Al-Mustaqbal university

Engineering technical college

Department of Building
&Construction Engineering

Mathematics

First class

Lecture No.10

Assist. Lecture

Alaa Hussein AbdUlameer

Integral

- Define محددة
- Indefine غير محددة

$$\int f(x) \cdot dx = g(x) + C$$

$$\int_a^b f(x) \cdot dx = [g(x)]_a^b = g(b) - g(a) = \text{Value}$$

Rules

1- $\int x^n \, dx = \frac{x^{n+1}}{n+1} + C \quad , n \neq -1$

2- $\int \underbrace{(ax+b)^n}_{\text{linear}} \, dx = \frac{(ax+b)^{n+1}}{(n+1) \cdot a}$ → مشتقه معامل X

3- $\int k \, dx = k \int dx = kx + C$

k → constant

4- $\int \underbrace{k^{(ax+b)}}_{e^x \text{ دالة}} \, dx = \frac{k^{ax+b}}{a \cdot \ln(k)} + C$

شرط الاس يكون linear

5- $\int \frac{\text{مشتقه المقام}}{\text{المقام}} \, dx = \ln |\text{المقام}| + C$

$$\int \frac{f'(x)}{f(x)} \, dx = \ln |f(x)| + C$$

6- $\int \frac{\overset{\text{ثابت}}{N}}{\underset{\text{معادلة خطية}}{\text{linear}}} dx = \frac{N}{\text{معامل } x} * \ln|\text{linear}| + c$

Ex: $\int \frac{3}{x-4} dx$

$= \frac{3}{1} * \ln|x-4| + c$

Ex: Evaluate the following integrals

1- $\int 5x^3 + \frac{3}{x^2} - \frac{4}{x} + 2 \, dx$

$= \int 5x^3 - 3x^{-2} - 4/x^{+1} + 2 \, dx$

$= \frac{5}{4} x^4 - 3 \frac{x^{-1}}{-1} - 4 \ln x + 2x + c$

2- $\int e^{2x+1} + 3^x \, dx$

* لازم نتأكد من القوة
بشرط تكون linear

$= \frac{e^{2x+1}}{2 \underbrace{\ln e}_{=1}} + \frac{3^x}{(1)\ln 3} + c$

$= \frac{1}{2} e^{2x+1} + \frac{1}{\ln 3} 3^x + c$

Ex: $\int \frac{2X}{3 + X^2} \, dx$

$= \ln|3 + X^2| + C$

Ex: $\int \frac{X}{5 - X^2}$

$= \frac{1}{-2} \int \frac{(-2)X}{5 - X^2} \, dx$

$= \frac{-1}{2} \ln|5 - X^2| + C$

Trigonometric functions:

1- $\int \sin X \, dx = \frac{-\cos X}{(1)} + C \Rightarrow -\cos X + C$

مشتقة الزاوية

2- $\int \cos X \, dx = \frac{\sin X}{(1)} + C$

3- $\int \tan x \, dx = -\int \frac{-\sin(X)}{\cos(X)} \, dx = -1 \ln|\cos(x)| + c$

or $\ln|\sec(x)| + c$

4- $\int \cot x \, dx = \int \frac{\cos x}{\sin x} \, dx = \ln \sin(x) + C$

5- $\int \sec(x) \, dx * \frac{\sec(x) + \tan(x)}{\sec(x) + \tan(x)} = \ln|\sec(x) + \tan(x)| + C$

$$6-\int \csc(x)\,dx = \ln|\csc(x)-\cot(x)|+c$$

$$7-\int \sec^2(x)\,dx = \tan(x)+c$$

$$8-\int \csc^2(x)\,dx = -\cot(x)+c$$

$$9-\int \tan^2(x)\,dx = \tan(x)-x+c$$

$$10-\int \sin^2(x)\,dx = \int \frac{1}{2}(1-\cos(2x)dx$$

قانون ضعف الزاوية

$$= \int \frac{1}{2}(x-\frac{\sin(2x)}{2})+c$$

$$11-\int \cos^2(x)\,dx = \int \frac{1}{2}(1+\cos(2x)dx$$

$$= \frac{1}{2}(x+\frac{\sin(2x)}{2})+c$$

$$12-\int \sec(x)\cdot\tan(x)\,dx = \sec(x)+c$$

$$13-\int \csc(x)\cdot\cot(x)\,dx = -\csc(x)+c$$

Ex: Find the following integral

1- $\displaystyle\int \frac{1}{\cos^2(x)}\,dx = \int \sec^2(x)\,dx$

$= \tan(x) + c$

2- $\displaystyle\int 2\sin(3x)\,dx = 2\int \sin(3x)\,dx$

$= 2\,\dfrac{-\cos(3x)}{3} + c$

3- $\displaystyle\int (5 + 5\cot^2(x))\,dx = 5\int (1+\cot^2(x))\,dx$

$= 5\displaystyle\int \csc^2(x)\,dx$

$= -5\cot(x) + c$

4- $\displaystyle\int \sec(2x)\cdot(\sec(2x) - 3\tan(2x) + 4\cos(2x))\,dx$

$= \displaystyle\int \sec^2(2(x)) - 3\sec(2x)\tan(2x) + 4\sec(2x)\cos(2x)\,dx$

$\dfrac{1}{\cos(2x)}$

$= \dfrac{\tan(2x)}{2} - \dfrac{3\sec(2x)}{2} + 4x + c$

* يجب ان تكون جميع الزوايا خطيه للدوال المثلثيه

Inverse Trigonometric functions

1- $$\int \frac{1}{1+x^2}\,dx = \tan^{-1}(x) + c$$

* $$\int \frac{b}{a^2 + k^2x^2}\,dx = \frac{b}{\underbrace{a\,k}} \cdot \tan^{-1}\left(\frac{kx}{a}\right) + c$$

جذر الاعداد

2- $$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}(x) + c$$

* $$\int \frac{b}{\sqrt{a^2 - k^2x^2}}\,dx = \frac{b}{k}\,\sin^{-1}\left(\frac{kx}{a}\right) + c$$

3- $$\int \sinh(x)\,dx = \cosh(x) + c$$

- $$\int \cosh(x)\,dx = \sinh(x) + c$$

- $$\int \tanh(x)\,dx = \int \frac{\sinh(x)}{\cosh(x)}\,dx$$

$$= \ln|\cosh(x)| + c$$

$$6-\int \frac{-1}{\sqrt{1-x^2}}\,dx = \cos^{-1}(x)+c$$

$$7-\int \frac{-1}{\sqrt{1+x^2}}\,dx = \cot^{-1}(x)+c$$

$$8-\int \frac{1}{x\sqrt{x^2-1}}\,dx = \sec^{-1}(x)+c$$

$$9-\int \frac{-1}{x\sqrt{x^2-1}}\,dx = \csc^{-1}(x)+c$$

Ex:

$$1-\int \frac{x-5}{1+x^2}\,dx$$

يتم توزيع البسط على المقام :.

$$=\frac{1}{2}\int \frac{(2)x}{1+x^2}\,dx-\int \frac{5}{1+x^2}\,dx$$

$$=\frac{1}{2}\ln|1+x^2|-5\int \frac{1}{1+x^2}\,dx$$

$$=\frac{1}{2}\ln|1+x^2|-5\tan^{-1}(x)+c$$

2- $\int \frac{dx}{9+4x^2}$

* $\int \frac{b}{a^2+k^2x^2}\,dx = \frac{b}{a.k}\tan^{-1}\left(\frac{kx}{a}\right)+c$

$= \frac{1}{3.(2)}\tan^{-1}\left(\frac{2x}{3}\right)+c$

$= \frac{1}{6}\tan^{-1}\left(\frac{2x}{3}\right)+C$

3- $\int \frac{4}{\sqrt{1-x^2}}\,dx$

$= 4\int \frac{1}{\sqrt{1-x^2}}\,dx$

$= 4\sin^{-1}(x)+C$

3- $\int \frac{1-x^2}{1-x^4}\,dx$

$= \int \frac{\cancel{(1-x^2)}}{\cancel{(1-x^2)}(1+x^2)}\,dx$

$= \int \frac{1}{1+x^2}\,dx$

$= \tan^{-1}(x)+c$