EE 433 | HW #2 | Anderson Master

3.4 Given: 20 MHz total spectrum is allocated for a duplex wireless system, each channel 25 kHz Bandwidth

Find: a) The number of duplex channels
b) Total number of Channels per cell site for $N=4$ reuse

a) Frequency Division Duplexing

Each Duplex channel will be 2 simplex channels, or 50 kHz

$$S = \frac{20\text{ MHz}}{50\text{ kHz}} = 400 \text{ Duplex Channels}$$

b) $S = kN$

$$k = \frac{S}{N} = \frac{400 \text{ Total Channels}}{4} = 100 \text{ channels/cell}$$

a) $S = 400$ channels
b) $k = 100$ channels/cell

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3.10 Given: 24 MHz Bandwidth allocated to a FDD system that uses 2 30kHz simplex channels.
Each user generates 0.1 Erlangs traffic, Erlang B

Find:
a) # Channels per cell with $N=4$ reuse
b) Each cell offers 90% capacity, find max # of users
c) Blocking probability when max # users in pool
d) Each cell uses 120° sectors, what is new blocking probability
e) Each cell covers 5km², how many users in 50km x 50 km area? omni
f) Each cell covers 5km², how many users in 50km x 50km? 120° sectors

a) Total # full duplex channels

$$S = \frac{24\text{ MHz}}{2(30\text{ kHz})} = 400 \text{ channels}$$

2 freqs. required FDD

$$K = \frac{S}{N} = \frac{400 \text{ channels}}{4 \text{ cells}} = 100 \text{ channels/cell}$$

b) Perfect scheduling occurs when $A = C \Rightarrow$ All channels are always occupied, no blocking.

90% of perfect scheduling means $A = 0.9C$

In this case, with 100 channels, perfect scheduling implies $C = A = 100$

90% perfect scheduling implies $A = 0.9(100) = 90$ Erlangs Traffic

Now, since I'm only providing 90 Erlangs capacity:

$A = U A_u$

$90 \text{ Erlangs} = U(0.1 \text{ Erlangs/user})$

$900 \text{ users} = U$

c) Using Erlang B chart

Traffic intensity = 90 Erlangs
Trunked Channels = 100

$GOS = 2.7\% \approx 3\%$ [erlangb.m(100,90)] Matlab

d) 120° sector antennas means $k_{sector} = \frac{100 \text{ channels/cell}}{3 \text{ sectors/cell}} = 33$ channels/sector

$GOS = 2.7\%$, $A \approx 26$ Erlangs/sector (Erlang B chart)

$A = U_s A_u$ per sector

26 Erlangs = U_s(0.1 Erlangs/user)

$260 = U_s$

Total $U_{cell} = 3U_s = 3(260) = 780$ users

e) Total Area: 50km × 50km = 2500 km²

Total Cells = $\frac{\text{Total Area}}{\text{Area/cell}} = \frac{2500 \text{ km}^2}{5 \text{ km}^2} = 500$ cells

Total Users = Total cells × Users/cell
= (500 cells)(900 users/cell)
= 450,000 users

f) Total cells still 500

Total Users = (500 cells)(780 users/sectored cell)
= 390,000 users.

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3.11 Given: $N=7$ system with $GOS = 1\%$, $H = 2$ min
$C = 57$ channels, $\lambda = 1$ call/hour

Find: Capacity loss when going from omni to sectored antennas.

First, find capacity of the omni system

$A_u = \lambda H = (1 \text{ call/hour})(0.033 \text{ hours/call})$
$A_u = 0.033$ Erlangs

$C = 57$ channels
$GOS = 0.01$
$A = 44$ Erlangs $\Leftarrow$ Erlang B chart

$A = U A_u$
44 Erlangs $= U(0.033 \text{ Erlangs/user})$
$U = 1333$ users/cell

$A = 44$ Erlangs omni
$U = 1333$ omni

Next, look at capacity of sectored system

60° sectors means 6 sectors/cell

Channels per sector $C_s = \frac{57}{6} = 9.5$ ⇏ Call it 9 and reserve a few for inter-cell handoffs

$C_s = 9$ chanels
$GOS = 0.01$
$A = 3.8$ Erlangs/sector

$A_{cell} = 6A = 6(3.8 \text{ Erlangs/sector}) = 22.8$ Erlangs/cell

$A = U A_u$
22.8 Erlangs $= U(0.033 \text{ Erlangs/user})$
690 users $= U$

$A = 22.8$ Erlangs sectored
$U = 690$ sectored

Loss $= A_{omni} - A_{sector} = 21.2$ Erlangs
Loss $= U_{omni} - U_{sector} = 643$ Users

3.15: Given: Exercises in trunking theory

Find: a) Maximum capacity (total & per channel) when providing a 2% GOS with 4, 20, and 40 channels.

b) How many users can be supported with 40 channels $GOS = 2\%$, $H = 105$ sec, $\lambda = 1$ call/hr

c) Using traffic intensity from part (a), find GOS in Erlang C system for delay $t > 20$ sec $H = 105$ sec, $C = 4, 20, 40$

d) Does (c) perform better than (a)?

a) $C = 4$
$GOS = 2\%$
Find A

Using chart
$A = 1.1$ Erlangs
$A_c = \frac{1.1}{4} = 0.275$ Erlangs

$C = 20$
$GOS = 2\%$
Find A

Using chart
$A = 13.2$
$A_c = 0.66$ Erlangs

$C = 40$
$GOS = 2\%$
Find A

Using Chart
$A = 31$
$A_c = 0.775$ Erlangs

b) $A_u = \lambda H$
$A_u = (105 \text{ sec})(1 \text{ call}/3600 \text{ sec})$
$A_u = 0.0292$ Erlangs

$A = U A_u$
$31 \text{ Erlangs} = U (0.0292 \text{ Erlangs})$
$1062 \text{ users} = U$

$\boxed{U = 1062 \text{ users}}$

c) $C = 4$
$A = 1.1$ Erlangs
Find GOS

Using Chart
$P_r[\text{delay} > 0] = 2.8\%$
[erlangc(4, 1.1)]

$$P_r[t > 20\text{sec}] = P_r[t > 0]\, e^{\frac{-(C-A)t}{H}}$$

$$= (0.028)\, e^{-\frac{(4 \text{ channels} - 1.1 \text{ Erlangs})(20 \text{ sec})}{105 \text{ sec}}}$$

$$= 0.0161$$

$\underline{\underline{P_r[t > 20 \text{sec}] = 1.61\%}}$

c (cont.)

$C = 20$ channels
$A = 13.2$ Erlangs

Using Chart
$Pr[t>0] = 5.7\%$

$$Pr[t > 20\,sec] = Pr[t>0]\, e^{-\frac{(C-A)t}{H}}$$

$$= (0.057)\, e^{-\frac{(20\text{ channels} - 13.2\text{ Erlangs})(20\,sec)}{105\,sec}}$$

$$= 0.0156$$

$Pr[t>20\,sec] = 1.56\%$

$C = 40$ channels
$A = 31$ Erlangs

Using Chart
$Pr[t>0] = 8.3\%$

$$Pr[t>20\,sec] = Pr[t>0]\, e^{-\frac{(C-A)t}{H}}$$

$$= (0.083)\, e^{-\frac{(40\text{ channels} - 31\text{ Erlangs})(20\,sec)}{105\,sec}}$$

$$= 0.0149$$

$Pr[t>20\,sec] = 1.49\%$

d) The blocked calls delayed system does perform better than the blocked calls cleared system.

Molisch 25.17 #1

Given: Analog Cellular system with 250 Duplex Channels
$Q = D/R = 7$
Cell Radius $= 2km$
$\lambda = 1$ call per Hour
$H = 2$ minutes
Erlang B, GoS = 3%

Find: a) Max Number Subscribers per cell
Network capacity in Erlangs/sq km
b) System changes to 125 Duplex Channels, $Q = D/R = 4$
Max # subscribers
Network capacity in Erlangs/sq km
c) Decrease cell Radius to 1 km
How Much is capacity increased
How Many more BTS Required

a) $Q = \sqrt{3N} \Rightarrow N = 16.33 \rightarrow$ Next Allowed Value is 19

Channels Per Cell $C = \frac{250}{19} = 13.2 = 13$

$A_u = \lambda H = (1\,\text{call}/\text{Hr})(2/60\ \text{Hrs/call}) = 0.0333$ Erlangs/user

From Erlang B chart: $A = 8$ Erlangs

Thus $A = UA_u \Rightarrow U = \frac{A}{A_u} = \frac{8\ \text{Erlangs}}{0.0333\ \text{Erlangs/user}} = 240$ users/cell

Net Capacity: $A_{net} = \frac{8\ \text{Erlangs}}{\pi(2km)^2} = 0.637$ Erlangs/sq.km

$$\boxed{\begin{aligned} &a)\ U = 240\ \text{users/cell} \\ &U_{net} = 0.637\ \text{Erlangs/sq km}\end{aligned}}$$ $\rightarrow$ 19 users/sq.km

b) $Q = \sqrt{3N} \doteq 4 \Rightarrow N = 5.33$, Next allowed value is 7

Channels per cell $C = \frac{125}{7} = 17.8 = 17$

$A_u = 0.0333$ Erlangs/user

From Erlang B chart: $A = 11.33$ Erlangs

Thus: $U = \frac{A}{A_u} = \frac{11.33 \text{ Erlangs}}{0.0333 \text{ Erlangs/user}} = 340$ users/cell

Net Capacity: $U_{Net} = \frac{11.33 \text{ Erlangs}}{\pi (2\text{km})^2} = 0.902$ Erlangs/sq.km

$$\boxed{\begin{array}{l} \text{b)}\ U = 340 \text{ users/cell} \\ U_{Net} = 0.902 \text{ Erlangs/sq km} \end{array}}$$

$\rightarrow$ 27 users/sq.km.

c) System in B covers:

$$(7 \text{ cells/cluster})(\pi(2\text{km})^2/\text{cell}) = 87.96 \text{ km}^2/\text{cluster}$$

System in C covers

$$(7 \text{ cells/cluster})(\pi(1\text{km})^2/\text{cell}) = 21.99 \text{ km}^2/\text{cluster}$$

$$U_{Net} = \frac{11.33 \text{ Erlangs}}{\pi(1\text{km})^2} = 3.61 \text{ Erlangs/sq.km}$$

Number Extra Clusters: $\frac{87.96 \text{ km}^2}{21.99 \text{ km}^2} = 4 \times$ Number of <u>Clusters</u>

With $N=7$, Need 28 Total BTS

$$\boxed{\begin{array}{l} \text{c)}\ U_{Net} = 3.61 \text{ Erlangs/sq.km} \\ \text{Need 21 New BTS} \end{array}}$$

$\rightarrow$ 108 users/sq km

25.17 #2 Given: Erlang-B system with
GOS = 5%
$U = 120$
10% Activity Level

Find: a) # channels Required For Single BTS
b) # channels Required For 3-sector BTS

a) Activity Level = 10%, users "on" 10% of time

Assume 1 call/Hour, $H = 0.1 \times 60\,\text{min} = 6\,\text{min} = 0.1\,\text{Hrs}$

$A_u = 0.1 \Rightarrow A = 12.0$

From Erlang B Chart, GOS = 5%, $A = 12.0$, #Channels = 17

b) Still have 12 Erlangs of traffic, but divided among 3 sectors of BTS.
Thus, each sector has

$A_{sector} = \frac{A}{3} = \frac{12}{3} = 4$ Erlangs of Traffic

From Erlang B chart, GoS = 5%, $A = 4.0$, # Channels = 8

Total # Channels for BTS is $8 \times 3 = 24$

$\boxed{\begin{array}{l} \text{a) \# Channels} = 17 \\ \text{b) \# Channels} = 24 \end{array}}$