## Chapter 4

Q1/ Answer the following branches:
A) Define the Brewster's angle.
B) Calculate the Brewster angle for a wave impinging on ground having a permittivity of $\varepsilon_{r}=5$.

## Solution:

A) Brewster's angle (also known as the polarization angle) is an angle of incidence at which the wave with a particular polarization is perfectly transmitted through a dielectric surface, with no reflection (reflection coefficient is equal to zero).
B)


$$
\begin{array}{r}
\sin \theta_{B}=\frac{\sqrt{\varepsilon_{\mathrm{r}}-1}}{\sqrt{\varepsilon_{\mathrm{r}}^{2}-1}}=\frac{\sqrt{5-1}}{\sqrt{25-1}}=\frac{\sqrt{4}}{\sqrt{24}}=\frac{2}{4.89}=0.4 \\
\theta_{B}=\sin ^{-1}(0.4)=23.5^{0}
\end{array}
$$

Q2/A mobile is located 4 km away from a base station and uses a vertical $\lambda / 4$ monopole antenna with a gain of 2.75 dB to receive cellular radio signals. The E-field at 1 km from the transmitter is measured to be $2 \times 10^{-3} \mathrm{~V} / \mathrm{m}$. The carrier frequency used for this system is 900 MHz .
a) Find the length and the effective aperture $\mathrm{A}_{e}$ of the receiving antenna.
b) Find the received power at the mobile using the 2-ray ground reflection model assuming the height of the transmitting antenna is 60 m and the receiving antenna is 2 m above ground.

## Solution:

$$
\begin{aligned}
& \mathrm{d}=4 \mathrm{~km} \\
& \lambda=\frac{c}{f}=\frac{3 \times 10^{8}}{900 \times 10^{6}}=0.333 \mathrm{~m}
\end{aligned}
$$

Length of the antenna $=\lambda / 4=0.333 / 4=0.0833 \mathrm{~m}=8.33 \mathrm{~cm}$.
$\mathrm{G}=10^{(2.75 / 10)}=1.884$

$$
\begin{aligned}
& \begin{aligned}
A_{e}=\frac{G \lambda^{2}}{4 \pi} & =\frac{1.884(0.333)^{2}}{4 \pi}=0.0166 \mathrm{~m}^{2}=166 \mathrm{~cm}^{2} \\
E_{\text {TOT }}(d) & \approx \frac{2 E_{0} d_{0}}{d} \frac{2 \pi h_{t} h_{r}}{\lambda d} \quad V / \mathrm{m} \\
& =\frac{2 \times 2 \times 10^{-3} \times 1 \times 10^{3}}{4 \times 10^{3}}\left[\frac{2 \pi \times 60 \times 2}{0.333 \times 4 \times 10^{3}}\right]=5.66 \times 10^{-4} \mathrm{~V} / \mathrm{m}
\end{aligned}
\end{aligned}
$$

The received power at a distance $d$ can be obtained using

$$
\begin{aligned}
& P_{r}(d)=\frac{|E|^{2}}{120 \pi} A_{e}=\frac{\left(5.66 \times 10^{-4}\right)^{2}}{377}(0.0166) \\
& =1.41 \times 10^{-11} \mathrm{~W}=-108.5 \mathrm{dBW} \text { or }-78.5 \mathrm{dBm}
\end{aligned}
$$

Q3/ If 65 watts transmitter power is applied to a unity gain antenna with a 900 MHz carrier frequency. Assume unity gain for the receiver antenna.
A) Express the transmitter power in units of $d B m$ and $d B W$.
B) Find the received power in $d B m$ at a free space distance of 100 m from the antenna.
C) What is $\operatorname{Pr}(12 \mathrm{~km})$ ?
D) Find the effective aperture $A_{e}$ of the transmitter antenna.

## Solution:

A)

$$
\begin{aligned}
& P_{t}=65 \mathrm{~W}=65 \times 10^{3} \mathrm{~mW} \\
& P_{t}(d B m)=10 \log \left[P_{t}(\mathrm{~mW})\right]=10 \log \left[65 \times 10^{3}\right]=48 \mathrm{dBm} \\
& P_{t}(d B m)=10 \log \left[P_{t}(\mathrm{~W})\right]=10 \log [65]=18 \mathrm{dBW}
\end{aligned}
$$

B)

$$
\begin{array}{ll}
\lambda=\frac{c}{f}=\frac{3 \times 10^{8}}{900 \times 10^{6}}=0.33 m \\
& \\
\qquad P_{r}=\frac{P_{t} G_{t} G_{r}}{L}\left(\frac{\lambda}{4 \pi d}\right)^{2}
\end{array}
$$

$$
\begin{aligned}
& P_{r}=\frac{65 \times 1 \times 1}{1}\left(\frac{0.33}{4 \pi \times 100}\right)^{2}=4.48 \times 10^{-6} \mathrm{~W}=4.48 \times 10^{-3} \mathrm{~mW} \\
& P_{r(d B m)}=10 \log \left(P_{r}(\mathrm{~mW})\right)=10 \mathrm{log}\left(4.48 \times 10^{-3}\right)=-23.4 \mathrm{dBm}
\end{aligned}
$$

c)

$$
\begin{aligned}
& P_{r}(12 \mathrm{~km})=P_{r(d B m)}(100)+20 \log \left(\frac{100}{12000}\right)=-23.4-41.58=-65 \mathrm{dBm} \\
& A_{e}=\frac{G \lambda^{2}}{4 \pi}=\frac{1(0.33)^{2}}{4 \pi}=0.00867 \mathrm{~m}^{2}=86.7 \mathrm{~cm}^{2}
\end{aligned}
$$

c)

Q4/ If 60 wattsis applied to a 1.5 gain antenna with a 0.9 GHz carrier frequency, find the received power in $d B m$ at a free space distance of 150 m from the antenna. What is $\operatorname{Pr}$ ( 9 $\mathrm{km})$ ? Assume gain=2 for the receiver antenna.

## Solution:

$\lambda=\frac{C}{f}=\frac{3 \times 10^{8}}{0.9 \times 10^{9}}=0.333 \mathrm{~m}$
$P_{r}(d)=\frac{P_{t} G_{t} G_{r}}{L}\left(\frac{\lambda}{4 \pi d}\right)^{2}$
$P_{r}(150)=\frac{60 \times 1.5 \times 2}{1}\left(\frac{0.333}{4 \pi 150}\right)^{2}=5.61 \times 10^{-3} \mathrm{~mW}$
$=10 \log \left(5.61 \times 10^{-3}\right)=-22.5 \mathrm{dBm}$
$P_{r}(9 \mathrm{~km})=P_{r(d B m)}(150)+20 \log \left(\frac{150}{9000}\right)=-22.5 \mathrm{dBm}-35.56 \mathrm{~dB}=-58.06 \mathrm{dBm}$

Q5/ Find the far-field distance for an antenna with maximum dimension of $3 m$ and operating frequency of 950 MHz .

## Solution:

$$
\begin{aligned}
& \lambda=\frac{c}{f}=\frac{3 \times 10^{8}}{950}=0.315 \mathrm{~m} \\
& d_{f}=\frac{2 \times D^{2}}{\lambda}=\frac{2 \times 3^{2}}{0.315}=57.14 \mathrm{~m}
\end{aligned}
$$

Q6/ Calculate the link budget from one side only, when the 39920 km GEO satellite communication link with a transmitter power of 2 kW that is applied to the transmitter antenna gain of 40 dBi . The satellite receiver is connected to an antenna with 35 dBi gain and a receive sensitivity of -100 dBm . The cables in both systems are short, with a loss of $1 d B$ at each side at the 12 GHz frequency of operation.

## Solution:

Tx Power $=10 \log \left(2000 \times 10^{3} \mathrm{~mW}\right)=63 \mathrm{dBm}$

$$
\begin{gathered}
\text { Total gain }=\text { Tx Power }+ \text { Tx Antenna Gain }- \text { Cable } \operatorname{loss}(T x) \\
\\
+ \text { Rx Antenna gain }- \text { Cable loss }(R x) \\
=63 d B+40 d B i-1 d B+35 d B i-1 d B=136 d B
\end{gathered}
$$

$$
\lambda=\frac{c}{f}=\frac{3 \times 10^{8}}{12 \times 10^{9}}=0.025 \mathrm{~m}
$$

$$
\operatorname{Path} \operatorname{Loss}(P L)=-20 \log \left(\frac{\lambda}{4 \pi d}\right)=-20 \log \left(\frac{0.025}{4 \pi \times 39920 \times 10^{3}}\right)=206 d B
$$

$$
\text { Expected } P_{r}=\text { Total gain }- \text { Free space loss }
$$

$$
=136 d B-206 d B=-70 d B m
$$

$$
\text { Link Margin }=\text { Expected } P_{r}-\text { Sensitivity of Client }
$$

$$
=-70-(-100)=30 d B
$$

Q7/ A- Calculate the link budget ( only from Tx to Rx ) for the 10 km transmitting distance of $20 w$ power that is applied to the transmitter with antenna gain of 20 dBi . The receiver is connected to an antenna with 25 dBi gain and a receive sensitivity of -80 dBm . The cables in both systems are short, with a loss of $1 d B$ at each side at the 13 GHz frequency of operation.

B- Label on the following figure the calculated and given parameter on the bellow figure.


## Solution:

A)

Tx Power $=10 \log \left(20 \times 10^{3} \mathrm{~mW}\right)=43 \mathrm{dBm}$
Total gain $=T x$ Power $+T x$ Antenna Gain - Cable loss $(T x)$
$+R x$ Antenna gain - Cable loss $(R x)$

$$
=43 d B+20 d B i-1 d B+25 d B i-1 d B=86 d B
$$

$$
\lambda=\frac{c}{f}=\frac{3 \times 10^{8}}{13 \times 10^{9}}=0.023 \mathrm{~m}
$$

$$
\text { Path Loss }(P L)=-20 \log \left(\frac{\lambda}{4 \pi d}\right)=-20 \log \left(\frac{0.023}{4 \pi \times 10000}\right)=134.7 d B
$$

Expected $P_{r}=$ Total gain - Free space loss
$=86 d B-134.7 d B=-48.7 \mathrm{dBm}$
Link Margin $=$ Expected $P_{r}-$ Sensitivity of Client

$$
=-48.7-(-80)=31.3 \mathrm{~dB}
$$

B-


Q8/ Calculate the link budget for the 10 km transmitting distance of $20 w$ power that is applied to the transmitter with antenna gain of 20 dBi and transmitter sensitivity of -85 $d B m$. The receiver $(\mathrm{Rx}=18 \mathrm{w})$ is connected to an antenna with $25 d B i$ gain and a receive sensitivity of -80 dBm . The cables in both systems are short, with a loss of $1 d B$ at each side at the 13 GHz frequency of operation.
A) from $T x$ to $R x$

Tx Power $=10 \log \left(20 \times 10^{3} \mathrm{~mW}\right)=43 \mathrm{dBm}$
Total gain $=T x$ Power + Tx Antenna Gain - Cable loss $(T x)$

$$
\begin{gathered}
\quad+\text { Rx Antenna gain - Cable loss }(R x) \\
=43 d B+20 d B i-1 d B+25 d B i-1 d B=86 d B
\end{gathered}
$$

$$
\lambda=\frac{c}{f}=\frac{3 \times 10^{8}}{13 \times 10^{9}}=0.023 \mathrm{~m}
$$

$$
\text { Path Loss }(P L)=-20 \log \left(\frac{\lambda}{4 \pi d}\right)=-20 \log \left(\frac{0.023}{4 \pi \times 10000}\right)=134.7 d B
$$

Expected $P_{r}=$ Total gain - Free space loss

$$
\begin{aligned}
= & 86 \mathrm{~dB}-134.7 \mathrm{~dB}=-48.7 \mathrm{dBm} \\
& \text { Link Margin }=\text { Expected } P_{r}-\text { Sensitivity of Client }
\end{aligned}
$$

$$
=-48.7-(-80)=31.3 d B
$$

## b) from Rx TO Tx

Total gain $=R x$ Power $+R x$ Antenna Gain - Cable loss $(R x)$

$$
+ \text { Tx Antenna gain - Cable loss }(T x)
$$

$$
=42.55 d B+25 d B i-1 d B+20 d B i-1 d B=85.55 d B
$$

Expected $P_{r}=$ Total gain - Free space loss

$$
=85.55 d B-134.7 d B=-49.14 \mathrm{dBm}
$$

Link Margin $=$ Expected $P_{r}-$ Sensitivity of Client

$$
=-49.14-(-85)=35.85 d B
$$

Q9\ Find the far-field distance for an antenna with maximum dimension of $3 m$ and operating frequency of 950 MHz .

## Solution:

$D=3 \mathrm{~m}, f=950 \mathrm{MHz}$,
$\lambda=\frac{c}{f}=\frac{3 \times 10^{8}}{950}=0.315 \mathrm{~m}$
$d_{f}=\frac{2 \times D^{2}}{\lambda}=\frac{2 \times 3^{2}}{0.315}=57.14 \mathrm{~m}$

Q10 Determine the isotropic free space loss at 5 GHz for 5 km path distance between the transmitter and the receiver.

## Solution:

$$
\begin{aligned}
& \lambda=\frac{c}{f}=\frac{3 \times 10^{8}}{5 \times 10^{9}}=0.06 \mathrm{~m} \\
& P L(d B)=-20 \log \left(\frac{0.06}{4 \pi \times 5 \times 10^{9}}\right)=60.3 \mathrm{~dB}
\end{aligned}
$$

