2-4- **Hyperbolic functions** : Hyperbolic functions are used to describe the motions of waves in elastic solids ; the shapes of electric power lines ; temperature distributions in metal fins that cool pipes …etc.

The hyperbolic sine (Sinh) and hyperbolic cosine (Cosh) are defined by the following equations :

1. $Sinh u = \frac{1}{2}(e^{u} - e^{-u})$ and $Cosh u = \frac{1}{2}(e^{u} + e^{-u})$
2. $tanh u = \frac{Sinh u}{Cosh u} = \frac{e^{u} - e^{-u}}{e^{u} + e^{-u}}$ and $Coth u = \frac{Cosh u}{Sinh u} = \frac{e^{u} + e^{-u}}{e^{u} - e^{-u}}$
3. $Sech u = \frac{1}{Cosh u} = \frac{2}{e^{u} + e^{-u}}$ and $Csch u = \frac{1}{Sinh u} = \frac{2}{e^{u} - e^{-u}}$
4. $Cosh^2 u - Sinh^2 u = 1$
5. $tanh^2 u + Sech^2 u = 1$ and $Coth^2 u - Csch^2 u = 1$
6. $Cosh u + Sinh u = e^{u}$ and $Cosh u - Sinh u = e^{-u}$
7. $Cosh(-u) = Cosh u$ and $Sinh(-u) = -Sinh u$
8. $Cosh 0 = 1$ and $Sinh 0 = 0$
9. $Sinh(x + y) = Sinh x . Cosh y + Cosh x . Sinh y$
10. $Cosh(x + y) = Cosh x . Cosh y + Sinh x . Sinh y$
11. $Sinh 2x = 2 . Sinh x . Cosh x$
12. $Cosh 2x = Cosh^2 x + Sinh^2 x$
13. $Cosh^2 x = \frac{Cosh 2x + 1}{2}$ and $Sinh^2 x = \frac{Cosh 2x - 1}{2}$

EX-16- Let $tanh\ u = -7/25$, determine the values of the remaining five hyperbolic functions .

Sol.-

$$Cothu = \frac{1}{tanh\,u} = -\frac{25}{7}$$

$$tanh^2 u + Sech^2 u = 1 \quad \Rightarrow \frac{49}{625} + Sech^2 u = 1 \Rightarrow Sechu = \frac{24}{25}$$

$$Coshu = \frac{1}{Sechu} = \frac{25}{24}$$

$$tanh\,u = \frac{Sinhu}{Coshu} \qquad \Rightarrow -\frac{7}{25} = \frac{Sinhu}{\frac{25}{24}} \Rightarrow Sinhu = -\frac{7}{24}$$

$$Cschu = \frac{1}{Sinhu} = -\frac{24}{7}$$

EX-17- Rewrite the following expressions in terms of exponentials . Write the final result as simply as you can :

$a)\ 2Cosh(\ln x)$ $\qquad b)\ tanh(\ln x)$

$c)\ Cosh5x + Sinh5x$ $\qquad d)\ (Sinhx + Coshx)^4$

Sol.-

$$a)\quad 2Cosh(\ln x) = 2.\frac{e^{\ln x} + e^{-\ln x}}{2} = x + \frac{1}{x}$$

$$b)\quad tanh(\ln x) = \frac{e^{\ln x} - e^{-\ln x}}{e^{\ln x} + e^{-\ln x}} = \frac{x - \frac{1}{x}}{x + \frac{1}{x}} = \frac{x^2 - 1}{x^2 + 1}$$

$$c)\quad Cosh5x + Sinh5x = \frac{e^{5x} + e^{-5x}}{2} + \frac{e^{5x} - e^{-5x}}{2} = e^{5x}$$

$$d)\quad (Sinhx + Coshx)^4 = \left(\frac{e^x - e^{-x}}{2} + \frac{e^x + e^{-x}}{2}\right)^4 = e^{4x}$$

EX-18- **Solve the equation for** x **:** $Cosh\, x = Sinh\, x + 1/2$.

Sol. - $Coshx - Sinhx = \frac{1}{2} \Rightarrow e^{-x} = \frac{1}{2} \Rightarrow -x = \ln 1 - \ln 2 \Rightarrow x = \ln 2$

EX-19 – **Verify the following identity :**

a) ***Sinh(u+v)=Sinh u. Cosh v + Cosh u.Sinh v***

b) then verify ***Sinh(u-v)=Sinh u. Cosh v - Cosh u.Sinh v***

Sol.-

$$a)\ R.H.S. = Sinhu.Coshv + Coshu.Sinhv$$

$$= \frac{e^{u} - e^{-u}}{2} \cdot \frac{e^{v} + e^{-v}}{2} + \frac{e^{u} + e^{-u}}{2} \cdot \frac{e^{v} - e^{-v}}{2}$$

$$= \frac{e^{u+v} - e^{-(u+v)}}{2} = Sinh(u+v) = L.H.S.$$

$$b)\ L.H.S. = Sinh(u + (-v)) = Sinhu.Cosh(-v) + Coshu.Sinh(-v)$$

$$= Sinhu.Coshv - Coshu.Sinhv = R.H.S.$$