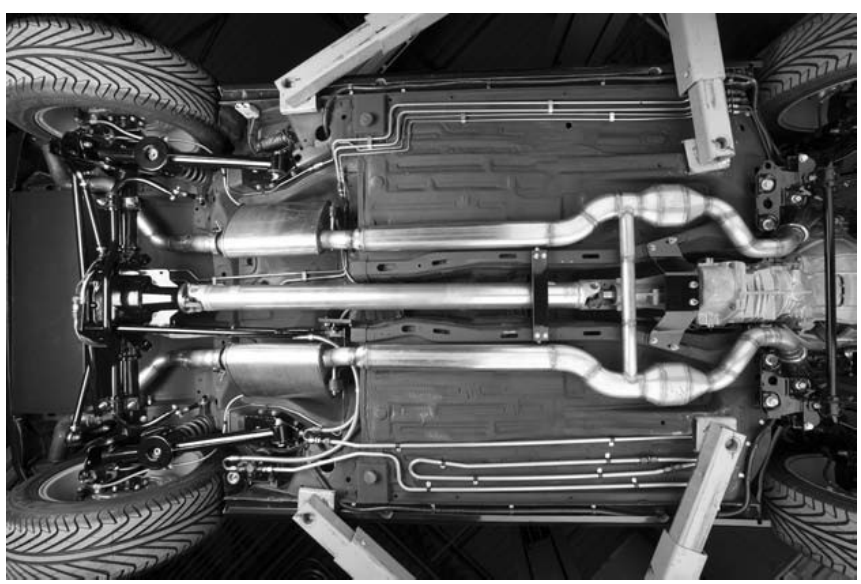
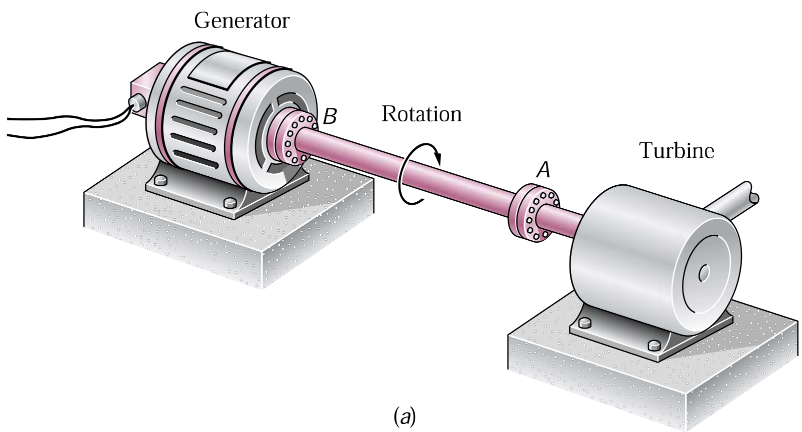
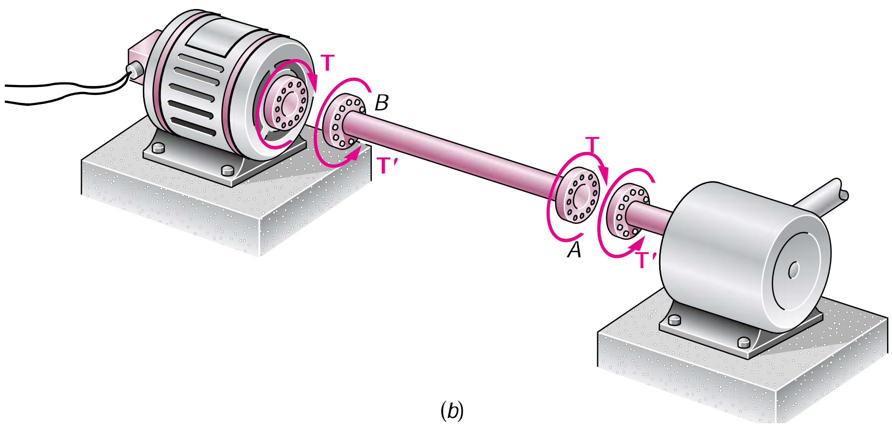


Fig. 3.2 In the automotive power train shown, the shaft transmits power from the engine to the rear wheels.



(*a*)



(*b*)

Fig. 3.3

Consider the system shown in Fig. 3.3*a*, which consists of a steam turbine *A* and an electric generator *B* connected by a transmission shaft *AB*. By breaking the system into its three component parts (Fig. 3.3*b*), you can see that the turbine exerts a twisting couple or torque **T** on the shaft and that the shaft exerts an equal torque on the generator. The generator reacts by exerting the equal and opposite torque **T**′ on the shaft, and the shaft by exerting the torque **T**′ on the turbine.

You will first analyze the stresses and deformations that take place in circular shafts. In Sec. 3.3, an important property of circular shafts is demonstrated: *When a circular shaft is subjected to torsion, every cross section remains plane and undistorted.* In other words, while the various cross sections along the shaft rotate through different angles, each cross section rotates as a solid rigid slab. This property will enable you to determine the *distribution of shearing strains in a circular shaft and to conclude that the shearing strain varies linearly with the distance from the axis of the shaft.*

Considering deformations in the *elastic range* and using Hooke's law for shearing stress and strain, you will determine the *distribution of shearing stresses* in a circular shaft and derive the *elastic torsion formulas* (Sec. 3.4).

In Sec. 3.5, you will learn how to find the *angle of twist* of a circular shaft subjected to a given torque, assuming again elastic deformations. The solution of problems involving *statically indeterminate shafts* is considered in Sec. 3.6.

In Sec. 3.7, you will study the *design of transmission shafts*. In order to accomplish the design, you will learn to determine the required physical characteristics of a shaft in terms of its speed of rotation and the power to be transmitted.

The torsion formulas cannot be used to determine stresses near sections where the loading couples are applied or near a section where an abrupt change in the diameter of the shaft occurs. Moreover, these formulas apply only within the elastic range of the material.

In Sec. 3.8, you will learn how to account for stress concentrations where an abrupt change in diameter of the shaft occurs. In Secs. 3.9 to 3.11, you will consider stresses and deformations in circular shafts made of a ductile material when the yield point of the material is exceeded. You will then learn how to determine the permanent *plastic deformations* and *residual stresses* that remain in a shaft after it has been loaded beyond the yield point of the material.

In the last sections of this chapter, you will study the torsion of noncircular members (Sec. 3.12) and analyze the distribution of stresses in thin-walled hollow noncircular shafts (Sec. 3.13).

3.2. PRELIMINARY DISCUSSION OF THE STRESSES IN A SHAFT

Considering a shaft *AB* subjected at *A* and *B* to equal and opposite torques **T** and **T′**, we pass a section perpendicular to the axis of the shaft through some arbitrary point *C* (Fig. 3.4). The free-body diagram of the portion *BC* of the shaft must include the elementary shearing forces *d***F,** perpendicular to the radius of the shaft, that portion *AC* ex-

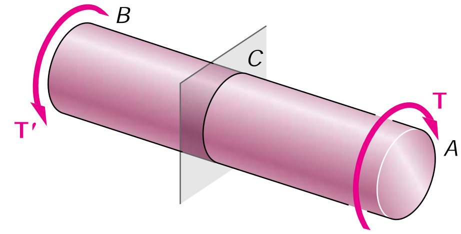


Fig. 3.4

erts on BC as the shaft is twisted (Fig. 3.5*a*). But the conditions of equilibrium for BC require that the system of these elementary forces be equivalent to an internal torque **T**, equal and opposite to **T**′ (Fig. 3.5*b*). Denoting by ρ the perpendicular distance from the force $d\mathbf{F}$ to the axis of the shaft, and expressing that the sum of the moments of the shearing forces $d\mathbf{F}$ about the axis of the shaft is equal in magnitude to the torque **T**, we write

$$\int \rho \, dF = T$$

or, since $dF = \tau \, dA$, where τ is the shearing stress on the element of area dA,

$$\int \rho(\tau \, dA) = T \qquad (3.1)$$

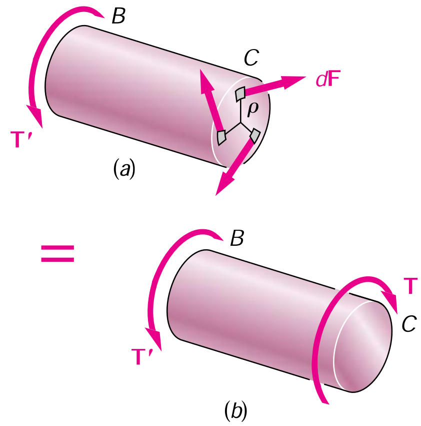


Fig. 3.5

While the relation obtained expresses an important condition that must be satisfied by the shearing stresses in any given cross section of the shaft, it does *not* tell us how these stresses are distributed in the cross section. We thus observe, as we already did in Sec. 1.5, that the actual distribution of stresses under a given load is *statically indeterminate,* i.e., this distribution *cannot be determined by the methods of statics*. However, having assumed in Sec. 1.5 that the normal stresses produced by an axial centric load were uniformly distributed, we found later (Sec. 2.17) that this assumption was justified, except in the neighborhood of concentrated loads. A similar assumption with respect to the distribution of shearing stresses in an elastic shaft *would be wrong*. We must withhold any judgment regarding the distribution of stresses in a shaft until we have analyzed the *deformations* that are produced in the shaft. This will be done in the next section.

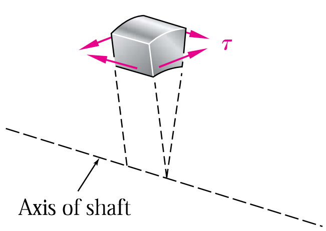


Fig. 3.6

One more observation should be made at this point. As was indicated in Sec. 1.12, shear cannot take place in one plane only. Consider the very small element of shaft shown in Fig. 3.6. We know that the torque applied to the shaft produces shearing stresses τ on the faces perpendicular to the axis of the shaft. But the conditions of equilibrium discussed in Sec. 1.12 require the existence of equal stresses on the faces formed by the two planes containing the axis of the shaft. That such shearing stresses actually occur in torsion can be demonstrated.

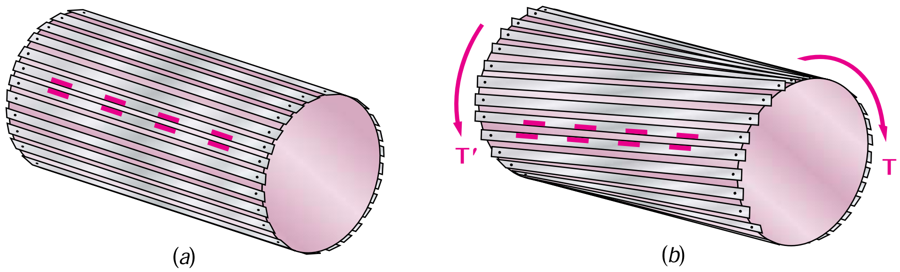


Fig. 3.7

by considering a "shaft" made of separate slats pinned at both ends to disks as shown in Fig. 3.7*a*. If markings have been painted on two adjoining slats, it is observed that the slats slide with respect to each other when equal and opposite torques are applied to the ends of the "shaft" (Fig. 3.7*b*). While sliding will not actually take place in a shaft made of a homogeneous and cohesive material, the tendency for sliding will exist, showing that stresses occur on longitudinal planes as well as on planes perpendicular to the axis of the shaft.†

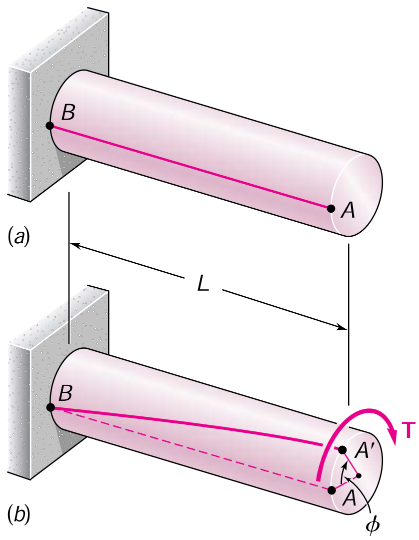


Fig. 3.8

3.3. DEFORMATIONS IN A CIRCULAR SHAFT

Consider a circular shaft that is attached to a fixed support at one end (Fig. 3.8*a*). If a torque **T** is applied to the other end, the shaft will twist, with its free end rotating through an angle ϕ called *the angle of twist* (Fig. 3.8*b*). Observation shows that, within a certain range of values of T, the angle of twist ϕ is proportional to T. It also shows that ϕ is proportional to the length L of the shaft. In other words, the angle of twist for a shaft of the same material and same cross section, but twice as long, will be twice as large under the same torque **T.** One purpose of our analysis will be to find the specific relation existing among ϕ, L, and T; another purpose will be to determine the distribution of shearing stresses in the shaft, which we were unable to obtain in the preceding section on the basis of statics alone.

At this point, an important property of circular shafts should be noted: When a circular shaft is subjected to torsion, *every cross section remains plane and undistorted*. In other words, while the various cross sections along the shaft rotate through different amounts, each cross section rotates as a solid rigid slab. This is illustrated in Fig. 3.9*a*, which shows the deformations in a rubber model subjected to torsion. The property we are discussing is characteristic of circular shafts, whether solid or hollow; it is not enjoyed by members of noncircular cross section. For example, when a bar of square cross section is subjected to torsion, its various cross sections warp and do not remain plane (Fig. 3.9*b*).

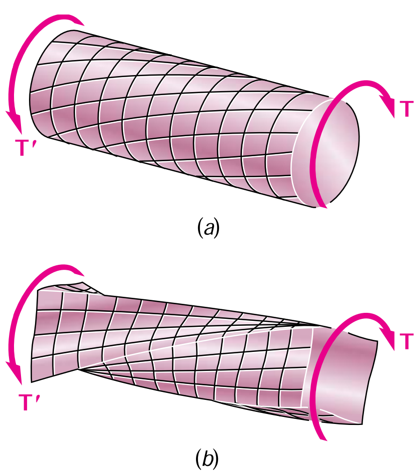


Fig. 3.9

†The twisting of a cardboard tube that has been slit lengthwise provides another demonstration of the existence of shearing stresses on longitudinal planes.

The cross sections of a circular shaft remain plane and undistorted because a circular shaft is *axisymmetric,* i.e., its appearance remains the same when it is viewed from a fixed position and rotated about its axis through an arbitrary angle. (Square bars, on the other hand, retain the same appearance only if they are rotated through 90° or 180°.) As we will see presently, the axisymmetry of circular shafts may be used to prove theoretically that their cross sections remain plane and undistorted.

Consider the points C and D located on the circumference of a given cross section of the shaft, and let C' and D' be the positions they will occupy after the shaft has been twisted (Fig. 3.10*a*). The axisymmetry of the shaft and of the loading requires that the rotation which would have brought D into C should now bring D' into C'. Thus C' and D' must lie on the circumference of a circle, and the arc $C'D'$ must be equal to the arc CD (Fig. 3.10*b*). We will now examine whether the circle on which C' and D' lie is different from the original circle. Let us assume that C' and D' do lie on a different circle and that the new circle is located to the left of the original circle, as shown in Fig. 3.10*b*. The same situation will prevail for any other cross section, since all the cross sections of the shaft are subjected to the same internal torque T, and an observer looking at the shaft from its end A will conclude that the loading causes any given circle drawn on the shaft to move *away*. But an observer located at B, to whom the given loading looks the same (a clockwise couple in the foreground and a counterclockwise couple in the background) will reach the opposite conclusion, i.e., that the circle moves *toward* him. This contradiction proves that our assumption is wrong and that C' and D' lie on the same circle as C and D. Thus, as the shaft is twisted, the original circle just rotates in its own plane. Since the same reasoning may be applied to any smaller, concentric circle located in the cross section under consideration, we conclude that the entire cross section remains plane (Fig. 3.11).

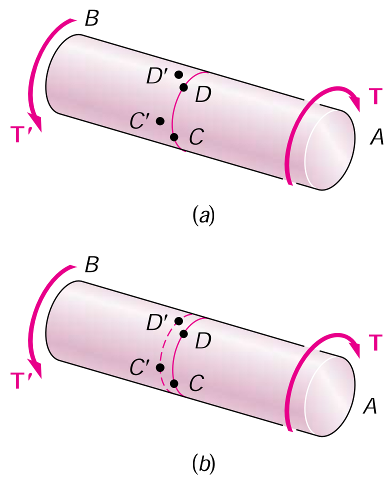


Fig. 3.10

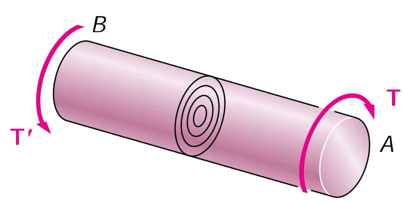


Fig. 3.11

The above argument does not preclude the possibility for the various concentric circles of Fig. 3.11 to rotate by different amounts when the shaft is twisted. But if that were so, a given diameter of the cross section would be distorted into a curve which might look as shown in Fig. 3.12*a*. An observer looking at this curve from A would conclude that the outer layers of the shaft get more twisted than the inner ones, while an observer looking from B would reach the opposite conclusion (Fig. 3.12*b*). This inconsistency leads us to conclude that any diameter of a given cross section remains straight (Fig. 3.12*c*) and, therefore, that any given cross section of a circular shaft remains plane and undistorted.

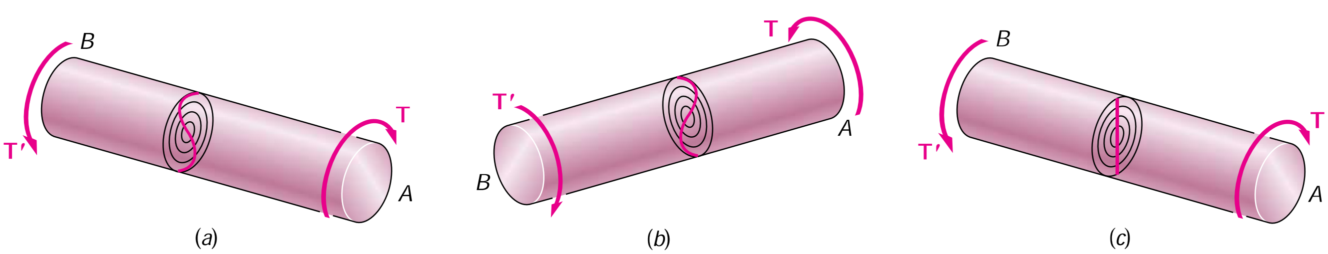


Fig. 3.12

Our discussion so far has ignored the mode of application of the twisting couples **T** and **T′**. If *all* sections of the shaft, from one end to the other, are to remain plane and undistorted, we must make sure that the couples are applied in such a way that the ends of the shaft themselves remain plane and undistorted. This may be accomplished by applying the couples **T** and **T′** to rigid plates, which are solidly attached to the ends of the shaft (Fig. 3.13a). We can then be sure that all sections will remain plane and undistorted when the loading is applied, and that the resulting deformations will occur in a uniform fashion throughout the entire length of the shaft. All of the equally spaced circles shown in Fig. 3.13*a* will rotate by the same amount relative to their neighbors, and each of the straight lines will be transformed into a curve (helix) intersecting the various circles at the same angle (Fig. 3.13*b*).

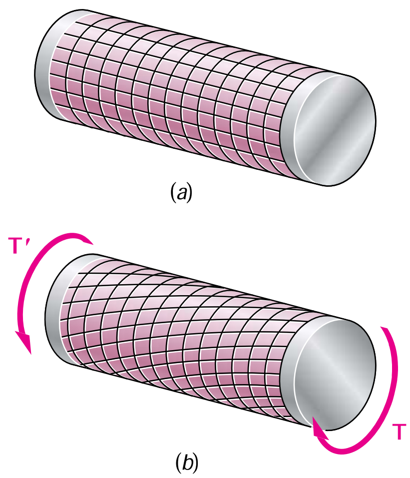


Fig. 3.13

The derivations given in this and the following sections will be based on the assumption of rigid end plates. Loading conditions encountered in practice may differ appreciably from those corresponding to the model of Fig. 3.13. The chief merit of this model is that it helps us define a torsion problem for which we can obtain an exact solution, just as the rigid-end-plates model of Sec. 2.17 made it possible for us to define an axial-load problem which could be easily and accurately solved. By virtue of Saint-Venant's principle, the results obtained for our idealized model may be extended to most engineering applications. However, we should keep these results associated in our mind with the specific model shown in Fig. 3.13.

We will now determine the distribution of *shearing strains* in a circular shaft of length L and radius c which has been twisted through an angle $\mathbf{f}$ (Fig. 3.14*a*). Detaching from the shaft a cylinder of radius $\mathbf{r}$, we consider the small square element formed by two adjacent circles and two adjacent straight lines traced on the surface of the cylinder before any load is applied (Fig. 3.14*b*). As the shaft is subjected to a torsional load, the element deforms into a rhombus (Fig. 3.14*c*). We now recall from Sec. 2.14 that the shearing strain $\mathbf{g}$ in a given element is measured by the change in the angles formed by the sides of that element. Since the circles defining two of the sides of the element considered here remain unchanged, the shearing strain $\mathbf{g}$ must be equal to the angle between lines AB and $A'B$. (We recall that $\mathbf{g}$ should be expressed in radians.)

We observe from Fig. 3.14*c* that, for small values of $\mathbf{g}$, we can express the arc length AA' as $AA' = L\mathbf{g}$. But, on the other hand, we have $AA' = \mathbf{rf}$. It follows that $L\mathbf{g} = \mathbf{rf}$, or

$$\mathbf{g} = \frac{\mathbf{rf}}{L} \qquad (3.2)$$

where $\mathbf{g}$ and $\mathbf{f}$ are both expressed in radians. The equation obtained shows, as we could have anticipated, that the shearing strain $\mathbf{g}$ at a given point of a shaft in torsion is proportional to the angle of twist $\mathbf{f}$. It also shows that $\mathbf{g}$ is proportional to the distance $\mathbf{r}$ from the axis of the shaft to the point under consideration. Thus, *the shearing strain in a circular shaft varies linearly with the distance from the axis of the shaft.*

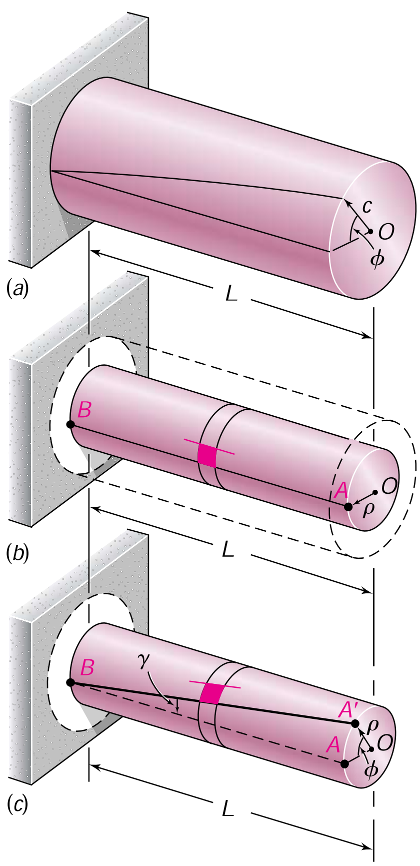


Fig. 3.14

It follows from Eq. (3.2) that the shearing strain is maximum on the surface of the shaft, where $\rho = c$. We have

$$\gamma_{max} = \frac{c\phi}{L} \tag{3.3}$$

Eliminating ϕ from Eqs. (3.2) and (3.3), we can express the shearing strain γ at a distance ρ from the axis of the shaft as

$$\gamma = \frac{\rho}{c}\gamma_{max} \tag{3.4}$$

3.4. STRESSES IN THE ELASTIC RANGE

No particular stress-strain relationship has been assumed so far in our discussion of circular shafts in torsion. Let us now consider the case when the torque **T** is such that all shearing stresses in the shaft remain below the yield strength τ_Y. We know from Chap. 2 that, for all practical purposes, this means that the stresses in the shaft will remain below the proportional limit and below the elastic limit as well. Thus, Hooke's law will apply and there will be no permanent deformation.

Recalling Hooke's law for shearing stress and strain from Sec. 2.14, we write

$$\tau = G\gamma \tag{3.5}$$

where G is the modulus of rigidity or shear modulus of the material. Multiplying both members of Eq. (3.4) by G, we write

$$G\gamma = \frac{\rho}{c}G\gamma_{max}$$

or, making use of Eq. (3.5),

$$\tau = \frac{\rho}{c}\tau_{max} \tag{3.6}$$

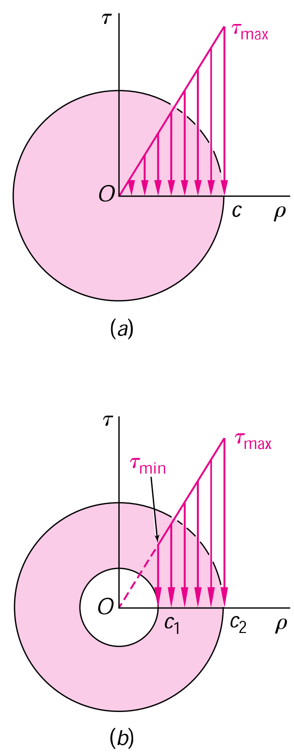


Fig. 3.15

The equation obtained shows that, as long as the yield strength (or proportional limit) is not exceeded in any part of a circular shaft, *the shearing stress in the shaft varies linearly with the distance* ρ *from the axis of the shaft*. Figure 3.15*a* shows the stress distribution in a solid circular shaft of radius c, and Fig. 3.15*b* in a hollow circular shaft of inner radius c_1 and outer radius c_2. From Eq. (3.6), we find that, in the latter case,

$$\tau_{min} = \frac{c_1}{c_2}\tau_{max} \tag{3.7}$$

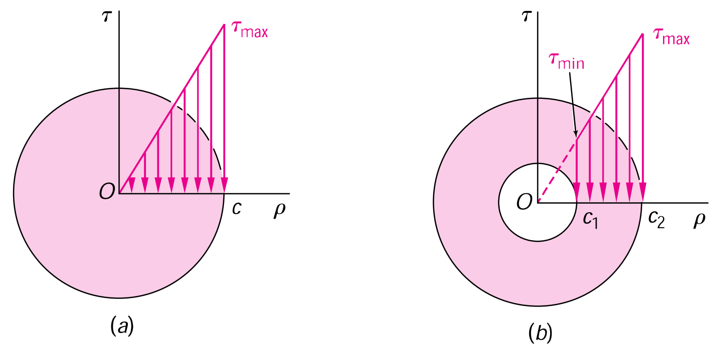


Fig. 3.15 (repeated)

We now recall from Sec. 3.2 that the sum of the moments of the elementary forces exerted on any cross section of the shaft must be equal to the magnitude T of the torque exerted on the shaft:

$$\int \rho(\tau \, dA) = T \tag{3.1}$$

Substituting for τ from (3.6) into (3.1), we write

$$T = \int \rho\tau \, dA = \frac{\tau_{max}}{c} \int \rho^2 \, dA$$

But the integral in the last member represents the polar moment of inertia J of the cross section with respect to its center O. We have therefore

$$T = \frac{\tau_{max} J}{c} \tag{3.8}$$

or, solving for τ_{max},

$$\tau_{max} = \frac{Tc}{J} \tag{3.9}$$

Substituting for τ_{max} from (3.9) into (3.6), we express the shearing stress at any distance ρ from the axis of the shaft as

$$\tau = \frac{T\rho}{J} \tag{3.10}$$

Equations (3.9) and (3.10) are known as the *elastic torsion formulas*. We recall from statics that the polar moment of inertia of a circle of radius c is $J = \frac{1}{2}\pi c^4$. In the case of a hollow circular shaft of inner radius c_1 and outer radius c_2, the polar moment of inertia is

$$J = \tfrac{1}{2}\pi c_2^4 - \tfrac{1}{2}\pi c_1^4 = \tfrac{1}{2}\pi(c_2^4 - c_1^4) \tag{3.11}$$

We note that, if SI metric units are used in Eq. (3.9) or (3.10), T will be expressed in N · m, c or ρ in meters, and J in m^4; we check that the resulting shearing stress will be expressed in N/m^2, that is, pascals (Pa). If U.S. customary units are used, T should be expressed in lb · in., c or ρ in inches, and J in in^4, with the resulting shearing stress expressed in psi.

EXAMPLE 3.01

A hollow cylindrical steel shaft is 1.5 m long and has inner and outer diameters respectively equal to 40 and 60 mm (Fig. 3.16). (*a*) What is the largest torque that can be applied to the shaft if the shearing stress is not to exceed 120 MPa? (*b*) What is the corresponding minimum value of the shearing stress in the shaft?

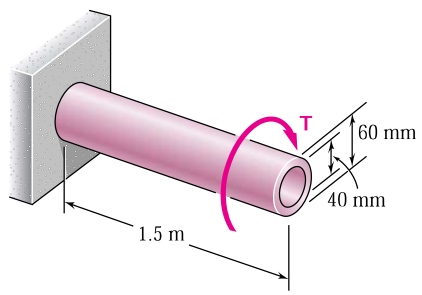


Fig. 3.16

(a) Largest Permissible Torque. The largest torque **T** that can be applied to the shaft is the torque for which $\tau_{max} = 120$ MPa. Since this value is less than the yield strength for steel, we can use Eq. (3.9). Solving this equation for T, we have

$$T = \frac{J\tau_{max}}{c} \tag{3.12}$$

Recalling that the polar moment of inertia J of the cross section is given by Eq. (3.11), where $c_1 = \frac{1}{2}(40 \text{ mm}) = 0.02$ m and $c_2 = \frac{1}{2}(60 \text{ mm}) = 0.03$ m, we write

$$J = \tfrac{1}{2}\pi(c_2^4 - c_1^4) = \tfrac{1}{2}\pi(0.03^4 - 0.02^4) = 1.021 \times 10^{-6} \text{ m}^4$$

Substituting for J and τ_{max} into (3.12), and letting $c = c_2 = 0.03$ m, we have

$$T = \frac{J\tau_{max}}{c} = \frac{(1.021 \times 10^{-6} \text{ m}^4)(120 \times 10^6 \text{ Pa})}{0.03 \text{ m}}$$
$$= 4.08 \text{ kN} \cdot \text{m}$$

(b) Minimum Shearing Stress. The minimum value of the shearing stress occurs on the inner surface of the shaft. It is obtained from Eq. (3.7), which expresses that τ_{min} and τ_{max} are respectively proportional to c_1 and c_2:

$$\tau_{min} = \frac{c_1}{c_2}\tau_{max} = \frac{0.02 \text{ m}}{0.03 \text{ m}}(120 \text{ MPa}) = 80 \text{ MPa}$$

The torsion formulas (3.9) and (3.10) were derived for a shaft of uniform circular cross section subjected to torques at its ends. However, they can also be used for a shaft of variable cross section or for a shaft subjected to torques at locations other than its ends (Fig. 3.17*a*). The distribution of shearing stresses in a given cross section S of the shaft is obtained from Eq. (3.9), where J denotes the polar moment of inertia of that section, and where T represents the *internal torque* in that section. The value of T is obtained by drawing the free-body diagram of the portion of shaft located on one side of the section (Fig. 3.17*b*) and writing that the sum of the torques applied to that portion, including the internal torque **T,** is zero (see Sample Prob. 3.1).

Up to this point, our analysis of stresses in a shaft has been limited to shearing stresses. This is due to the fact that the element we had selected was oriented in such a way that its faces were either parallel or perpendicular to the axis of the shaft (Fig. 3.6). We know from earlier discussions (Secs. 1.11 and 1.12) that normal stresses, shearing stresses, or a combination of both may be found under the same loading condition, depending upon the orientation of the element which has

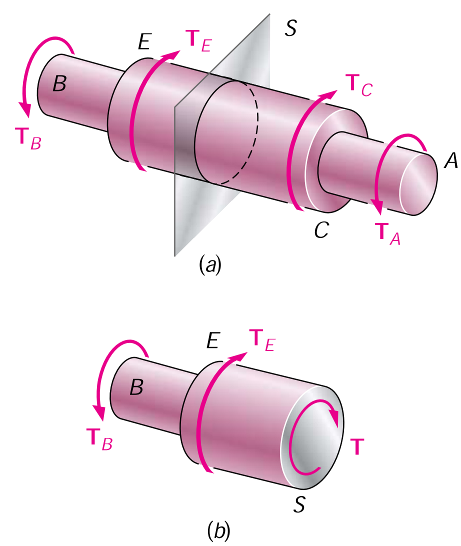


Fig. 3.17

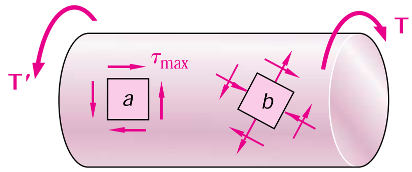


Fig. 3.18

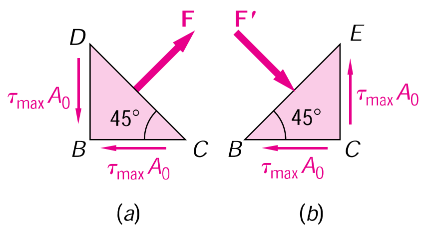


Fig. 3.19

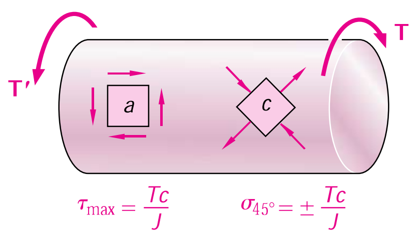


Fig. 3.20

Fig. 3.21

been chosen. Consider the two elements a and b located on the surface of a circular shaft subjected to torsion (Fig. 3.18). Since the faces of element a are respectively parallel and perpendicular to the axis of the shaft, the only stresses on the element will be the shearing stresses defined by formula (3.9), namely $\tau_{max} = Tc/J$. On the other hand, the faces of element b, which form arbitrary angles with the axis of the shaft, will be subjected to a combination of normal and shearing stresses.

Let us consider the particular case of an element c (not shown) at 45° to the axis of the shaft. In order to determine the stresses on the faces of this element, we consider the two triangular elements shown in Fig. 3.19 and draw their free-body diagrams. In the case of the element of Fig. 3.19*a*, we know that the stresses exerted on the faces BC and BD are the shearing stresses $\tau_{max} = Tc/J$. The magnitude of the corresponding shearing forces is thus $\tau_{max}A_0$, where A_0 denotes the area of the face. Observing that the components along DC of the two shearing forces are equal and opposite, we conclude that the force **F** exerted on DC must be perpendicular to that face. It is a tensile force, and its magnitude is

$$F = 2(\tau_{max}A_0)\cos 45° = \tau_{max}A_0\sqrt{2} \tag{3.13}$$

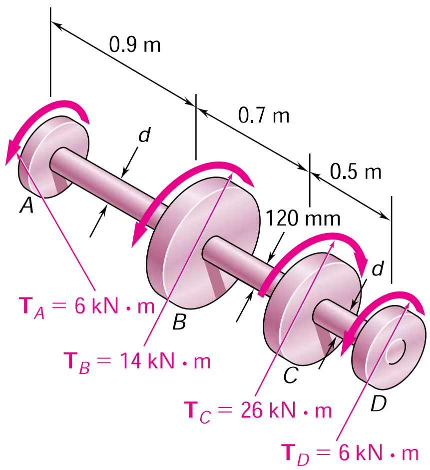
The corresponding stress is obtained by dividing the force F by the area A of face DC. Observing that $A = A_0\sqrt{2}$, we write

$$\sigma = \frac{F}{A} = \frac{\tau_{max}A_0\sqrt{2}}{A_0\sqrt{2}} = \tau_{max} \tag{3.14}$$

A similar analysis of the element of Fig. 3.19*b* shows that the stress on the face BE is $\sigma = -\tau_{max}$. We conclude that the stresses exerted on the faces of an element c at 45° to the axis of the shaft (Fig. 3.20) are normal stresses equal to $\pm\tau_{max}$. Thus, while the element a in Fig. 3.20 is in pure shear, the element c in the same figure is subjected to a tensile stress on two of its faces, and to a compressive stress on the other two. We also note that all the stresses involved have the same magnitude, Tc/J.†

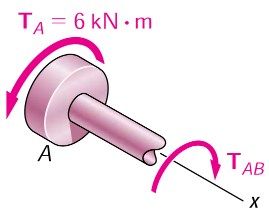
As you learned in Sec. 2.3, ductile materials generally fail in shear. Therefore, when subjected to torsion, a specimen J made of a ductile material breaks along a plane perpendicular to its longitudinal axis (Fig. 3.21*a*). On the other hand, brittle materials are weaker in tension than in shear. Thus, when subjected to torsion, a specimen made of a brittle material tends to break along surfaces which are perpendicular to the direction in which tension is maximum, i.e., along surfaces forming a 45° angle with the longitudinal axis of the specimen (Fig. 3.21*b*).

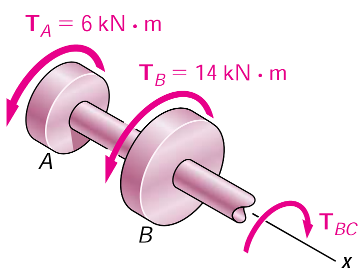
†Stresses on elements of arbitrary orientation, such as element b of Fig. 3.18, will be discussed in Chap. 7.

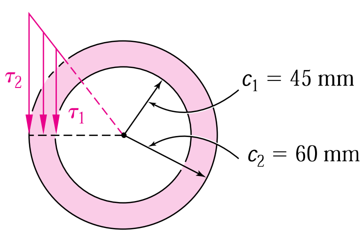


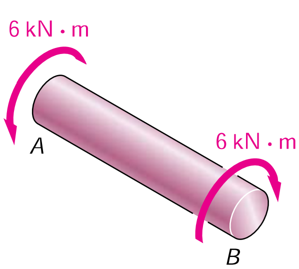
SAMPLE PROBLEM 3.1

Shaft *BC* is hollow with inner and outer diameters of 90 mm and 120 mm, respectively. Shafts *AB* and *CD* are solid and of diameter *d*. For the loading shown, determine (*a*) the maximum and minimum shearing stress in shaft *BC*, (*b*) the required diameter *d* of shafts *AB* and *CD* if the allowable shearing stress in these shafts is 65 MPa.









SOLUTION

Equations of Statics. Denoting by $\mathbf{T}_{AB}$ the torque in shaft *AB*, we pass a section through shaft *AB* and, for the free body shown, we write

$$\Sigma M_x = 0: \qquad (6 \text{ kN} \cdot \text{m}) - T_{AB} = 0 \qquad T_{AB} = 6 \text{ kN} \cdot \text{m}$$

We now pass a section through shaft *BC* and, for the free body shown, we have

$$\Sigma M_x = 0: \qquad (6 \text{ kN} \cdot \text{m}) + (14 \text{ kN} \cdot \text{m}) - T_{BC} = 0 \qquad T_{BC} = 20 \text{ kN} \cdot \text{m}$$

***a.* Shaft *BC*.** For this hollow shaft we have

$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}[(0.060)^4 - (0.045)^4] = 13.92 \times 10^{-6} \text{ m}^4$$

Maximum Shearing Stress. On the outer surface, we have

$$\tau_{max} = \tau_2 = \frac{T_{BC} c_2}{J} = \frac{(20 \text{ kN} \cdot \text{m})(0.060 \text{ m})}{13.92 \times 10^{-6} \text{ m}^4} \qquad \tau_{max} = 86.2 \text{ MPa} \blacktriangleleft$$

Minimum Shearing Stress. We write that the stresses are proportional to the distance from the axis of the shaft.

$$\frac{\tau_{min}}{\tau_{max}} = \frac{c_1}{c_2} \qquad \frac{\tau_{min}}{86.2 \text{ MPa}} = \frac{45 \text{ mm}}{60 \text{ mm}} \qquad \tau_{min} = 64.7 \text{ MPa} \blacktriangleleft$$

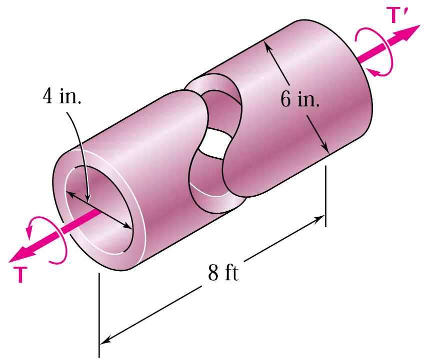
***b.* Shafts *AB* and *CD*.** We note that in both of these shafts the magnitude of the torque is $T = 6$ kN · m and $\tau_{all} = 65$ MPa. Denoting by *c* the radius of the shafts, we write

$$\tau = \frac{Tc}{J} \qquad 65 \text{ MPa} = \frac{(6 \text{ kN} \cdot \text{m})c}{\frac{\pi}{2} c^4}$$

$$c^3 = 58.8 \times 10^{-6} \text{ m}^3 \qquad c = 38.9 \times 10^{-3} \text{ m}$$

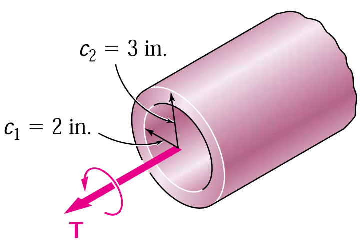
$$d = 2c = 2(38.9 \text{ mm}) \qquad d = 77.8 \text{ mm} \blacktriangleleft$$

SAMPLE PROBLEM 3.2



The preliminary design of a large shaft connecting a motor to a generator calls for the use of a hollow shaft with inner and outer diameters of 4 in. and 6 in., respectively. Knowing that the allowable shearing stress is 12 ksi, determine the maximum torque that can be transmitted (*a*) by the shaft as designed, (*b*) by a solid shaft of the same weight, (*c*) by a hollow shaft of the same weight and of 8-in. outer diameter.

SOLUTION

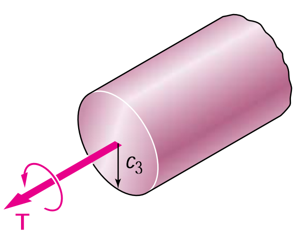


***a*. Hollow Shaft as Designed.** For the hollow shaft we have

$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}[(3 \text{ in.})^4 - (2 \text{ in.})^4] = 102.1 \text{ in}^4$$

Using Eq. (3.9), we write

$$\tau_{max} = \frac{Tc_2}{J} \qquad 12 \text{ ksi} = \frac{T(3 \text{ in.})}{102.1 \text{ in}^4} \qquad T = 408 \text{ kip} \cdot \text{in.} \blacktriangleleft$$



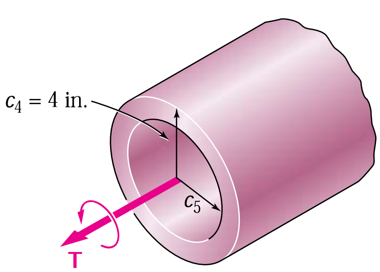
***b*. Solid Shaft of Equal Weight.** For the shaft as designed and this solid shaft to have the same weight and length, their cross-sectional areas must be equal.

$$A_{(a)} = A_{(b)}$$

$$\pi[(3 \text{ in.})^2 - (2 \text{ in.})^2] = \pi c_3^2 \qquad c_3 = 2.24 \text{ in.}$$

Since $\tau_{all} = 12$ ksi, we write

$$\tau_{max} = \frac{Tc_3}{J} \qquad 12 \text{ ksi} = \frac{T(2.24 \text{ in.})}{\frac{\pi}{2}(2.24 \text{ in.})^4} \qquad T = 211 \text{ kip} \cdot \text{in.} \blacktriangleleft$$



***c*. Hollow Shaft of 8-in. Diameter.** For equal weight, the cross-sectional areas again must be equal. We determine the inside diameter of the shaft by writing

$$A_{(a)} = A_{(c)}$$

$$\pi[(3 \text{ in.})^2 - (2 \text{ in.})^2] = \pi[(4 \text{ in.})^2 - c_5^2] \qquad c_5 = 3.317 \text{ in.}$$

For $c_5 = 3.317$ in. and $c_4 = 4$ in.,

$$J = \frac{\pi}{2}[(4 \text{ in.})^4 - (3.317 \text{ in.})^4] = 212 \text{ in}^4$$

With $\tau_{all} = 12$ ksi and $c_4 = 4$ in.,

$$\tau_{max} = \frac{Tc_4}{J} \qquad 12 \text{ ksi} = \frac{T(4 \text{ in.})}{212 \text{ in}^4} \qquad T = 636 \text{ kip} \cdot \text{in.} \blacktriangleleft$$