LOW TEMPERATURE REFRIGERATION (CRYOGENICS)

Cryogenic refers to the technique of reaching very low temperatures or, in other words, reaching temperatures near absolute zero

Joule – Thompson for real gas

When gas is expansion freely without doing a work, its temperature was changes although the enthalpy of the gas remains constant. It is represent the ratio between temperature changes to pressure change when the enthalpy of the gas remains constant at passing through an expansion valve.

$$\mu = \left.\frac{\partial T}{\partial p}\right)_h$$

Cascade Refrigeration system

Example 14.1. *A cascade refrigeration system is designed to supply 10 tonnes of refrigeration at an evaporator temperature of – 60°C and a condenser temperature of 25°C. The load at – 60°C is absorbed by a unit using R-22 as the refrigerant and is rejected to a cascade condenser at – 20°C. The cascade condenser is cooled by a unit using R-12 as the refrigerant and operating between – 30°C evaporating temperature and 25°C condenser temperature. The refrigerant leaving the R-12 condenser is subcooled to 20°C but there is no subcooling of R-22 refrigerant. The gas leaving both the evaporators is dry and saturated and the compressions are isentropic. Neglecting losses, determine : 1. Compression ratio for each unit ; 2. Quantity of refrigerant circulated per minute for each unit; 3. C.O.P. for each unit; 4. C.O.P. of the whole system; and 5. Theoretical power required to run the system.*

Solution. Given : Q = 10 TR ; t_E (R–22) = – 60°C ; t_c (R–12) = 25°C ; t_c (R–22) = –20°C; t_E (R–12) = –30°C ; t_7 = 20°C

The schematic diagram of a two-stage cascade refrigerating system using R-22 and R-12 is shown in Fig. 14.3 (*a*). The corresponding *p-h* diagram of the system is shown in Fig. 14.3 (*b*). The cycle 1-2-3-4 is for R-22 unit whereas the cycle 5-6-7-8 is for R-12 unit. Both these cycles are superimposed in the *p-h* diagram.

From the *p-h* diagram for R-22, we find that the pressure at point 1 corresponding to – 60°C,

$$p_1 = 0.3745 \text{ bar}$$

Enthalpy of saturated vapour refrigerant at point 1,

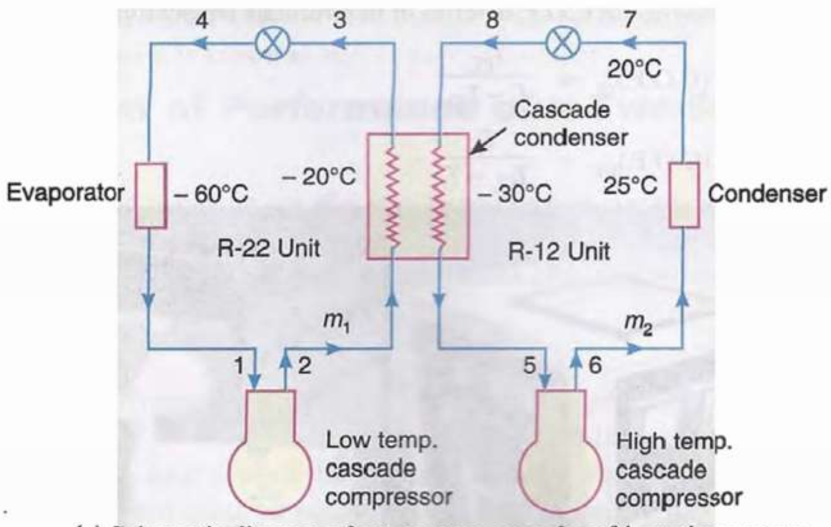
$$h_1 = 223.7 \text{ kJ/kg}$$

Entropy of saturated vapour refrigerant at point 1,

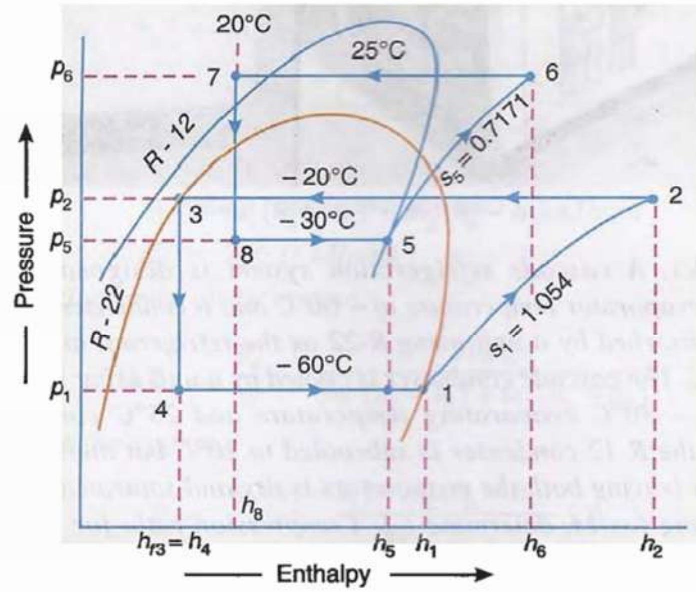
$$s_1 = 1.054 \text{ kJ/kg K}$$

Enthalpy of superheated vapour refrigerant at point 2,

$$h_2 = 275 \text{ kJ/kg}$$



(*a*) Schematic diagram of a two-stage cascade refrigerating system.



(*b*) *p-h* diagram.

Fig. 14.3

Pressure at point 2 corresponding to – 20°C,

$$p_2 = 2.458 \text{ bar}$$

and enthalpy of saturated liquid refrigerant at point 3 (corresponding to – 20°C)

$$h_{f3} = h_4 = 22.2 \text{ kJ/kg}$$

Now from the *p-h* diagram for R-12, we find that the pressure at point 5 corresponding to – 30°C,

$$p_5 = 1.044 \text{ bar}$$

Enthalpy of saturated vapour refrigerant at point 5,

$$h_5 = 174.2 \text{ kJ/kg}$$

Entropy of saturated vapour refrigerant at point 5,

$$s_5 = 0.7171 \text{ kJ/kg K}$$

Enthalpy of superheated vapour refrigerant at point 6,

$$h_6 = 207 \text{ kJ/kg}$$

Pressure at point 6 corresponding to 25°C,

$$p_6 = 6.518 \text{ bar}$$

and enthalpy of liquid refrigerant at point 7 corresponding to 20°C,

$$h_{f7} = h_8 = 54.9 \text{ kJ/kg}$$

1. Compression ratio for each unit

We know that the compression ratio for R-22 unit

$$= \frac{p_2}{p_1} = \frac{2.458}{0.3745} = 6.56 \textbf{ Ans.}$$

Similarly, compression ratio for R-12 unit

$$= \frac{p_6}{p_5} = \frac{6.518}{1.044} = 6.24 \textbf{ Ans.}$$

2. Quantity of refrigerant circulated per minute for each unit

We know that mass of refrigerant circulated for R-22 unit,

$$m_1 = \frac{210\,Q}{h_1 - h_4} = \frac{210 \times 10}{223.7 - 22.2} = 10.4 \text{ kg/min} \textbf{ Ans.}$$

and mass of refrigerant circulated for R-12 unit,

$$m_2 = \frac{m_1(h_2 - h_4)}{h_5 - h_8} = \frac{10.4(275 - 22.2)}{174.2 - 54.9} = 22.04 \text{ kg/min} \textbf{ Ans.}$$

3. C.O.P. for each unit

We know that refrigerating effect for R-22 unit,

$$R_{E1} = m_1(h_1 - h_4) = 210\,Q = 210 \times 10 = 2100 \text{ kJ/min}$$

Refrigerating effect for R-12 unit,

$$R_{E2} = m_2(h_5 - h_8) = 22.04\,(174.2 - 54.9) = 2629.4 \text{ kJ/min}$$

Work done in the compressor for R-22 unit,

$$W_1 = m_1(h_2 - h_1) = 10.4\,(275 - 223.7) = 533.5 \text{ kJ/min}$$

and work done in the compressor for R-12 unit,

$$W_2 = m_2(h_6 - h_5) = 22.04\,(207 - 174.2) = 723 \text{ kJ/min}$$

$\therefore$ C.O.P. for R-22 unit, $= \frac{R_{E1}}{W_1} = \frac{2100}{533.5} = 3.93$ **Ans.**

and C.O.P. for R-12 unit $= \frac{R_{E2}}{W_2} = \frac{2629.4}{723} = 3.64$ **Ans.**

4. C.O.P. of the whole system

We know that C.O.P. of the whole system

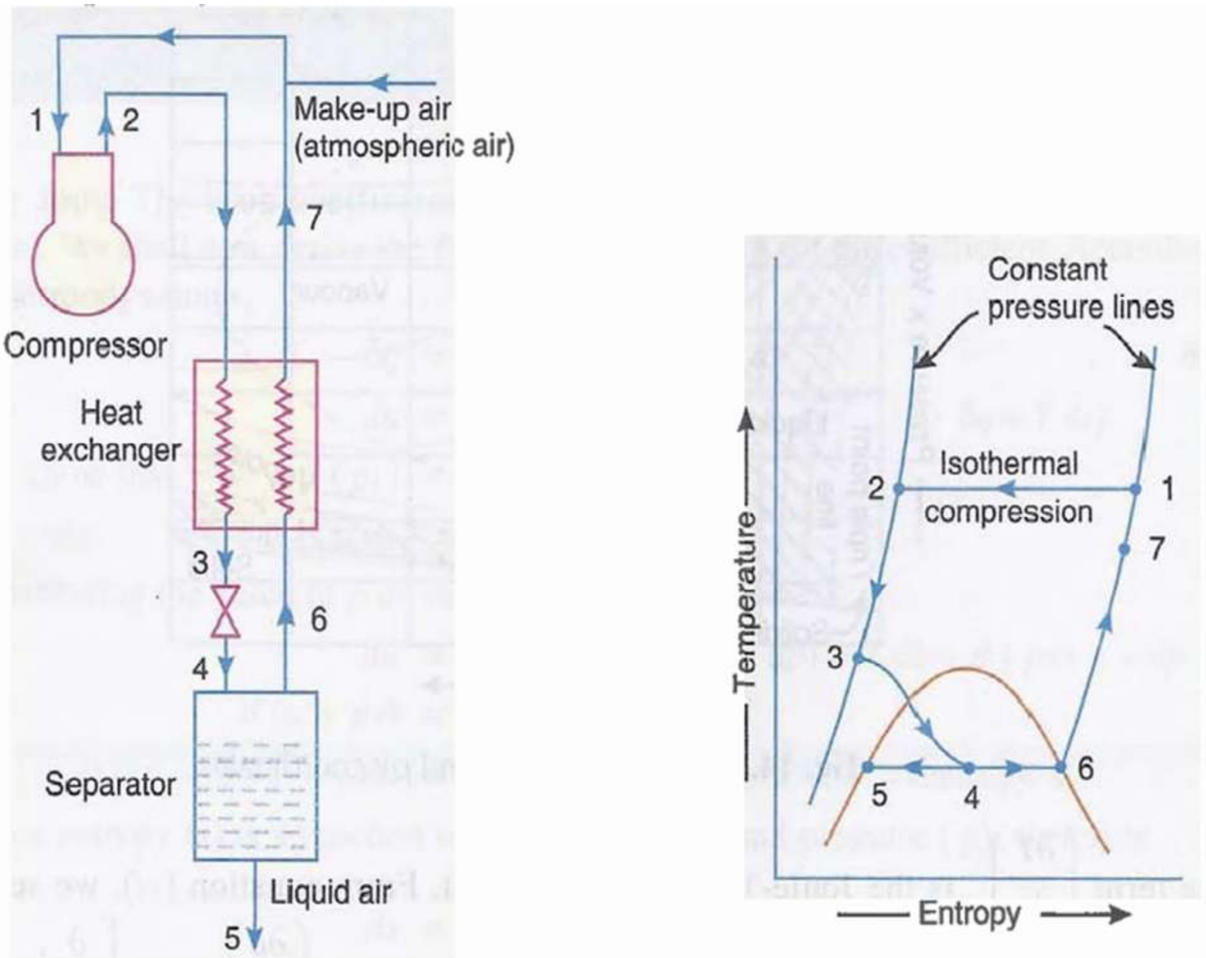
$$= \frac{R_E}{W} = \frac{210\,Q}{W_1 + W_2} = \frac{210 \times 10}{533.5 + 723} = 1.67 \textbf{ Ans.}$$

5. Theoretical power required to run the system

We know that the theoretical power required to run the system,

$$P = \frac{W_1 + W_2}{60} = \frac{533.5 + 723}{60} = 20.94 \text{ kW} \textbf{ Ans.}$$

Linde System for Liquefaction of Air or Hampson System



(*a*) Linde system. (*b*) *T-s* diagram.

Now for the heat balance of the heat exchanger,

$$m_2 (h_2 - h_3) = m_6 (h_1 - h_6)$$

$$= (m_2 - m_5)(h_1 - h_6) \quad \ldots (\because \; m_6 = m_2 - m_5)$$

and for the heat balance of the separator,

$$m_2 h_4 = m_5 h_5 + m_6 h_6 = m_5 h_5 + (m_2 - m_5) h_6$$

Since the process 3-4 is a throttling process, therefore

$$h_3 = h_4$$

Example 14.3. *Dry air at 20°C and 1 bar is to be liquefied by the simple* **Linde system the** *air is isothermally compressed at 20°C to 170 bar. The make-up air is supplied to* **the system at** *20°C and 1 bar. Find the yield of liquid air in kg per kg of air compressed and the temperature of air before throttling.*

Solution. Given : $t_1 = t_2 = 20$°C ; $p_1 = p_6 = 1$ bar ; $p_2 = 170$ bar

The *T-s* diagram of air for simple Linde system is shown in Fig. 14.14. From the diagram, we find that enthalpy at point 1,

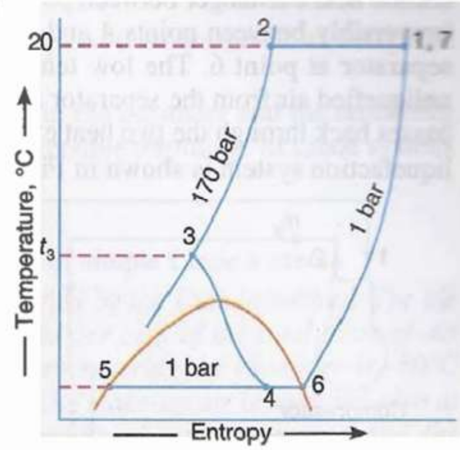


Fig. 14.14

$$h_1 = 506 \text{ kJ/kg}$$

Enthalpy at point 2, $h_2 = 473$ kJ/kg

Enthalpy at point 5, $h_5 = 92$ kJ/kg

and enthalpy at point 6, $h_6 = 292$ kJ/kg

Yield of liquid air per kg of air compressed

Let m_5 = Yield of liquid air in kg per kg of air compressed, and

m_2 = Mass of air compressed = 1 kg ... (Given)

We know that for the heat balance of the heat exchanger,

$$m_2(h_2 - h_3) = (m_2 - m_5)(h_1 - h_6)$$
$$1(473 - h_3) = (1 - m_5)(506 - 292)$$
$$473 - h_3 = 214 - 214\, m_5$$
$$\therefore \quad h_3 = 259 + 214\, m_5 \quad \ldots (i)$$

Now for the heat balance of the separator,

$$m_2 h_4 = m_5 h_5 + (m_2 - m_5) h_6$$
$$1 \times h_3 = m_5 \times 92 + (1 - m_5)\, 292 \quad \ldots (\because h_4 = h_3)$$
$$\therefore \quad h_3 = 92\, m_5 + 292 - 292\, m_5$$
$$= 292 - 200\, m_5 \quad \ldots (ii)$$

Equating equations (*i*) and (*ii*), we have

$$259 + 214\, m_5 = 292 - 200\, m_5$$
$$214\, m_5 + 200\, m_5 = 292 - 259 = 33$$
$$\therefore \quad m_5 = 0.08 \text{ kg}$$ **Ans.**

Temperature of air before throttling

Let t_3 = Temperature of air before throttling.

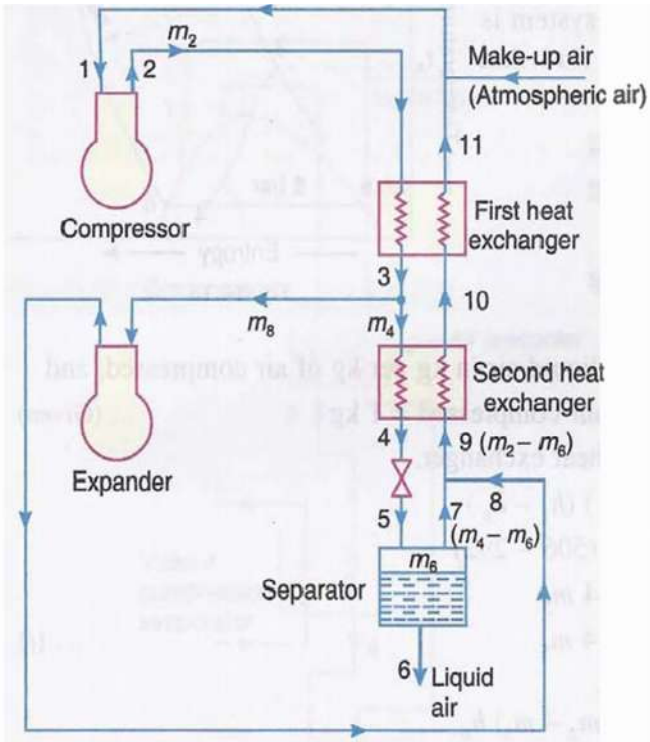
Substituting the value of m_5 in equation (*i*), we have

$$h_3 = 259 + 214 \times 0.08 = 276.1 \text{ kJ/kg}$$

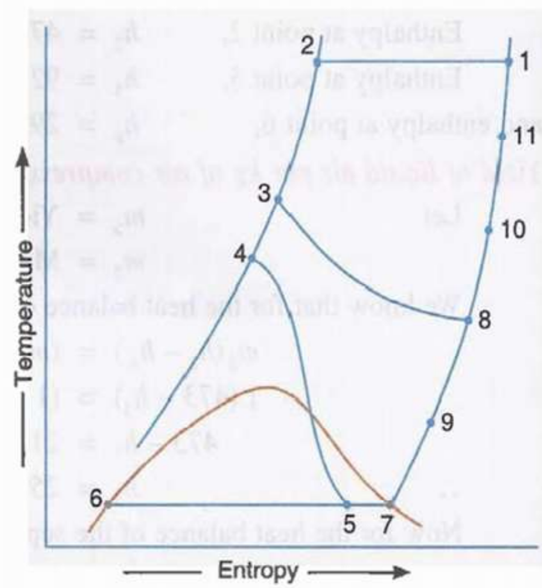
From the *T-s* diagram for air, we find that the temperature of air corresponding to 276.1 kJ/kg is

$$t_3 = 175 \text{ K} = -98\text{°C}$$ **Ans.**

Claude System for Liquefaction of Air



(*a*) Claude system for air liquefaction.



(*b*) *T-s* diagram for Claude air liquefaction system.

Now for the heat balance of the first heat exchanger,

$$m_2 (h_2 - h_3) = (m_2 - m_6)(h_1 - h_{10})$$

For the heat balance of the second heat exchanger

$$m_4 (h_3 - h_4) = (m_2 - m_6)(h_{10} - h_9)$$

and for the heat balance of the separator,

$$m_4 h_5 = m_6 h_6 + (m_4 - m_6) h_7$$

The enthalpy at point 9 is given by

$$(m_4 - m_6 + m_8) h_9 = m_8 h_8 + (m_4 - m_6) h_7$$

$$h_9 = \frac{m_8 h_8 + (m_4 - m_6) h_7}{m_4 - m_6 + m_8} = \frac{m_8 h_8 + (m_4 - m_6) h_7}{m_2 - m_6} \quad \ldots (iv)$$

$$\ldots (\because m_4 = m_2 - m_8)$$

Example 14.4. *Dry air at 20°C and 1 bar is to be liquefied by the Claude method. The air is compressed isothermally at 20°C to 170 bar. Assume that 80 per cent of the total mass of air compressed passes through the expander. The temperature of air entering the expander is –80°C while the temperature of air leaving the expander is –140°C. The make-up air is supplied also at 20°C and 1 bar. Determine the yield of liquid air in kg per kg of air compressed and the temperature of air before throttling.*

Solution. Given : $t_1 = t_2 = 20°C$; $p_1 = 1$ bar ; $p_2 = 170$ bar ; $m_8 = 0.8\, m_2$; $t_3 = -80°C$; $t_8 = -140°C$

The *T-s* diagram of air for Claude system is shown in Fig. 14.16. From the diagram, we find that enthalpy at point 1,

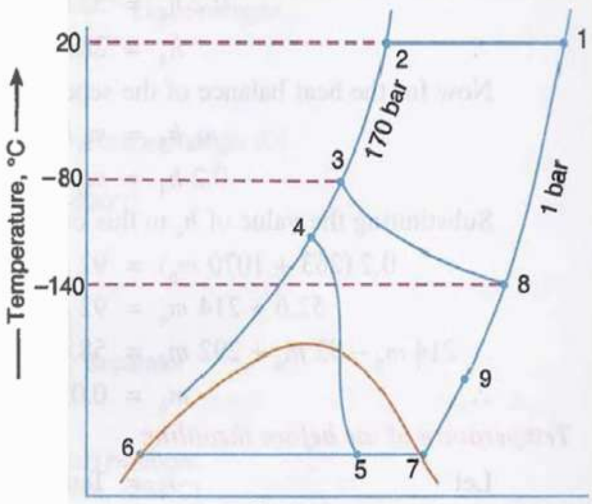


Fig. 14.16

$h_1 = 506$ kJ/kg

Enthalpy at point 2, $h_2 = 473$ kJ/kg

Enthalpy at point 3, $h_3 = 309$ kJ/kg

Enthalpy at point 6, $h_6 = 92$ kJ/kg

Enthalpy at point 7, $h_7 = 292$ kJ/kg

and enthalpy at point 8, $h_8 = 343$ kJ/kg

Yield of liquid air in kg per kg of air compressed

Let m_6 = Yield of liquid air in kg per kg of air compressed,

m_2 = Mass of air compressed = 1 kg, ... (Given)

m_8 = Mass of air bypassed to expander, ... (Given)

$= 0.8\, m_2 = 0.8$ kg,

m_4 = Mass of air passing through the second heat exchanger.

$= m_2 - m_8 = 1 - 0.8 = 0.2$ kg

The air from the expander at point 8 is mixed with unliquefied air from the separator at point 7. The condition of air after mixing is represented by point 9 on the *T-s* diagram. The enthaply at point 9 is given by

$$h_9 = \frac{m_8 h_8 + (m_4 - m_6) h_7}{m_2 - m_6} = \frac{0.8 \times 343 + (0.2 - m_6)292}{1 - m_6}$$

$$= \frac{274.4 + 58.4 - 292\, m_6}{1 - m_6} = \frac{332.8 - 292\, m_6}{1 - m_6} \quad \text{...(i)}$$

We know that for the heat balance of the first heat exchanger,

$$m_2(h_2 - h_3) = (m_2 - m_6)(h_1 - h_{10})$$

$$1(473 - 309) = (1 - m_6)(506 - h_{10})$$

$$164 = 506 - 506\, m_6 - h_{10}(1 - m_6)$$

$$h_{10}(1 - m_6) = 342 - 506\, m_6 \qquad \dots (ii)$$

For the heat balance of the second heat exchanger,

$$m_4(h_3 - h_4) = (m_2 - m_6)(h_{10} - h_9)$$

$$0.2(309 - h_4) = (1 - m_6)\left(h_{10} - \frac{332.8 - 292\, m_6}{1 - m_6}\right)$$

... [Substituting the value of h_9 from equation (*i*)]

$$61.8 - 0.2\, h_4 = (1 - m_6)\, h_{10} - 332.8 + 292\, m_6$$

$$h_{10}(1 - m_6) = 394.6 - 292\, m_6 - 0.2\, h_4 \qquad \dots (iii)$$

Equating equations (*ii*) and (*iii*), we have

$$342 - 596\, m_6 = 394.6 - 292\, m_6 - 0.2\, h_4$$

$$0.2\, h_4 = 52.6 + 214\, m_6$$

$$\therefore \quad h_4 = 263 + 1070\, m_6 \qquad \dots (iv)$$

Now for the heat balance of the separator,

$$m_4 h_5 = m_6 h_6 + (m_4 - m_6)\, h_7$$

$$0.2\, h_4 = m_6 \times 92 + (0.2 - m_6)\, 292 \qquad \dots (\because h_5 = h_4)$$

Substituting the value of h_4 in this expression, we have

$$0.2(263 + 1070\, m_6) = 92\, m_6 + (0.2 - m_6)\, 292$$

$$52.6 + 214\, m_6 = 92\, m_6 + 58.4 - 292\, m_6$$

$$214\, m_6 - 92\, m_6 + 292\, m_6 = 58.4 - 52.6 = 5.8$$

$$\therefore \quad m_6 = 0.014 \text{ kg}$$ **Ans.**

Temperature of air before throttling

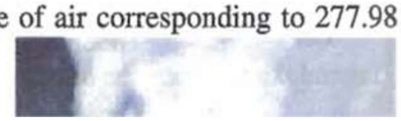
Let t_4 = Temperature of air before throttling.

Substituting the value of m_6 in equation (*iv*),

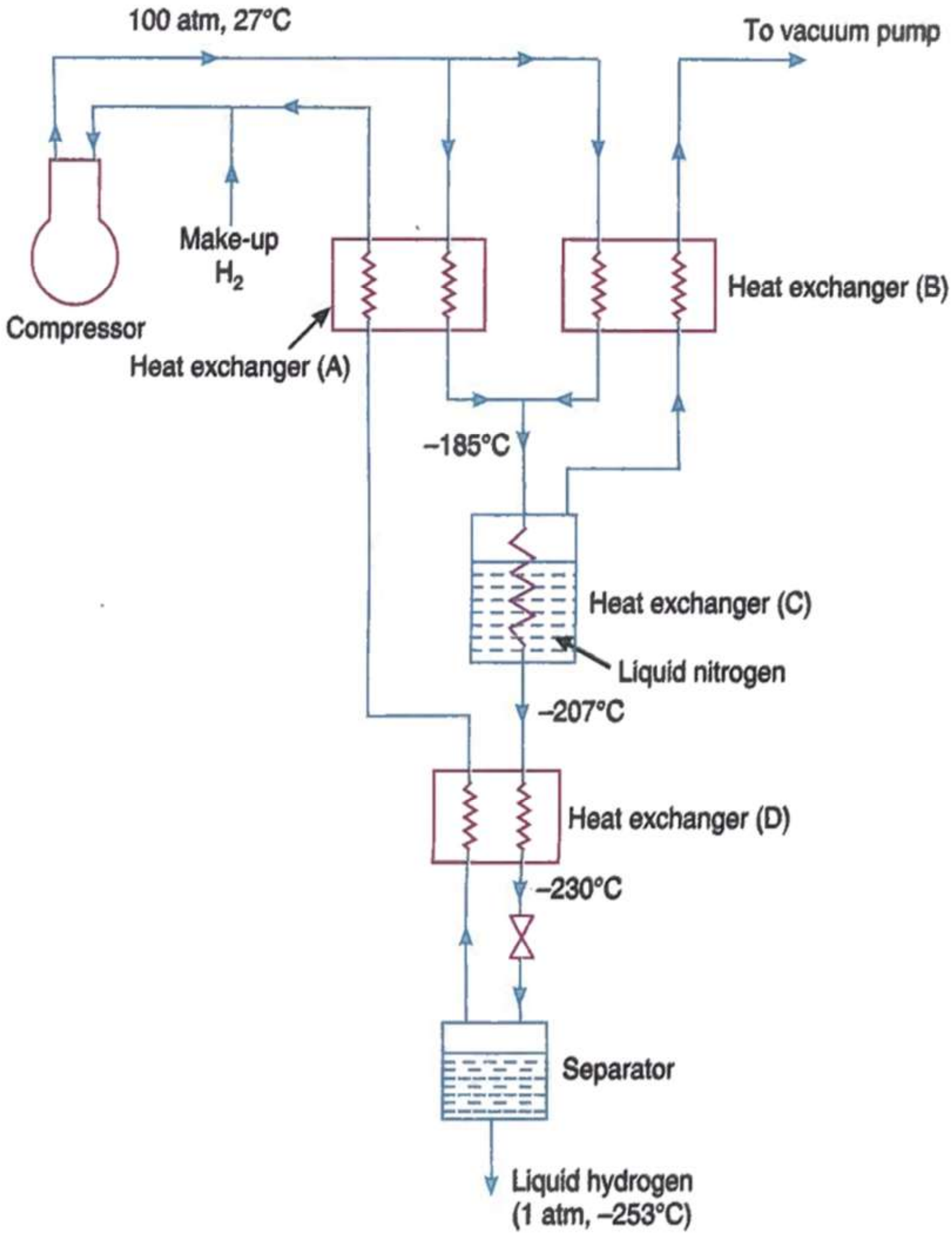
$$h_4 = 263 + 1070 \times 0.014 = 277.98 \text{ kJ/kg}$$

From the *T-s* diagram for air, we find that the temperature of air corresponding to 277.98 kJ/kg is

$$t_4 = -100°\text{C}$$ **Ans.**



Liquefaction of Hydrogen



Liquefaction of Helium