1.1. INTRODUCTION

The main objective of the study of the mechanics of materials is to provide the future engineer with the means of analyzing and designing various machines and load-bearing structures.

Both the analysis and the design of a given structure involve the determination of *stresses* and *deformations*. This first chapter is devoted to the concept of *stress*.

Section 1.2 is devoted to a short review of the basic methods of statics and to their application to the determination of the forces in the members of a simple structure consisting of pin-connected members. Section 1.3 will introduce you to the concept of *stress* in a member of a structure, and you will be shown how that stress can be determined from the *force* in the member. After a short discussion of engineering analysis and design (Sec. 1.4), you will consider successively the *normal stresses* in a member under axial loading (Sec. 1.5), the *shearing stresses* caused by the application of equal and opposite transverse forces (Sec. 1.6), and the *bearing stresses* created by bolts and pins in the members they connect (Sec. 1.7). These various concepts will be applied in Sec. 1.8 to the determination of the stresses in the members of the simple structure considered earlier in Sec. 1.2.

The first part of the chapter ends with a description of the method you should use in the solution of an assigned problem (Sec. 1.9) and with a discussion of the numerical accuracy appropriate in engineering calculations (Sec. 1.10).

In Sec. 1.11, where a two-force member under axial loading is considered again, it will be observed that the stresses on an *oblique* plane include both *normal* and *shearing* stresses, while in Sec. 1.12 you will note that *six components* are required to describe the state of stress at a point in a body under the most general loading conditions.

Finally, Sec. 1.13 will be devoted to the determination from test specimens of the *ultimate strength* of a given material and to the use of a *factor of safety* in the computation of the *allowable load* for a structural component made of that material.

1.2. A SHORT REVIEW OF THE METHODS OF STATICS

In this section you will review the basic methods of statics while determining the forces in the members of a simple structure.

Consider the structure shown in Fig. 1.1, which was designed to support a 30-kN load. It consists of a boom *AB* with a 30 × 50-mm rectangular cross section and of a rod *BC* with a 20-mm-diameter circular cross section. The boom and the rod are connected by a pin at *B* and are supported by pins and brackets at *A* and *C*, respectively. Our first step should be to draw a *free-body diagram* of the structure by detaching it from its supports at *A* and *C*, and showing the reactions that these supports exert on the structure (Fig. 1.2). Note that the sketch of the structure has been simplified by omitting all unnecessary details. Many of you may have recognized at this point that *AB* and *BC* are *two-force members*. For those of you who have not, we will pursue our analysis, ignoring that fact and assuming that the directions of the reactions at *A* and *C* are unknown. Each of these reactions, therefore, will

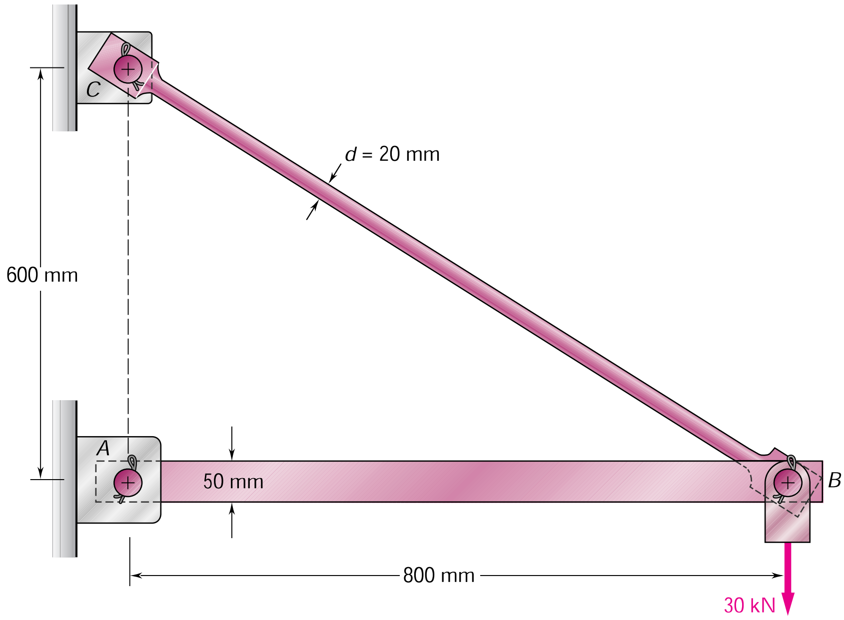

Fig. 1.1

be represented by two components, $\mathbf{A}_x$ and $\mathbf{A}_y$ at A, and $\mathbf{C}_x$ and $\mathbf{C}_y$ at C. We write the following equilibrium equations:

$$+\circlearrowleft \Sigma M_C = 0: \quad A_x(0.6 \text{ m}) - (30 \text{ kN})(0.8 \text{ m}) = 0$$

$$A_x = +40 \text{ kN} \tag{1.1}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0: \quad A_x + C_x = 0$$

$$C_x = -A_x \qquad C_x = -40 \text{ kN} \tag{1.2}$$

$$+\uparrow \Sigma F_y = 0: \quad A_y + C_y - 30 \text{ kN} = 0$$

$$A_y + C_y = +30 \text{ kN} \tag{1.3}$$

We have found two of the four unknowns, but cannot determine the other two from these equations, and no additional independent equation can be obtained from the free-body diagram of the structure. We must now dismember the structure. Considering the free-body diagram of the boom AB (Fig. 1.3), we write the following equilibrium equation:

$$+\circlearrowleft \Sigma M_B = 0: \quad -A_y(0.8 \text{ m}) = 0 \qquad A_y = 0 \tag{1.4}$$

Substituting for A_y from (1.4) into (1.3), we obtain $C_y = +30$ kN. Expressing the results obtained for the reactions at A and C in vector form, we have

$$\mathbf{A} = 40 \text{ kN} \rightarrow \qquad \mathbf{C}_x = 40 \text{ kN} \leftarrow, \mathbf{C}_y = 30 \text{ kN}\uparrow$$

We note that the reaction at A is directed along the axis of the boom AB and causes compression in that member. Observing that the components C_x and C_y of the reaction at C are, respectively, proportional to the horizontal and vertical components of the distance from B to C, we conclude that the reaction at C is equal to 50 kN, is directed along the axis of the rod BC, and causes tension in that member.

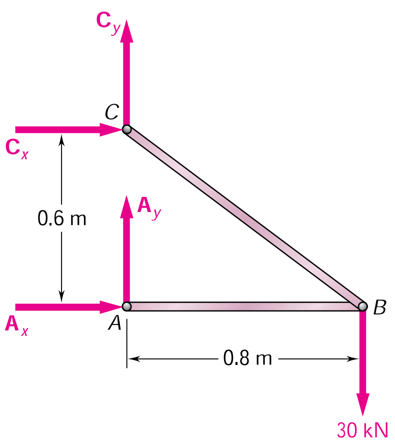

Fig. 1.2

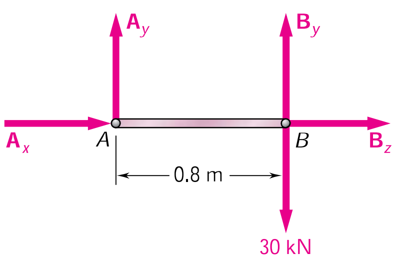

Fig. 1.3

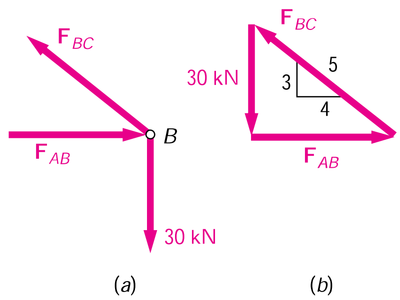

Fig. 1.4

These results could have been anticipated by recognizing that *AB* and *BC* are two-force members, i.e., members that are subjected to forces at only two points, these points being *A* and *B* for member *AB*, and *B* and *C* for member *BC*. Indeed, for a two-force member the lines of action of the resultants of the forces acting at each of the two points are equal and opposite and pass through both points. Using this property, we could have obtained a simpler solution by considering the free-body diagram of pin *B*. The forces on pin *B* are the forces $\mathbf{F}_{AB}$ and $\mathbf{F}_{BC}$ exerted, respectively, by members *AB* and *BC*, and the 30-kN load (Fig. 1.4*a*). We can express that pin *B* is in equilibrium by drawing the corresponding force triangle (Fig. 1.4*b*).

Since the force $\mathbf{F}_{BC}$ is directed along member *BC*, its slope is the same as that of *BC*, namely, 3/4. We can, therefore, write the proportion

$$\frac{F_{AB}}{4} = \frac{F_{BC}}{5} = \frac{30 \text{ kN}}{3}$$

from which we obtain

$$F_{AB} = 40 \text{ kN} \qquad F_{BC} = 50 \text{ kN}$$

The forces $\mathbf{F}'_{AB}$ and $\mathbf{F}'_{BC}$ exerted by pin *B*, respectively, on boom *AB* and rod *BC* are equal and opposite to $\mathbf{F}_{AB}$ and $\mathbf{F}_{BC}$ (Fig. 1.5).

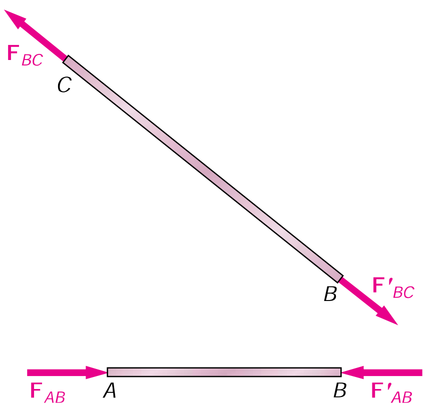

Fig. 1.5

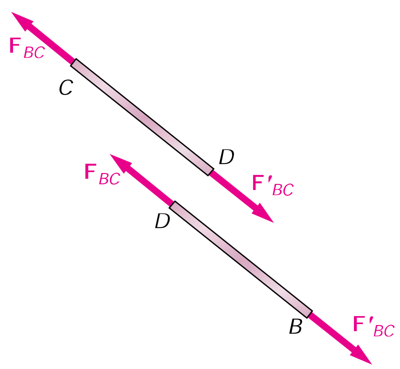

Fig. 1.6

Knowing the forces at the ends of each of the members, we can now determine the internal forces in these members. Passing a section at some arbitrary point *D* of rod *BC*, we obtain two portions *BD* and *CD* (Fig. 1.6). Since 50-kN forces must be applied at *D* to both portions of the rod to keep them in equilibrium, we conclude that an internal force of 50 kN is produced in rod *BC* when a 30-kN load is applied at *B*. We further check from the directions of the forces $\mathbf{F}_{BC}$ and $\mathbf{F}'_{BC}$ in Fig. 1.6 that the rod is in tension. A similar procedure would enable us to determine that the internal force in boom *AB* is 40 kN and that the boom is in compression.

1.3. STRESSES IN THE MEMBERS OF A STRUCTURE

While the results obtained in the preceding section represent a first and necessary step in the analysis of the given structure, they do not tell us whether the given load can be safely supported. Whether rod BC, for example, will break or not under this loading depends not only upon the value found for the internal force F_{BC}, but also upon the cross-sectional area of the rod and the material of which the rod is made. Indeed, the internal force F_{BC} actually represents the resultant of elementary forces distributed over the entire area A of the cross section (Fig. 1.7) and the average intensity of these distributed forces is equal to the force per unit area, F_{BC}/A, in the section. Whether or not the rod will break under the given loading clearly depends upon the ability of the material to withstand the corresponding value F_{BC}/A of the intensity of the distributed internal forces. It thus depends upon the force F_{BC}, the cross-sectional area A, and the material of the rod.

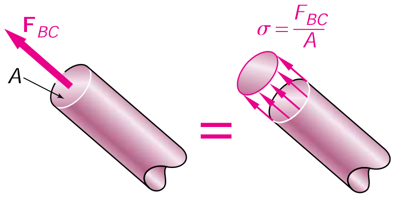

Fig. 1.7

The force per unit area, or intensity of the forces distributed over a given section, is called the *stress* on that section and is denoted by the Greek letter σ (sigma). The stress in a member of cross-sectional area A subjected to an axial load $\mathbf{P}$ (Fig. 1.8) is therefore obtained by dividing the magnitude P of the load by the area A:

$$\sigma = \frac{P}{A} \tag{1.5}$$

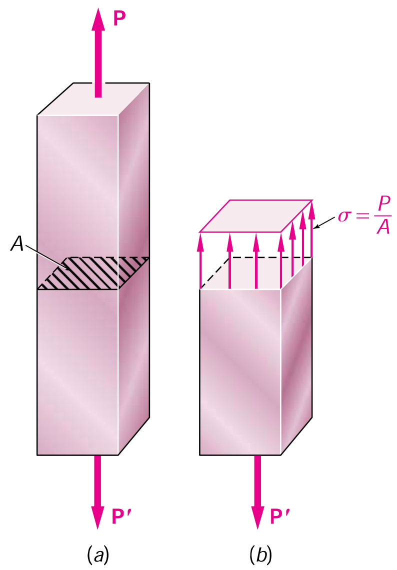

Fig. 1.8

A positive sign will be used to indicate a tensile stress (member in tension) and a negative sign to indicate a compressive stress (member in compression).

Since SI metric units are used in this discussion, with P expressed in newtons (N) and A in square meters (m^2), the stress σ will be expressed in N/m^2. This unit is called a *pascal* (Pa). However, one finds that the pascal is an exceedingly small quantity and that, in practice, multiples of this unit must be used, namely, the kilopascal (kPa), the megapascal (MPa), and the gigapascal (GPa). We have

$$1\ \text{kPa} = 10^3\ \text{Pa} = 10^3\ \text{N/m}^2$$

$$1\ \text{MPa} = 10^6\ \text{Pa} = 10^6\ \text{N/m}^2$$

$$1\ \text{GPa} = 10^9\ \text{Pa} = 10^9\ \text{N/m}^2$$

When U.S. customary units are used, the force P is usually expressed in pounds (lb) or kilopounds (kip), and the cross-sectional area A in square inches (in^2). The stress σ will then be expressed in pounds per square inch (psi) or kilopounds per square inch (ksi).†

†The principal SI and U.S. customary units used in mechanics are listed in tables inside the front cover of this book. From the table on the right-hand side, we note that 1 psi is approximately equal to 7 kPa, and 1 ksi approximately equal to 7 MPa.

1.4. ANALYSIS AND DESIGN

Considering again the structure of Fig. 1.1, let us assume that rod *BC* is made of a steel with a maximum allowable stress $\sigma_{all} = 165$ MPa. Can rod *BC* safely support the load to which it will be subjected? The magnitude of the force F_{BC} in the rod was found earlier to be 50 kN. Recalling that the diameter of the rod is 20 mm, we use Eq. (1.5) to determine the stress created in the rod by the given loading. We have

$$P = F_{BC} = +50 \text{ kN} = +50 \times 10^3 \text{ N}$$
$$A = \pi r^2 = \pi\left(\frac{20 \text{ mm}}{2}\right)^2 = \pi(10 \times 10^{-3} \text{ m})^2 = 314 \times 10^{-6} \text{ m}^2$$
$$\sigma = \frac{P}{A} = \frac{+50 \times 10^3 \text{ N}}{314 \times 10^{-6} \text{ m}^2} = +159 \times 10^6 \text{ Pa} = +159 \text{ MPa}$$

Since the value obtained for σ is smaller than the value σ_{all} of the allowable stress in the steel used, we conclude that rod *BC* can safely support the load to which it will be subjected. To be complete, our analysis of the given structure should also include the determination of the compressive stress in boom *AB*, as well as an investigation of the stresses produced in the pins and their bearings. This will be discussed later in this chapter. We should also determine whether the deformations produced by the given loading are acceptable. The study of deformations under axial loads will be the subject of Chap. 2. An additional consideration required for members in compression involves the *stability* of the member, i.e., its ability to support a given load without experiencing a sudden change in configuration. This will be discussed in Chap. 10.

The engineer's role is not limited to the analysis of existing structures and machines subjected to given loading conditions. Of even greater importance to the engineer is the *design* of new structures and machines, that is, the selection of appropriate components to perform a given task. As an example of design, let us return to the structure of Fig. 1.1, and assume that aluminum with an allowable stress $\sigma_{all} = 100$ MPa is to be used. Since the force in rod *BC* will still be $P = F_{BC} = 50$ kN under the given loading, we must have, from Eq. (1.5),

$$\sigma_{all} = \frac{P}{A} \qquad A = \frac{P}{\sigma_{all}} = \frac{50 \times 10^3 \text{ N}}{100 \times 10^6 \text{ Pa}} = 500 \times 10^{-6} \text{ m}^2$$

and, since $A = \pi r^2$,

$$r = \sqrt{\frac{A}{\pi}} = \sqrt{\frac{500 \times 10^{-6} \text{ m}^2}{\pi}} = 12.62 \times 10^{-3} \text{ m} = 12.62 \text{ mm}$$

$$d = 2r = 25.2 \text{ mm}$$

We conclude that an aluminum rod 26 mm or more in diameter will be adequate.

1.5. AXIAL LOADING; NORMAL STRESS

As we have already indicated, rod BC of the example considered in the preceding section is a two-force member and, therefore, the forces $\mathbf{F}_{BC}$ and $\mathbf{F}'_{BC}$ acting on its ends B and C (Fig. 1.5) are directed along the axis of the rod. We say that the rod is under *axial loading*. An actual example of structural members under axial loading is provided by the members of the bridge truss shown in Fig. 1.9.

Fig. 1.9 This bridge truss consists of two-force members that may be in tension or in compression.

Returning to rod BC of Fig. 1.5, we recall that the section we passed through the rod to determine the internal force in the rod and the corresponding stress was perpendicular to the axis of the rod; the internal force was therefore normal to the plane of the section (Fig. 1.7) and the corresponding stress is described as a *normal stress*. Thus, formula (1.5) gives us the *normal stress in a member under axial loading*:

$$\sigma = \frac{P}{A} \tag{1.5}$$

We should also note that, in formula (1.5), σ is obtained by dividing the magnitude P of the resultant of the internal forces distributed over the cross section by the area A of the cross section; it represents, therefore, the *average value* of the stress over the cross section, rather than the stress at a specific point of the cross section.

To define the stress at a given point Q of the cross section, we should consider a small area ΔA (Fig. 1.10). Dividing the magnitude of $\Delta\mathbf{F}$ by ΔA, we obtain the average value of the stress over ΔA. Letting ΔA approach zero, we obtain the stress at point Q:

$$\boxed{\sigma = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A}} \tag{1.6}$$

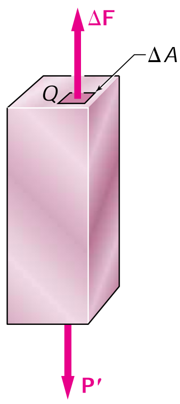

Fig. 1.10

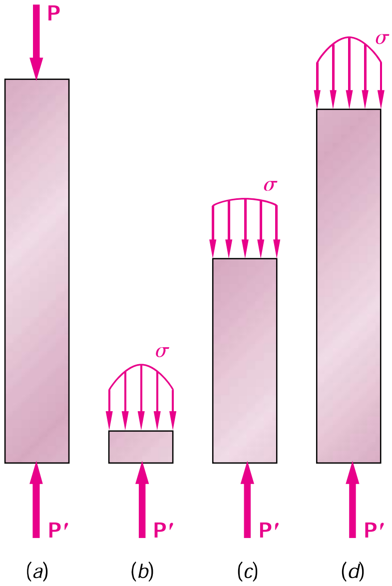

Fig. 1.11

In general, the value obtained for the stress $\mathbf{s}$ at a given point Q of the section is different from the value of the average stress given by formula (1.5), and $\mathbf{s}$ is found to vary across the section. In a slender rod subjected to equal and opposite concentrated loads **P** and **P′** (Fig. 1.11*a*), this variation is small in a section away from the points of application of the concentrated loads (Fig. 1.11*c*), but it is quite noticeable in the neighborhood of these points (Fig. 1.11*b* and *d*).

It follows from Eq. (1.6) that the magnitude of the resultant of the distributed internal forces is

$$\int dF = \int_A \mathbf{s}\, dA$$

But the conditions of equilibrium of each of the portions of rod shown in Fig. 1.11 require that this magnitude be equal to the magnitude P of the concentrated loads. We have, therefore,

$$P = \int dF = \int_A \mathbf{s}\, dA \tag{1.7}$$

which means that the volume under each of the stress surfaces in Fig. 1.11 must be equal to the magnitude P of the loads. This, however, is the only information that we can derive from our knowledge of statics, regarding the distribution of normal stresses in the various sections of the rod. The actual distribution of stresses in any given section is *statically indeterminate*. To learn more about this distribution, it is necessary to consider the deformations resulting from the particular mode of application of the loads at the ends of the rod. This will be discussed further in Chap. 2.

In practice, it will be assumed that the distribution of normal stresses in an axially loaded member is uniform, except in the immediate vicinity of the points of application of the loads. The value $\mathbf{s}$ of the stress is then equal to $\mathbf{s}_{\text{ave}}$ and can be obtained from formula (1.5). However, we should realize that, when we assume a uniform distribution of stresses in the section, i.e., when we assume that the internal forces are uniformly distributed across the section, it follows from elementary statics† that the resultant **P** of the internal forces must be applied at the centroid C of the section (Fig. 1.12). This means that *a uniform distribution of stress is possible only if the line of action of the concentrated loads* **P** *and* **P′** *passes through the centroid of the section considered* (Fig. 1.13). This type of loading is called *centric loading* and will be assumed to take place in all straight two-force members found in trusses and pin-connected structures, such as the one considered in Fig. 1.1.

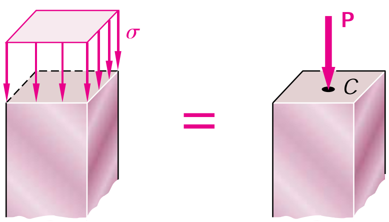

Fig. 1.12

†See Ferdinand P. Beer and E. Russell Johnston, Jr., *Mechanics for Engineers,* 4th ed., McGraw-Hill, New York, 1987, or *Vector Mechanics for Engineers,* 6th ed., McGraw-Hill, New York, 1996, secs. 5.2 and 5.3.